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The modified branch and bound algorithm and dotted board model for triangular shape items

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Abstract. Cutting Stock Problem (CSP) is a problem of cutting stocks into items with certain cutting rules. This study used the data where the rectangular stocks were cut into triangular shape items with various order sizes. This study used the Modified Branch and Bound Algorithm (MBBA) to determine the optimal cutting pattern then formulated it into a Dotted Board model. Based on the results, it showed that the MBBA produced three optimal cutting patterns, which used 6 times, 8 times, and 4 times respectively to fulfill the consumer demand. Then the cutting patterns were formulated into the Dotted Board model. Minimum trim loss of the three models are $1,774 \text{ cm}^2$, $1,720 \text{ cm}^2$ dan 980 cm^2 .

1. Introduction

Efficiency in production is very important where currently the available natural resources are less and the market demand is higher. Industry players must strive to overcome these problems in order to compete in a globalization world. One effort to improve production efficiency is by optimizing the use of raw materials, namely minimizing the remaining cutting (trim loss). Many types of raw materials used in industry including wood, paper, glass, steel, marble, and others. The raw material used will be cut according to the specified cutting pattern.

The problem of cutting in optimization in order to optimize raw materials is known as Cutting Stock Problem (CSP). CSP is divided into three parts based on the number of dimensions, namely one-dimensional CSP, two-dimensional CSP, and three-dimensional CSP. This study examines two-dimensional CSP, wherein cutting only based on the width and length of the raw material.

CSP has been extensively developed by researchers with various problem-solving algorithms starting from generating patterns, formulating models, and solving methods. Research that discussed the pattern generation such as [1] who formulated CSP in a linear program model. The formula introduced by [1] was used to solve one-dimensional CSP. Furthermore, according to [2] two-dimensional CSP can be classified into regular and irregular items. Then [3] completed a two-dimensional CSP by generating patterns using the Modified Branch and Bound algorithm. This method can be used in cutting large quantities but limited only to rectangular items. Whereas [4] made cuts on rectangular-shaped items into triangles of various sizes.

A study of two-dimensional CSP for irregular-shaped items by proposing a model called the Dotted Board [5]. The packing problem and the cutting of raw materials which are also irregular in shape was conducted by [6]. The optimal solution is obtained from several models including the Dotted Board model and the 3-Phase Matheuristic model. Whereas [7] conducted a study on how to design and create



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applications for cutting pattern generation in two-dimensional CSP. Furthermore, [8] implemented the Pattern Generation Algorithm on Gilmore and Gomory models in two-dimensional CSP and [9] formulated the Gilmore and Gomory models in two-dimensional multiple stock size cutting stock problem whereas [10] implemented branch and cut method on n-sheet model in solving two dimensional cutting stock problem.

Based on this background, this research looks for cutting patterns for triangular items. Patterns were searched using the Modified Branch and Bound method. Furthermore, the pattern obtained was modeled into the Dotted Board model which in previous studies was used for irregular shaped items. This study uses research data from [4] where the raw materials used were rectangular and then cut into triangles of various sizes. Previously, [4] used the Gilmore and Gomory models, therefore another model will be formed using the Dotted Board model.

2. Literature Review

2.1. Modified Branch and Bound Algorithm (CSP)

Modified Branch and Bound Algorithm (MBBA) is one of the method used to form cutting patterns in CSP. In this research, the rectangular raw material is cut into triangular items.

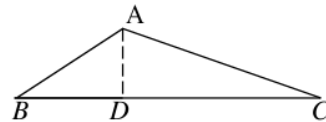


Figure 1. Triangular

Based on Fig. 1, $BC \geq AB$ and $BC \geq AC$ with $BC = l_i$ (length of the i^{th} item), $AD = w_i$ (width of the i^{th} item) and $BD = e_i$ where $(i = 1, 2, \dots, m)$. The length and width were denoted by L and W respectively. The steps of MBBA [4]:

Step 1: Arrange lengths, l_i ($i = 1, 2, \dots, m$) in decreasing order, ie $l_1 > l_2 > \dots > l_m$ where m is number of item. Arrange required widths,

w_i ($i = 1, 2, \dots, m$) and lengths e_i ($i = 1, 2, \dots, m$) according to the corresponding lengths.

