

# Implementation of Arc Flow Model Incapacitated Multi-Period Cutting Stock Problem with The Pattern Set Up Cost to Minimize The Trim Loss

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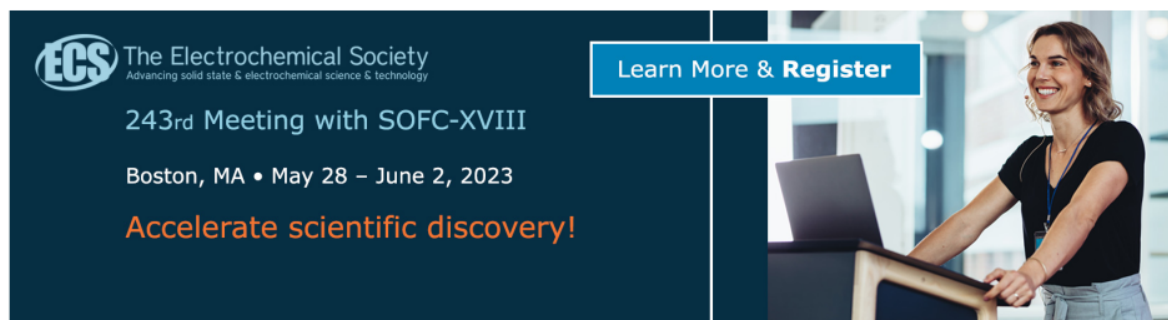
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# Implementation of arc flow model incapacitated multi-period cutting stock problem with the pattern set up cost to minimize the trim loss

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**Abstract.** Two-dimensional Cutting Stock Problem (CSP) is a problem in cutting raw materials where the trim loss is on two sides, namely the width and length sides. This research implemented the Arc Flow model incapacitated multi-period with set up cost to minimize the trim loss in the cutting of paper. The cutting patterns were generated by the Pattern Generation (PG) algorithm. Furthermore, it was formulated to a linear Arc Flow model where the constraints indicated the number of demands per item. The solution of the model was completed using the LINGO 13.0 application. The optimal solution of the Arc Flow model showed that the quantity demands for the second and third types of items were fulfilled. The maximum amount of inventory contained in the second type of item for the second period was 132,517 sheets. Excess inventory will become a surplus. Based on the arc flow model solution, it turned out that no trim loss was produced, or in other words, trim loss is equal to zero.

## 1. Introduction

The raw material used in the printing industry can be paper of various sizes. The available raw materials usually do not meet the demand because the large standard size must be cut according to the demand. Determining how to cut raw material in optimization is known as the Cutting Stock Problem (CSP). CSP is divided into three parts based on the number of dimensions, namely one-dimensional CSP, two-dimensional CSP, and three-dimensional CSP. This study discusses two-dimensional CSP, where cutting only considers the width and length of the raw material.

A heuristic algorithm was used to solve CSP<sup>[1]-[4]</sup>. As the number of demand increases, the probability of the number of patterns and decision variables increases exponentially. This heuristic approach often yields integer solutions. Two-dimensional CSP is a problem to find patterns that meet demand with different lengths and cut from two sides, namely, width and length<sup>[4]</sup>. Two-dimensional CSP aims to minimize the remaining cut which is called trim loss.

A pattern generation algorithm was improved by formulating the Gilmore and Gomory model<sup>[5]</sup>. Constraints in the Gilmore and Gomory model were carried out to ensure that the strips cut in the first stage can be used in the second stage. The Branch and Cut method was used to obtain an optimal solution<sup>[6]</sup>. There were many pattern combinations when the optimal cutting pattern corresponding to the first stage was combined with the second stage. On the other side, the model of two-dimensional CSP for different stock sizes was formulated<sup>[7]</sup>. The first stage generated the patterns based on width, followed by the length in the next stage.

