

Gilmore and Gomory Model on Two Dimensional Multiple Stock Size Cutting Stock Problem

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Abstract. Two-dimensional Multiple Stock Size Cutting Stock Problem (MSS2DCSP) is an optimization problem in determining the optimal cutting pattern of materials with length and width variables. The optimal cutting pattern is needed to minimize the number of materials used. This research implemented the pattern generation algorithm to generate Gilmore and Gomory models on every variation of material size. The first stage generated the pattern based on the length, followed by a pattern generated based on the width on the next stage. Then, Gilmore and Gomory models were generated according to the size of the material, where the Gilmore and Gomory model constraints ensured that strips cut in the first stage cutting were used in the second stage cutting and met all product requirements. The Branch and Bound method were used to determine the minimum size of the material sheets needed.

1. Introduction

Manufacturing is one of the most prominent industry in Indonesia. The industry uses a great variety of materials including iron, leather, paper, glass, and wood, where some raw materials have a standardized size. Companies must cut the raw material to smaller sizes according to their needs. This process often produced residual cuts called trim loss. Trim loss is formed due to improper cutting patterns, resulting in an excessive use of raw materials. This is a problem to manufacturing companies, as they must minimize raw materials usage to maximize productivity.

Research on cutting patterns has been widely carried out including the Pattern Generation (PG) algorithm to solve CSP [1]. The implementation of the PG algorithm in a 2-dimensional multiple stock CSP was completed using the Column Generation Technique (CGT) [2]. On the other side, the implementation of PG algorithm in 2-dimensional CSP with predetermined cutting locations and the completion using Branch and Bound Algorithm have also been done [3]. CGT was also used to solve the Multiple Width CSP (MWCSP) [4].

Two dimensional CSP with different raw material sizes is then solved by a modified heuristic algorithm [5]. Further research on 2-dimensional CSP by guillotine cutting was also completed using Column Generation Heuristic [6]. While the research on 2-dimensional CSP with different raw material sizes and there was little damage to each raw material used was also completed using the heuristic algorithm [7].

Based on this background, this study expanded from the application of CGT on 1-dimensional CSP into 2-dimensional CSP with various sizes of raw materials. This study uses data of CSP based on a plate, where the plate provided consisted of multiple stock sizes and was completed using the Branch



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and Bound method. The importance of this model can be used to find the proper cutting pattern with minimal trim loss. This model can also be applied to another cutting stock problem.

2. Literature Review

Literary studies that support and relate to this research, including Cutting Stock Problem (CSP), Pattern Generation (PG), Gilmore and Gomory Model, and Branch and Bound Method.

2.1. Cutting Stock Problem (CSP)

Cutting Stock Problem (CSP) was first introduced in England in 1930 by Leonid Kantorovich. Later in 1960, Gilmore and Gomory completed CSP using the Linear Program (LP) approach. After that, the development of CSP completion methods was carried out. CSP is a problem of cutting raw materials with a standard size that will be cut according to the demanded size of the product with a minimum trim loss [9].

2.2. Pattern Generation (PG)

The stocks with standard length (l'_k) and standard width (w'_k) where k indicates the number of raw materials used and $k = 1, 2, 3, \dots$, are cut to the certain length (l_i) and width (w_i) with n sizes of demand and i is the number of demand sizes and $i = 1, 2, 3, \dots, n$. The objective is to find the cutting pattern with minimum trim loss and fulfill the demand.

The steps in PG algorithm according to [1] are as follows:

1. Sort the length l_i ($i = 1, 2, \dots, n$) in decreasing order.
2. Fill the first column ($j = 1$) of the matrix

$$a_{ijk} = \left\lfloor \frac{l'_k - \sum_{z=1}^{i-1} a_{zjk} l_z}{l_i} \right\rfloor \quad (1)$$

3. Count the cut loss from the cutting pattern

$$c_{jk} = l'_k - \sum_{i=1}^n a_{ijk} l_i \quad (2)$$

4. Set index level (row index) i to $n - 1$.

5. Check the current vertices in i level, e.g. vertex (i, j) . If the vertex has 0 value ($a_{ijk} = 0$), go to

Step 7. Otherwise, generate new column $j = j + 1$ with the following elements:

- a. Elements to fill the preceding vertex (i, j) is $a_{zjk} = a_{z(j-1)k}$ ($z = 1, 2, \dots, i - 1$). (3)
- b. Elements to fill vertex (i, j) is $a_{ijk} = a_{i(j-1)k} - 1$. (4)
- c. The remaining vertex from j , fill with Eq (1)

6. Count the cut loss from the j -th cutting pattern by using Eq(2). Go back to step 4.

7. Reduction of index i , with $i_f = i_{f-1} - 1$, where $f = 1, 2, 3, \dots, n - 1$. If $i_f > 0$ redo Step 5.

If $i_f = 0$, then STOP.