Step 2: For $(i = 1, 2, \dots, m)$ and $j = 1$ do Steps 3 to 5.

Step 3: Set $a_{11} = \left\lfloor \frac{L}{l_1} \right\rfloor$ (1)

$$a_{ij} = \left\lfloor \frac{L - \sum_{z=1}^{i-1} a_{zj} l_z}{l_i} \right\rfloor \quad (2)$$

where L is the length of the main sheet. a_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern along the length of the main sheet and $\lfloor y \rfloor$ is the greatest integer less than or equal to y .

Step 4: If $a_{ij} > 4$, then set $b_{ij} = \left\lfloor \frac{W}{w_i} \right\rfloor$ (2)

Else set $b_{ij} = 0$, where W is the width of the main sheet. b_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern in the main sheet.

Step 5: Set $P_{ij} = (2a_{ij} - 1)b_{ij}$ (4)

where P_{ij} is the number of pieces of the i^{th} item in the j^{th} pattern in the main sheet.

$$P_{ij} = \begin{cases} 0 & ; P_{ij} \leq 0 \\ 4x & ; P_{ij} > 0 \end{cases}$$

Step 6: Cutting loss

(i) Cut loss along the length of the main sheet:

$$C_u = (L - \sum_{i=1}^m a_{ij} l_i) W \quad (5)$$

For $= 1, 2, \dots, m$, If $(L - \sum_{i=1}^m a_{ij} l_i) \geq w_i$ and $W \geq l_i$ then set:

$$A_{ij} = \left\lfloor \frac{L - \sum_{i=1}^m a_{ij} l_i}{w_i} \right\rfloor \quad (6)$$

$$B_{ij} = \begin{cases} \left\lfloor \frac{w}{w_i} \right\rfloor & ; A_{ij} > 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (7)$$

$$P_{ij} = (2A_{ij}B_{ij}) - 1 \quad (8)$$

Else set :

$$A_{ij} = 0 \quad (9)$$

$$B_{ij} = 0 \quad (10)$$

$$P_{ij} = P_{ij} \quad (11)$$

$$\text{If } A_{ij} > 0, \text{ then set : } C_u = \left((L - \sum_{i=1}^m a_{ij} l_i) - A_{ij} w_i \right) B_{ij} l_i \quad (12)$$

$$C_v = \left(\sum_{i=1}^m a_{ij} l_i \right) \cdot (W - B_{ij} l_i) \quad (13)$$

$$C_t = \left(\frac{1}{2} l_i w_i \right) \quad (14)$$

where, A_{ij} and B_{ij} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the C_u rectangle respectively and C_u and C_v are the total cut loss area along the length and width of the C_u rectangle respectively.

(ii) Cut loss along the width of the main sheet: $C_v =$

$$(a_{ij} l_i) \cdot k_{ij} \quad (15)$$

$$k_{ij} = W - b_{ij} w_i \quad (16)$$

If $b_{ij} w_i = 0$ then set $k_{ij} = 0$

For $z \neq i$, If $(a_{ij} l_i) \geq l_z$ and $k_{ij} \geq w_z$ then set:

$$A_{zj} = \left\lfloor \frac{a_{ij} l_i}{l_z} \right\rfloor \quad (17)$$

$$B_{ij} = \begin{cases} \left\lfloor \frac{k_{ij}}{w_z} \right\rfloor & ; A_{zj} > 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (18)$$

$$P_{zj} = P_{zj} + (2A_{zj} - 1) b_{zj} \quad (19)$$

$$\text{Else set : } A_{ij} = 0 \quad (20)$$

$$B_{ij} = 0 \quad (21)$$

$$P_{ij} = P_{ij} \quad (22)$$

$$\text{If } A_{zj} > 0 \text{ then set: } C_u = (a_{ij} l_i - A_{zj} l_z) \cdot B_{zj} w_z \quad (23)$$

$$C_v = a_{ij} l_i \cdot (k_{ij} - B_{zj} w_z) \quad (24)$$

$$\text{Else } C_t = \left(\frac{1}{2} l_i w_i \right) \quad (25)$$

where, A_{zj} and B_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the C_v rectangle respectively and C_u and C_v are the total cut loss are along the length and width of the C_v rectangle respectively.