All possible cutting patterns generated by the Pattern Generation (PG) algorithm were examined and formulated to the n-sheet model [8]. The formulation of arc flow model with a number of constraints for damaged raw materials and a series of constraints which state that the number of items must be entered was done [9]. This model was strengthened by fixing several variables at the initial level, to reduce the symmetry of the solution space.

Many other types of model CSP were also done [10]-[13]. Arc flow model has a conservation set of flow constraints and a single set of demand constraints to ensure that the demands of each item are met [14]. Capacitated multi-period CSP with pattern determination costs was discussed in this study. Determining the pattern in each period aimed to minimize the total costs, including pattern setting, inventory storage, and the cost of materials used. Therefore, this research intends to implement the arc flow model incapacitated multi-period cutting stock problem with the pattern set up cost to minimize the trim loss. The patterns were formed using the PG algorithm and formulated into the Gilmore and Gomory model and the arc flow model. The data used in this study is in the form of raw size data and item sizes for cutting regular shapes.

## 2. Methods

The steps taken in this study are as follows:

- a. Describe the data needed in forming cutting patterns which include stock size (length and width), item's size, and the number of demand for each item.

The data used in this study was secondary data whereas the stock size is **3,500 cm × 3,500 cm** which consists of 3 types of items. The size of each type of item and the number of demand can be seen in Table 1.

**Table 1.** Size of items and number of demands

Item	Width (mm)	Length (mm)	Demand (pieces)
1	755	555	4
2	496	555	6
3	200	378	75

- b. Implement Pattern Generation algorithm to generate the pattern, whereas the first stage was cutting based on the width and the second stage was cutting based on the length.
- c. Resume the cutting pattern with the cut loss into the table.
- d. Formulate the Gilmore and Gomory model and find the optimal pattern.
- e. Formulate the arc flow model and find the optimal pattern.
- f. Analyze the final solution.

## 3. Result and Discussion

### 3.1. The Gilmore and Gomory Model

Based on the pattern generation algorithm [12], the cutting patterns which are according to the width and length can be seen in Table 2 and Table 3 respectively. From Table 1, we can see that there are 3 items with different width and length. The greatest number of demand is the third item and the smallest number is the first item. There are 23 cutting patterns based on the width and 28 cutting patterns based on the length. The cutting patterns that had been obtained from the Pattern Generation (PG) algorithm were formulated into the Gilmore and Gomory model.

Following are the steps to form a cutting pattern into Gilmore and Gomory's model:

- a. Defining the variables

The variables that we used are:

$z$  is the objective function. It states the minimum number of stocks.

$J_0$  is the set of patterns in the first stage.

$l_i$  is the length of  $i^{th}$  item, for  $i=1,2,3$ , so  $l_1 = 378$  mm,  $l_2 = 555$  mm, and  $l_3 = 555$  mm.  
 $w_i$  is the width of  $i^{th}$  item, for  $i=1,2,3$ , so  $w_1 = 200$  mm,  $w_2 = 496$  mm, and  $w_3 = 755$  mm.  
 $\lambda_j^0$  is the number of stocks in the first stage which are cut according to the  $j^{th}$  pattern.  
 $\lambda_j^s$  is the number of stripe with width  $W$  and length  $l_s$  with  $s \in \{1,2, \dots, m'\}$  which are cut according to the  $j^{th}$  pattern.  
 $b_i$  is the demand of  $i^{th}$  item.