2.3. Gilmore and Gomory Model

The two-dimensional CSP model was first proposed by Gilmore and Gomory in 1961. Gilmore and Gomory completed a two-dimensional CSP in two stages. The first stage produced strips that had been cut based on length or width, which then would be cut back into the demand item in the second stage. Gilmore and Gomory formulated this problem into the Model (5).

$$\text{Minimize } Z = \sum_{j \in j_0} \lambda_j^0 \quad (5)$$

Subject to:

$$M' \bar{\lambda} = \mathbf{0} \quad (5.a)$$

$$M'' \bar{\lambda} \geq \mathbf{b} \quad (5.b)$$

$$\bar{\lambda} \geq \mathbf{0} \text{ and integer} \quad (5.c)$$

Where as

j_0 is the feasible cutting pattern which used in the first stage, λ_j^0 is the j th cutting pattern which is used in the first stage, λ_j^s is the j th cutting pattern which corresponds to the set of s th cutting pattern in the second stage, where $s \in \{1, 2, \dots, m\}$, \mathbf{b} is the column vector where the entries are the number of

demand of i th item, $\bar{\lambda} = [\lambda_1^0 \dots \lambda_j^0 \lambda_1^1 \dots \lambda_j^1 \dots \lambda_1^{m'} \dots \lambda_j^{m'}]^T$, M' and M'' is the first row and last row from M matrix.

2.4. Branch and Bound Method

The Branch and Bound method is an approach to solve problems based on the division of all feasible solutions to a problem into smaller sub-problems [9]. Then, this sub-problem is solved systematically until the optimal solution is obtained.

In determining the vertex that must be investigated first and the vertex that does not cut there is the most common searching strategy, namely:

- a. Depth-First (Last-In, First-Out)
This method completes the last resolved sub-problem at branching.
- b. First-In, First-Out
This method completes the first subproblem solved in branching.
- c. Best-Bound-First
This method does branching on vertices with the largest Z value.

Steps in Branch and Bound method:

- a. Initialization
- b. Selection of a vertex
- c. Update lower bound
- d. Infeasible deduction
- e. Limit cutting
- f. Update upper bound and cutting optimality
- g. Branching

3. Methodology

This study implements PG algorithm into the Gilmore and Gomory model of the Two Dimensional Multiple Stock Size Cutting Stock Problem (2DMSSCSP). The steps in this study are as follows:

- a. Describe data. The data includes the length and width of the raw material, the length and width of the product, and the number of products demanded.
- b. Sort the size of the product from the largest to the smallest size, then implemented into the PG algorithm.
- c. Apply the PG algorithm to the 2-dimensional CSP to obtain the first cutting pattern based on the length, and a cutting pattern based on width for the second stage.
- d. Form a searching tree cutting pattern based on the PG algorithm and represent it in table form.
- e. Form a table of Gilmore and Gomory model.
- f. Form the Gilmore and Gomory model based on the first cutting pattern and the second cutting pattern.
- g. Complete the Gilmore and Gomory model using the Branch and Bound method and the Linear Interactive General Optimizer (LINGO) application 13.0.
- h. Analyze the final results.

4. Results

This research used the data with the plated material of the stock. The sizes of stocks can be seen in Table 1. The sizes and the number of demand for products can be seen in Table 2.

Table 1. The sizes of stocks

The i -th stocks	Length (inches)	Width (inches)
1	24	14
2	24	13
3	18	10
4	13	10

Table 2. The demand sizes

i -th product	Length (inches)	Width (inches)	Number of Demand
1	2	1	5
2	4	2	5
3	5	3	2
4	7	4	3
5	8	5	2

Based on Table 1, there are 4 stocks with different length and width. There are 5 products with different length and width and so number of demand can be seen in Table 2. By implementing the PG algorithm, there are 63, 29, and 14 cutting patterns which correspond to the length of 24 inches, 18 inches, and 13 inches respectively. By cutting based on the width of 14 inches, 13 inches, and 10 inches, there are 70, 57, and 29 cutting patterns. One of the searching tree for 24 inches stocks can be seen in Figure 1.