(iii) Cut loss within the triangular shape items in the main sheet:

$$\text{If } a_{ij} = 0 \text{ then set } C_t = \frac{1}{2} e_i w_i + \frac{1}{2} (l_i - e_i) w_i \text{ else set } C_t = 0 \quad (26)$$

$$\text{For } z \neq i, \text{ If } e_i \geq l_z \text{ and } \frac{(e_i - (l_z - e_z)) w_i}{e_i} \geq w_z \quad (27)$$

$$\text{Then } l_i - e_i \geq l_z \text{ and } \frac{(l_i - e_i - e_z) w_i}{(l_i - e_i)} \geq w_z \quad (28)$$

$$\text{set: } E_{zj} = \left\lfloor \frac{e_i}{l_z} \right\rfloor \quad (29)$$

$$F_{zj} = \begin{cases} \left\lfloor \frac{w_i}{w_z} \right\rfloor & ; \text{ if } E_{zj} > 0 \\ 0 & ; \text{ otherwise} \end{cases} \quad (30)$$

$$P_{zj} = P_{zj} + (2E_{zj} - 1) F_{zj} b_{ij} \quad (31)$$

$$\text{Else set : } E_{zj} = 0 \quad (32)$$

$$F_{zj} = 0 \quad (33)$$

$$P_{zj} = P_{zj} \quad (34)$$

$$\text{If } E_{zj} > 0 \text{ then set : } C_t = \left(\frac{1}{2} e_i w_i - \left(\frac{1}{2} E_{zj} l_z w_z \right) \right) \text{ else set } C_t = \frac{1}{2} e_i w_i \quad (35)$$

$$\text{For } z \neq i, \text{ If } (l_i - e_i) \geq l_z \text{ and } \frac{[(l_i - e_i) - e_z] w_i}{l_i - e_i} \geq w_z \quad (36)$$

$$\text{then : } E_{zj} = \left\lfloor \frac{l_i - e_i}{l_z} \right\rfloor \quad (37)$$

$$F_{zj} = \begin{cases} \left\lfloor \frac{w_i}{w_z} \right\rfloor & ; \text{ if } E_{zj} > 0 \\ 0 & ; \text{ otherwise} \end{cases} \quad (38)$$

$$P_{zj} = P_{zj} + (2E_{zj} - 1) F_{zj} b_{ij} \quad (39)$$

$$\text{Else set : } E_{zj} = 0 \quad (40)$$

$$F_{zj} = 0 \quad (41)$$

$$P_{zj} = P_{zj} \quad (42)$$

If $E_{zj} > 0$ then :

$$C_t = \left\lfloor \frac{1}{2} (l_i - e_i) \right\rfloor - \left(\frac{1}{2} E_{zj} l_z w_z \right); \text{ else set, } C_t = \frac{1}{2} (l_i - e_i) w_i \quad (43)$$

where, E_{zj} and F_{zj} are the number of pieces of the i^{th} item in the j^{th} pattern along the length and width of the C_t rectangle respectively and C_t is the total cut loss area of the triangular shapes.

$$\text{Step 7: Set } r = m - 1 \quad (44)$$

while $r > 0$ do Step 8

Step 8: While $a_{rj} > 0$ set $j = j + 1$, and do Step 9

Step 9: If $a_{rj} \geq b_{rj}$, then generate a new pattern according to the following conditions:

$$\text{For } z = 1, 2, \dots, r - 1, \text{ Set } a_{zj} = a_{zj-1} \text{ and } b_{zj} = b_{zj-1} \quad (45)$$

For $z = r$

$$\text{Set } a_{zj} = a_{zj-1} - 1$$

$$\text{If } a_{zj} > 0, \text{ then set } b_{zj} = \left\lfloor \frac{w}{w_z} \right\rfloor, \text{ else set } b_{zj} = 0 \quad (46)$$

For $z = r + 1, \dots, m$

Calculate a_{zj} and b_{zj} using Eq (1) and (2)

$$\text{4) For } i = 1, 2, \dots, m, \text{ Set } p_{ij} = a_{ij} b_{ij} \quad (47)$$