**Table 2.** The cutting patterns which are according to the width

The cutting pattern	Width of Items (mm)			Cut Loss (mm)
	755	496	200	
1	4	0	2	80
2	3	2	1	43
3	3	1	3	139
4	3	0	6	35
5	2	4	0	6
6	2	3	2	102
7	2	2	4	198
8	2	1	7	94
9	2	0	9	190
10	1	5	1	65
11	1	4	3	161
12	1	3	6	57
13	1	2	8	153
14	1	1	11	49
15	1	0	13	145
16	0	7	0	28
17	0	6	2	124
18	0	5	5	20
19	0	4	7	116
20	0	3	10	12
21	0	2	12	108
22	0	1	15	4
23	0	0	17	100

b. Determining the objective function and set of constraints

The objective function in this problem is to minimize the amount of stock to be cut but it can fulfill the demand for the items ordered. This model can be seen in Model (1)

Minimize

$$z = \sum_{i=0}^{23} \lambda_i^0 \tag{1}$$

Subject to

$$2\lambda_1^0 + \lambda_2^0 + 3\lambda_3^0 + 6\lambda_4^0 + 2\lambda_6^0 + 4\lambda_7^0 + 7\lambda_8^0 + 9\lambda_9^0 + \lambda_{10}^0 + 3\lambda_{11}^0 + 6\lambda_{12}^0 + 8\lambda_{13}^0 + 11\lambda_{14}^0 + 13\lambda_{15}^0 + 2\lambda_{17}^0 + 5\lambda_{18}^0 + 7\lambda_{19}^0 + 10\lambda_{20}^0 + 12\lambda_{21}^0 + 15\lambda_{22}^0 + 17\lambda_{23}^0 - \lambda_1^1 = 0 \tag{1.a}$$

$$2\lambda_2^0 + \lambda_3^0 + 4\lambda_5^0 + 3\lambda_6^0 + 2\lambda_7^0 + \lambda_8^0 + 5\lambda_{10}^0 + 4\lambda_{11}^0 + 3\lambda_{12}^0 + 2\lambda_{13}^0 + \lambda_{14}^0 + 7\lambda_{16}^0 + 6\lambda_{17}^0 + 5\lambda_{18}^0 + 4\lambda_{19}^0 + 3\lambda_{20}^0 + 2\lambda_{21}^0 + \lambda_{22}^0 - \sum_{i=1}^6 \lambda_i^2 = 0 \tag{1.b}$$

$$4\lambda_1^0 + 3\lambda_2^0 + 3\lambda_3^0 + 3\lambda_4^0 + 2\lambda_5^0 + 2\lambda_6^0 + 2\lambda_7^0 + 2\lambda_8^0 + 2\lambda_9^0 + \sum_{i=10}^{15} \lambda_i^0 - \sum_{i=1}^{21} \lambda_i^3 = 0 \tag{1.c}$$

$$9\lambda_1^1 + 7\lambda_1^2 + 6\lambda_2^2 + 4\lambda_3^2 + 3\lambda_4^2 + \lambda_5^2 + 7\lambda_1^3 + 6\lambda_2^3 + 4\lambda_4^3 + 3\lambda_5^3 + \lambda_6^3 + 6\lambda_8^3 + 4\lambda_9^3 + 3\lambda_{10}^3 + \lambda_{11}^3 + 4\lambda_{13}^3 + 3\lambda_{14}^3 + \lambda_{15}^3 + 3\lambda_{17}^3 + \lambda_{18}^3 + \lambda_{20}^3 \geq 75 \tag{1.d}$$

$$\lambda_1^2 + 2\lambda_2^2 + 3\lambda_3^2 + 4\lambda_4^2 + 5\lambda_5^2 + 6\lambda_6^2 + \lambda_2^3 + 2\lambda_3^3 + 3\lambda_4^3 + 4\lambda_5^3 + 5\lambda_6^3 + \lambda_8^3 + 2\lambda_9^3 + 3\lambda_{10}^3 + 4\lambda_{11}^3 + \lambda_{13}^3 + 2\lambda_{14}^3 + 3\lambda_{15}^3 + \lambda_{17}^3 + 2\lambda_{18}^3 + \lambda_{20}^3 \geq 6 \tag{1.e}$$