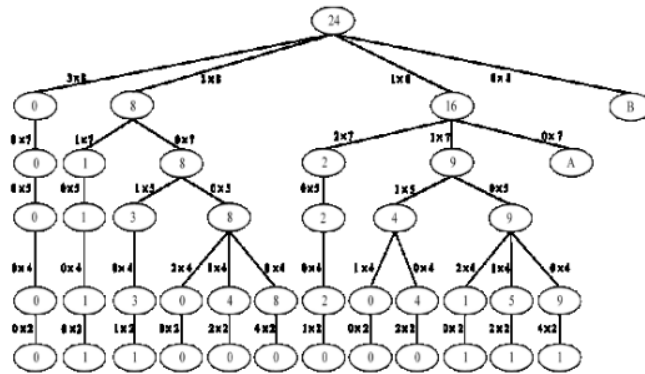


Figure 1. The searching tree of cutting patterns based on length

Based on the cutting patterns, the next step is forming Gilmore and Gomory model that can be seen in Eq. (6-10).

$$\begin{aligned} & \text{Minimize } \sum_{j=1}^{63} \lambda_j^0 & (6) \\ & \text{Subject to (5.a), (5.b) and (5.c)} \\ & \text{where } \bar{\lambda} = [\lambda_1^0 \dots \lambda_{63}^0 \lambda_1^1 \lambda_1^2 \dots \lambda_7^2 \lambda_1^3 \dots \lambda_{16}^3 \lambda_1^4 \dots \lambda_{23}^4 \lambda_1^5 \dots \lambda_{23}^5]^T \end{aligned}$$

$$\begin{aligned} & \text{Minimize } \sum_{j=1}^{63} \lambda_j^0 & (7) \\ & \text{Subject to (6.a), (6.b) and (6.c)} \\ & \text{where } \bar{\lambda} = [\lambda_1^0 \dots \lambda_{63}^0 \lambda_1^1 \lambda_1^2 \dots \lambda_6^2 \lambda_1^3 \dots \lambda_{14}^3 \lambda_1^4 \dots \lambda_{18}^4 \lambda_1^5 \dots \lambda_{18}^5]^T \end{aligned}$$

$$\begin{aligned} & \text{Minimize } \sum_{j=1}^{29} \lambda_j^0 & (8) \\ & \text{Subject to (6.a), (6.b) and (6.c)} \\ & \text{where } \bar{\lambda} = [\lambda_1^0 \dots \lambda_{29}^0 \lambda_1^1 \lambda_1^2 \dots \lambda_5^2 \lambda_1^3 \dots \lambda_8^3 \lambda_1^4 \dots \lambda_9^4 \lambda_1^5 \dots \lambda_6^5]^T \end{aligned}$$

$$\begin{aligned} & \text{Minimize } \sum_{j=1}^{14} \lambda_j^0 & (9) \\ & \text{Subject to (6.a), (6.b) and (6.c)} \\ & \text{Where } \bar{\lambda} = [\lambda_1^0 \dots \lambda_{14}^0 \lambda_1^1 \lambda_1^2 \dots \lambda_5^2 \lambda_1^3 \dots \lambda_8^3 \lambda_1^4 \dots \lambda_9^4 \lambda_1^5 \dots \lambda_6^5]^T \end{aligned}$$

Model (6) is Gilmore and Gomory Model for the first stock, Model (7) is for the second stock and Model (8) and (9) are for the third and fourth stock respectively.

The optimal solution by using Branch and Bound method and LINGO 13.0 shows that the value of $Z = 1$ with $\lambda_{10}^0 = 1, \lambda_5^2 = 2, \lambda_{21}^4 = 1,$ and $\lambda_{23}^5 = 1$ for the first stock, $Z = 1$ with $\lambda_8^0 = 1, \lambda_5^2 = 1, \lambda_{10}^3 = 1, \lambda_{18}^4 = 1,$ and $\lambda_{18}^5 = 1$ for the second stock, $Z = 2,$ with $\lambda_1^0 = 1, \lambda_9^0 = 1, \lambda_1^1 = 1, \lambda_5^2 = 1, \lambda_7^4 = 1, \lambda_8^4 = 1,$ and $\lambda_6^5 = 2$ for the third stock, and $Z = 3$ with $\lambda_2^0 = 1, \lambda_4^0 = 2, \lambda_3^2 = 1, \lambda_4^3 = 2, \lambda_8^4 = 1, \lambda_9^4 = 1, \lambda_6^5 = 1$ for the fourth stock.