Go to Step 5

Else generate a new pattern according to the following conditions ($a_{rj} \leq b_{rj}$):

$$\text{For } z = 1, 2, \dots, r - 1, \text{ Set } a_{zj} = a_{zj-1} \text{ and } b_{zj} = b_{zj-1} \quad (48)$$

$$\text{For } z = r, \text{ Set } a_{zj} = a_{zj-1} \text{ and } b_{zj} = b_{zj-1} - 1 \quad (49)$$

For $z = r + 1, \dots, m$, Calculate a_{zj} and b_{zj} using Eq (2) and (3)

$$\text{For } i = 1, 2, \dots, m, \text{ Set } p_{ij} = a_{ij} b_{ij} \quad (50)$$

Do Step 5

$$\text{Step 10: Set } r = r - 1 \quad (51)$$

Step 11: STOP.

2.2. Dotted Board Model

The Dotted Board model aims to minimize the use of board length and width by presenting a number of dots on board for each row and column. This method is done as a reference for laying items on a board based on the reference point. According to [6] reference points can only be positioned on the dot of set D that represents the board. The shape of the board used is a rectangular flat shape that has length L and width W . The advantages of the Dotted Board model are convex and non-convex items can be placed on the board in the same way. Items of type t and placed at point (δ_t^d) have three basic restrictions, namely:

- Each piece needs to be positioned entirely inside the board.
- All the pieces need to be positioned.
- The pieces may not overlap.

Inner-fit polygons (IFP) are met if each item that is positioned fully inside the board, while the nofit polygon (NFP) is met if each item does not overlap. The dotted board model for two dimensional Cutting Stock Problem according to [5] can be seen in Model (52).

Objective Function:

Minimize

$$z = ((c \cdot g_x + x_t^M) \cdot \delta_t^d) \quad \forall d \in IFP_t, \forall t \in T \quad (52)$$

Subject to:

$$(c \cdot g_x + x_t^M) \cdot \delta_t^d \leq z \quad \forall d \in IFP_t, \forall t \in T; \quad (52.a)$$

$$\sum_{d \in IFP_t} \delta_t^d = q_t \quad \forall t \in T; \quad (52.b)$$

$$\delta_u^e + \delta_t^d \leq 1 \quad \forall e \in NFP_{t,u}^d, \forall t, u \in T, \forall d \in IFP_t \quad (52.c)$$

$$\delta_t^d \in \{0,1\} \quad \forall d \in IFP_t, \forall t \in T; \quad (52.d)$$

$$z \geq 0 \quad (52.e)$$

where

$$\delta_t^d = \begin{cases} 1 & \text{if the reference point of a piece of type } t \text{ is positioned on dot } d \\ 0 & \text{otherwise.} \end{cases} \quad (53)$$

where z is objective function, c is board column, r is board row, g_x is grid resolution in x axis, x_t^M is horizontal distance from the reference point to the end of item, δ_t^d is binary decision variable, q_t is numbers of item type t that should be positioned, d, e are type of point on board, t, u are type of item, IFP is inner fit polygon and NFP is nofit polygon.

3. Method

The steps in this study are as follows:

- a. Describe the data which includes the length and width of the product along with the demand.
- b. Implement a Modified Branch and Bound Algorithm to form cutting pattern in a two-dimensional CSP by
 - Define the used variable such as length of item (l_i), one side of the item (e_i), length of raw material (L) and width of raw material (W).
 - Form the cutting pattern using Modified Branch and Bound Algorithm in accordance with predetermined variables by sorting the length (l_i), forming the cutting pattern and counting the cut loss along the length, width and in the form of a triangle on the main sheet.
- c. Formulate the Dotted Board Model by defining the used variable, formulating the objective function that minimizes the used of the board and a set of constraints that ensured all of the demand is fulfilled.
- d. Analyze the final solution.

4. Result and Discussion

This study used the data of rectangular marble slabs with a size of 50 cm × 15 cm, then the material was cut into triangle items according to the order size in Table 1. The Modified Branch and Bound Algorithm (MBBA) is implemented in Two Dimensional CSP for triangular shape items. The objective function of MBBA is to determine cutting patterns, maximize the number of items ordered according to consumer demand and minimize the use of stock.