$$\lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \lambda_4^3 + \lambda_5^3 + \lambda_6^3 + 2\lambda_7^3 + 2\lambda_8^3 + 2\lambda_9^3 + 2\lambda_{10}^3 + 2\lambda_{11}^3 + 3\lambda_{12}^3 + 3\lambda_{13}^3 + 3\lambda_{14}^3 + 3\lambda_{15}^3 + 4\lambda_{16}^3 + 4\lambda_{17}^3 + 4\lambda_{18}^3 + 5\lambda_{19}^3 + 5\lambda_{20}^3 + 6\lambda_{21}^3 \geq 4 \tag{1.f}$$

with  $\bar{\lambda} = [\lambda_1^0 \dots \lambda_j^0 \dots \lambda_1^1 \dots \lambda_j^1 \dots \lambda_1^2 \dots \lambda_j^2 \dots \lambda_1^{m'} \dots \lambda_j^{m'}]^T$

**Table 3.** The cutting patterns which are according to the length

The cutting pattern	Length of items (mm)			Cut Loss (mm)
	555	555	378	
1	6	0	0	170
2	5	1	0	170
3	5	0	1	347
4	4	2	0	170
5	4	1	1	347
6	4	0	3	146
7	3	3	0	170
8	3	2	1	347
9	3	1	3	146
10	3	0	4	323
11	2	4	0	170
12	2	3	1	347
13	2	2	3	146
14	2	1	4	323
15	2	0	6	122
16	1	5	0	170
17	1	4	1	347
18	1	3	3	146
19	1	2	4	323
20	1	1	6	122
21	1	0	7	299
22	0	6	0	170
23	0	5	1	347
24	0	4	3	146
25	0	3	4	323
26	0	2	6	122
27	0	1	7	299
28	0	0	9	98

Constraint (1.a) ensured that the strip with width of 200 mm that produced in the first stage was used in the second stage. Constraint (1.b-1.c) ensured that the strip with width of 496 mm and 755 mm that produced in the first stage were used in the second stage. Constraint (1.d) states that the demand for items measuring 200 mm × 378 mm was not less than 75 pieces. On the other side, the demand for items measuring 496 mm × 555 mm and 755 mm × 555 mm were not less than 6 and 4 pieces respectively.

Solutions of Model (1) by using LINGO 13.0 were  $\lambda_{13}^0 = 1$ ,  $\lambda_1^1 = 8$ ,  $\lambda_6^2 = 2$ , and  $\lambda_{17}^3 = 1$ . It meant we used the 13<sup>th</sup> pattern in the first stage and in the second stage we used the 1<sup>st</sup>, 6<sup>th</sup>, and 17<sup>th</sup> pattern. Based on the 13<sup>th</sup> pattern that was cut according to the width, there were 1 piece item of 755 mm, 2 pieces items of 496 mm, and 8 pieces items of 200 mm. The cut loss in this patterns was 153 mm. From the first pattern, there were 6 pieces items of 555 mm with 170 mm of trim loss. The sixth

pattern yielded 4 pieces items of 555 mm and 3 pieces items of 378 mm with 146 mm of trim loss. The 17<sup>th</sup> pattern yielded 5 pieces items of 555 mm and 1 piece item of 378 mm with 347 mm of trim loss.

3.2. The Arc Flow Model

The data that we used were as follows:

- $n$  is the number of items,  $n = 3$
- $T$  is the number of period,  $T = 3$
- $d_{it}$  is the demand of the  $i^{th}$  item
- $h_i$  is 0,01  $l_i$
- $Q_t$  is the production capacity in  $t$  period.
- $L$  is the length of stocks,  $L = 3500$
- $C$  is the cost of one item,  $C = L$
- $p$  is the number of pattern,  $p = 4$
- $t$  is the period,  $t = 3$
- $i$  is item,  $i = 3$
- $\beta$  is the pattern set up cost,  $\beta = 0,01 L$

The Arc flow model can be seen in Model (2).