Based on the counting there are some of optimal cutting patterns that shown in Figure 2 to 8.

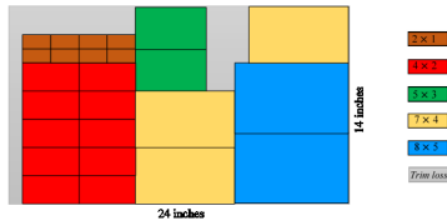


Figure 2. The combination of cutting patterns on first stock

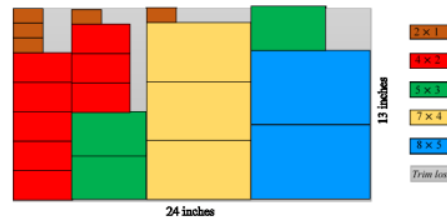


Figure 3. The combination of cutting patterns on second stock

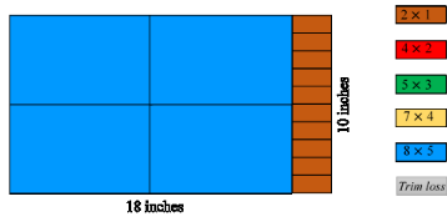


Figure 4. The first combination of cutting pattern on the third stock

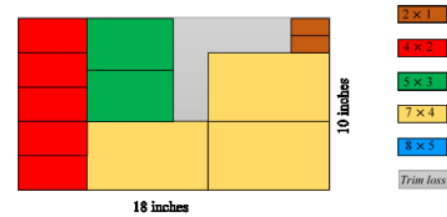


Figure 5. The second combination of cutting pattern on the third stock

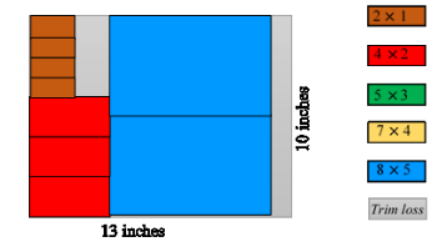


Figure 6. The first combination of cutting pattern on the fourth stock

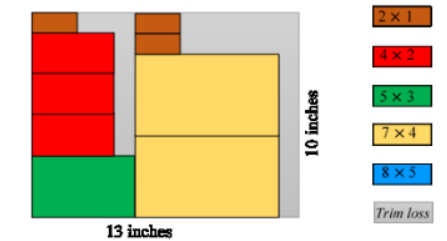


Figure 7. The second combination of cutting pattern on the fourth stock

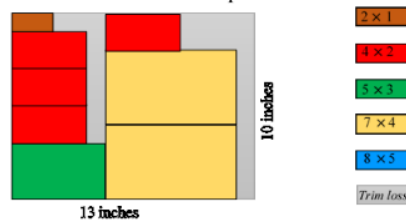


Figure 8. The third combination of cutting pattern on the fourth stock

Figure 2 - Figure 8, show the combination of cutting pattern for each stock. There are one combination of first and second stock, two combinations of third stock and three combinations of

fourth stock. The number of products that will be produced based on the combination of cutting patterns can be seen in Table 3.

Table 3.The number of products that will be produced

No.	The size of demand	Number of products (plates)
1.	2 inches× 1 inch	33
2.	4 inches× 2 inches	33
3.	5 inches× 3 inches	9
4.	7 inches× 4 inches	13
5.	8 inches× 5 inches	11

Based on the results that can be seen in Table 3, there are five sizes of demand with each number of products. The greatest number of products are for 2 inches 1 inch and 4 inches 2 inches, 33 plates respectively.

5. Conclusion

Based on the result, the solution of Gilmore and Gomory model in MSS2DCSP by using Branch and Bound shows that the minimum number of stocks which used to fulfill the demands are 1 plate of 24 inches 14 inches, 1 plate of 24 inches 13 inches, 2 plates of 18 inches 10 inches, and 3 plates of 13 inches 10 inches.

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