Table 1. The size of items and the number of demands.

Item	1	2	3	4
BC (cm)	40	25	8	4
AD (cm)	13	12	5	2
BD (cm)	30	24	2	2
Demand (d_1) (pieces)	6	30	125	500

Based on MBBA, there are 3 optimum cutting patterns, as shown in Table 2. This research assumed that the rotation of items are not allowed. The optimum patterns from MBBA were put in Dotted Board

as shown in Figure 2, 3 and 4 respectively. The Dotted Board model for each cutting pattern can be seen in Model (54), (55) and (56).

Table 2. Optimum cutting pattern.

Cutting Item	Optimum Patterns			Demand	Surplus
	14	17	20		
1	1	0	0	6	0
2	1	3	0	30	0
3	0	0	33	125	7
4	59	48	8	500	270
Cut loss (cm ²)	104	108	58	-	-
Number of usage	6	8	4	-	-

Minimize

$$z = 40\delta_1^{656} + 39\delta_2^{639} + 2\delta_4^{45} + 8\delta_4^{74} + 2\delta_4^{40} + 8\delta_4^{70} + 12\delta_4^{96} + 18\delta_4^{100} + 2\delta_4^{36} + 4\delta_4^{68} + 8\delta_4^{132} + 8\delta_4^{128} + 20\delta_4^{163} + 12\delta_4^{193} + 14\delta_4^{227} + 16\delta_4^{257} + 18\delta_4^{291} + 20\delta_4^{321} + 22\delta_4^{365} + 24\delta_4^{395} + 26\delta_4^{429} + 28\delta_4^{459} + 30\delta_4^{493} + 32\delta_4^{523} + 34\delta_4^{557} + 36\delta_4^{587} + 38\delta_4^{611} + 40\delta_4^{641} + 41\delta_4^{668} + 41\delta_4^{664} + 41\delta_4^{660} + 86\delta_4^{702} + 86\delta_4^{698} + 86\delta_4^{694} + 43\delta_4^{690} + 45\delta_4^{736} + 90\delta_4^{732} + 90\delta_4^{728} + 90\delta_4^{724} + 94\delta_4^{766} + 94\delta_4^{762} + 94\delta_4^{758} + 47\delta_4^{754} + 98\delta_4^{788} + 49\delta_4^{792}$$

subject to

$$\delta_1^{656} = 1 \tag{54}$$

$$\delta_2^{639} = 1$$

$$\delta_4^{45} + \delta_4^{74} + \delta_4^{40} + \delta_4^{70} + \delta_4^{96} + \delta_4^{100} + \delta_4^{36} + \delta_4^{68} + \delta_4^{132} + \delta_4^{128} + \delta_4^{163} + \delta_4^{193} + \delta_4^{227} + \delta_4^{257} + \delta_4^{291} + \delta_4^{321} + \delta_4^{365} + \delta_4^{395} + \delta_4^{429} + \delta_4^{459} + \delta_4^{493} + \delta_4^{523} + \delta_4^{557} + \delta_4^{587} + \delta_4^{611} + \delta_4^{641} + \delta_4^{668} + \delta_4^{664} + \delta_4^{660} + \delta_4^{702} + \delta_4^{698} + \delta_4^{694} + \delta_4^{690} + \delta_4^{736} + \delta_4^{732} + \delta_4^{728} + \delta_4^{724} + \delta_4^{766} + \delta_4^{762} + \delta_4^{758} + \delta_4^{754} + \delta_4^{788} + \delta_4^{792} = 43$$

$$\delta_u^e + \delta_t^d \leq 1$$

$$\delta_t^d \in \{0,1\}$$

$$z \geq 0$$

The Model (54) shows that the minimum trim loss is 1.774 cm².