Minimize

$$z_{arc\ flow} = 0,01l_1I_{11} + 0,01l_1I_{12} + 0,01l_1I_{13} + 0,01l_2I_{21} + 0,01l_2I_{22} + 0,01l_2I_{23} + 0,01l_3I_{31} + 0,01l_3I_{32} + 0,01l_3I_{33} + 3500z_1^1 + 3500z_2^1 + 3500z_3^1 + 35z_1^2 + 35z_2^2 + 35z_3^2 + 7000z_1^3 + 7000z_2^3 + 7000z_3^3 + 35z_1^4 + 35z_2^4 + 35z_3^4 + 10500z_1^3 + 10500z_2^3 + 10500z_3^3 + 35z_1^3 + 35z_2^3 + 35z_3^3 + 14000z_1^4 + 14000z_2^4 + 14000z_3^4 + 35z_1^4 + 35z_2^4 + 35z_3^4 \tag{2}$$

Subject to

$$9x_1^1 + 7x_1^2 + 6x_2^2 + 4x_3^2 + 3x_4^2 + x_5^2 + 7x_1^3 + 6x_2^3 + 4x_3^3 + 3x_4^3 + x_5^3 + 6x_6^3 + 4x_9^3 + 3x_{10}^3 + x_{11}^3 + 4x_{13}^3 + 3x_{14}^3 + x_{15}^3 + 3x_{17}^3 + x_{18}^3 + x_{20}^3 \geq 56625 \tag{2.a}$$

$$x_1^2 + 2x_2^2 + 3x_3^2 + 4x_4^2 + 5x_5^2 + 6x_6^2 + x_7^2 + 2x_8^2 + 3x_4^3 + 4x_5^3 + 5x_6^3 + x_8^3 + 2x_9^3 + 3x_{10}^3 + 4x_{11}^3 + x_{13}^3 + 2x_{14}^3 + 3x_{15}^3 + x_{17}^3 + 2x_{18}^3 + x_{20}^3 \geq 2976 \tag{2.b}$$

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + 2x_7^3 + 2x_8^3 + 2x_9^3 + 2x_{10}^3 + 2x_{11}^3 + 3x_{12}^3 + 3x_{13}^3 + 3x_{14}^3 + 3x_{15}^3 + 4x_{16}^3 + 4x_{17}^3 + 4x_{18}^3 + 5x_{19}^3 + 5x_{20}^3 + 6x_{21}^3 \geq 800 \tag{2.c}$$

$$9x_1^1 + 7x_1^2 + 6x_2^2 + 4x_3^2 + 3x_4^2 + x_5^2 + 7x_1^3 + 6x_2^3 + 4x_3^3 + 3x_4^3 + x_5^3 + 6x_6^3 + 4x_9^3 + 3x_{10}^3 + x_{11}^3 + 4x_{13}^3 + 3x_{14}^3 + x_{15}^3 + 3x_{17}^3 + x_{18}^3 + x_{20}^3 - I_{10} - I_{11} = d_{11} \tag{2.d}$$

$$x_1^2 + 2x_2^2 + 3x_3^2 + 4x_4^2 + 5x_5^2 + 6x_6^2 + x_7^2 + 2x_8^2 + 3x_4^3 + 4x_5^3 + 5x_6^3 + x_8^3 + 2x_9^3 + 3x_{10}^3 + 4x_{11}^3 + x_{13}^3 + 2x_{14}^3 + 3x_{15}^3 + x_{17}^3 + 2x_{18}^3 + x_{20}^3 - I_{21} - I_{22} = d_{22} \tag{2.e}$$