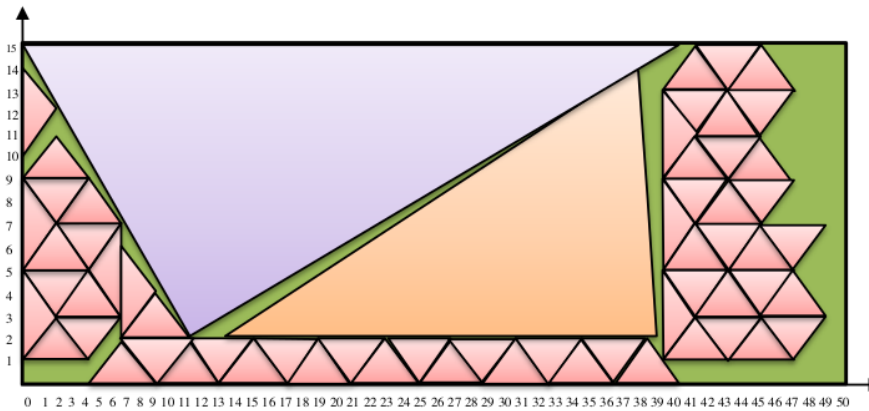


Figure 2. The dotted board of first cutting pattern.

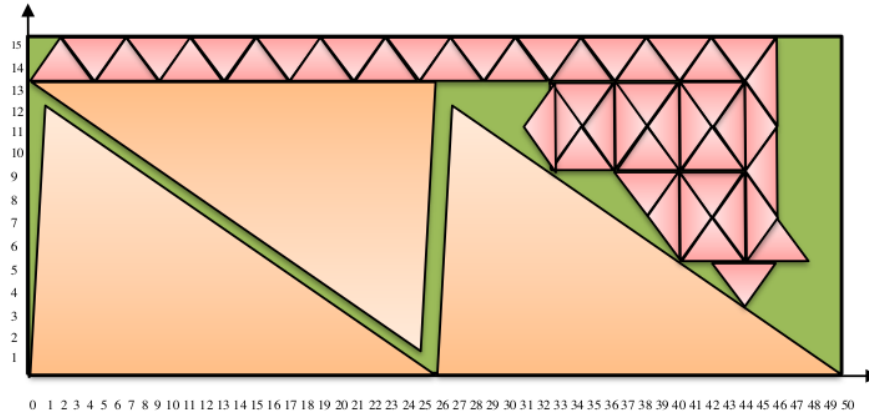


Figure 3. The dotted board of second cutting pattern.

Minimize

$$z = 25\delta_2^{401} + 25\delta_2^{454} + 50\delta_2^{801} + 4\delta_4^{78} + 6\delta_4^{112} + 8\delta_4^{142} + 10\delta_4^{176} + 12\delta_4^{206} + 14\delta_4^{240} + 16\delta_4^{270} + 18\delta_4^{304} + 20\delta_4^{334} + 22\delta_4^{368} + 24\delta_4^{398} + 26\delta_4^{432} + 28\delta_4^{462} + 30\delta_4^{496} + 96\delta_4^{526} + 34\delta_4^{560} + 108\delta_4^{590} + 38\delta_4^{624} + 120\delta_4^{654} + 42\delta_4^{688} + 132\delta_4^{718} + 92\delta_4^{752} + 34\delta_4^{556} + 38\delta_4^{620} + 42\delta_4^{684} + 92\delta_4^{748} + 36\delta_4^{586} + 120\delta_4^{650} + 132\delta_4^{714} + 42\delta_4^{680} + 46\delta_4^{744} + 44\delta_4^{710} + 48\delta_4^{774} + 46\delta_4^{742}$$

(55)

Subject to

$$\delta_2^{401} + \delta_2^{454} + \delta_2^{801} = 3$$

$$\delta_4^{78} + \delta_4^{112} + \delta_4^{142} + \delta_4^{176} + \delta_4^{206} + \delta_4^{240} + \delta_4^{270} + \delta_4^{304} + \delta_4^{334} + \delta_4^{368} + \delta_4^{398} + \delta_4^{432} + \delta_4^{462} + \delta_4^{496} + \delta_4^{526} + \delta_4^{560} + \delta_4^{590} + \delta_4^{624} + \delta_4^{654} + \delta_4^{688} + \delta_4^{718} + \delta_4^{752} + \delta_4^{556} + \delta_4^{620} + \delta_4^{684} + \delta_4^{748} + \delta_4^{586} + \delta_4^{650} + \delta_4^{714} + \delta_4^{680} + \delta_4^{744} + \delta_4^{710} + \delta_4^{774} + \delta_4^{742} = 34$$

$$\delta_u^e + \delta_t^d \leq 1$$

$$\delta_t^d \in \{0,1\}$$

$$z \geq 0, \text{ The minimum trim loss of Model (55) is } 1.720 \text{ cm}^2.$$

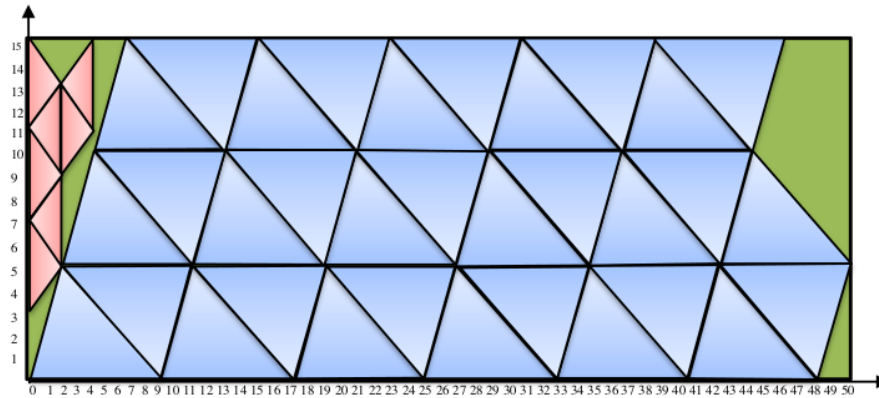


Figure 4. The dotted board of third cutting pattern.

$$\text{Minimize } z = 8 \delta_3^{129} + 20 \delta_3^{166} + 24 \delta_3^{203} + 14 \delta_3^{240} + 16 \delta_3^{257} + 36 \delta_3^{294} + 40 \delta_3^{331} + 22 \delta_3^{368} + 24 \delta_3^{385} + 52 \delta_3^{422} + 56 \delta_3^{459} + 30 \delta_3^{496} + 32 \delta_3^{513} + 68 \delta_3^{550} + 72 \delta_3^{587} + 38 \delta_3^{624} + 40 \delta_3^{641} + 84 \delta_3^{678} + 88 \delta_3^{715} + 46 \delta_3^{752} + 48 \delta_3^{769} + 100 \delta_3^{806} + 2 \delta_4^{38} + 4 \delta_4^{42} + 4 \delta_4^{46} + 4 \delta_4^{76} + 8 \delta_4^{80}$$

Subject to

$$\delta_3^{129} + \delta_3^{166} + \delta_3^{203} + \delta_3^{240} + \delta_3^{257} + \delta_3^{294} + \delta_3^{331} + \delta_3^{368} + \delta_3^{385} + \delta_3^{422} + \delta_3^{459} + \delta_3^{496} + \delta_3^{513} + \delta_3^{550} + \delta_3^{587} + \delta_3^{624} + \delta_3^{641} + \delta_3^{678} + \delta_3^{715} + \delta_3^{752} + \delta_3^{769} + \delta_3^{806} = 22 \quad (56)$$

$$\delta_4^{38} + \delta_4^{42} + \delta_4^{46} + \delta_4^{76} + \delta_4^{80} = 5$$

$$\delta_u^e + \delta_t^d \leq 1$$

$$\delta_t^d \in \{0,1\}$$

$$z \geq 0, \text{ the minimum trim loss of Model (56) is } 980 \text{ cm}^2.$$

5. Conclusion

Based on the results, it can be concluded that the Dotted Board Model is formed based on the cutting pattern obtained from the Modified Branch and Bound Algorithm (MBBA), which are three optimal patterns that meet each item demand. For item 1 requests are fulfilled using the first 6 cutting patterns. Item 2 request is fulfilled using 6 times the first pattern and 8 times the second pattern. Item 3 requests are fulfilled using 4 times the third pattern. And item 4 requests are fulfilled using the first 6 times, 8 times the second pattern, and 4 times the third pattern. Minimum trim loss of the three models are $1,774 \text{ cm}^2$, $1,720 \text{ cm}^2$ dan 980 cm^2 .

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