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + 2x_7^3 + 2x_8^3 + 2x_9^3 + 2x_{10}^3 + 2x_{11}^3 + 3x_{12}^3 + 3x_{13}^3 + 3x_{14}^3 + 3x_{15}^3 + 4x_{16}^3 + 4x_{17}^3 + 4x_{18}^3 + 5x_{19}^3 + 5x_{20}^3 + 6x_{21}^3 - I_{32} - I_{33} = d_{33} \tag{2.f}$$

$$z_1^1 + 2z_2^1 + 3z_3^1 + 4z_4^1 \leq Q_1 \tag{2.g}$$

$$z_2^1 + 2z_2^2 + 3z_3^2 + 4z_4^2 \leq Q_2 \tag{2.h}$$

$$z_3^1 + 2z_3^2 + 3z_3^3 + 4z_4^3 \leq Q_3 \tag{2.i}$$

$$I_{10} = 0 \tag{2.j}$$

$$I_{20} = 0 \tag{2.k}$$

$$I_{30} = 0 \tag{2.l}$$

The optimal solutions were  $I_{11} = 84019$  and  $I_{31} = 1$  which meant the stock of the 1<sup>st</sup> item and the 3<sup>rd</sup> item were 84,019 pieces and 1 piece respectively. From the arc flow model, it can be concluded that the demand for the first and third items were fulfilled. The maximum inventory for the second item in the second period was 132,517 pieces.

4. Conclusion

From the result and discussion, it can be concluded that the arc flow model can be formulated from the optimal pattern of the Gilmore and Gomory model. The solution from the arc flow model showed the

number of fulfilled items. The excess in inventory would become a surplus. Based on the arc flow model solution in Model (2) it turned out that no trim loss was generated or in other words, trim loss was equal to zero.

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#### References

- [1] Mobasher A and Ekici A 2013 Solution approaches for the cutting stock problem with setup cost *Computational Operational Research* **40** (1) 225–235
- [2] Jin M, Ge P, and Ren P 2015 A new heuristic algorithm for two-dimensional defective stock guillotine cutting stock problem with multiple stock sizes *Tehnicki Vjesnik* **22** (5) 1107–1116
- [3] MirHassani S A and Bashirzadeh A J 2015 A GRASP meta-heuristic for two-dimensional irregular cutting stock problem *International Journal of Advanced Manufacturing Technology* **81**(1–4) 455–464
- [4] Cherri L H, Carravilla M A, and Toledo F M B 2016 A model-based heuristic for the irregular strip packing problem *Pesquisa Operacional* **36** (3) 447–468
- [5] Rodrigo W N P, Daundasekera W B, and Perera A A I 2012 Pattern generation for two-dimensional cutting stock problem with location *Indian Journal of Mathematics Trends and Technology* **3** (2) 354–368
- [6] Octarina S, Radiana M, and Bangun P B J 2018 Implementation of pattern generation algorithm in forming Gilmore and Gomory model for two dimensional cutting stock problem *IOP Conference Series Material Science and Engineering* **300** (1)
- [7] Octarina S, Ananda V, and Yuliza E 2019 Gilmore and gomory model on two dimensional multiple stock size cutting stock problem *Journal of Physics: Conference Series* **1282** (1)
- [8] Bangun P B J, Octarina S, and Pertama A P 2019 Implementation of branch and cut method on n-sheet model in solving two dimensional cutting stock problem *Journal of Physics: Conference Series* **1282** (1)
- [9] Braga N, Alves C, Macedo R, and Carvalho J V D 2016 Combined cutting stock and scheduling: a matheuristic approach *International Journal of Innovative and Computational Application* **7**(3) 135–146
- [10] Rodrigo N and Shashikala S 2017 One-dimensional cutting stock problem with cartesian coordinate points *International Journal of Systems Science and Applied Mathematics* **2** (5) 99
- [11] Lomate S, Rajiv B, Pantawane P D, and Ahuja B B 2020 Greedy algorithm to generate cutting patterns for cutting stock problem (1D and 2D) *Industrial Engineering Journal* **13** (4) 1–11
- [12] Octarina S, Janna M, Cahyono E S, Bangun P B J, and Hanum L 2020 The modified branch and bound algorithm and dotted board model for triangular shape items *Journal of Physics: Conference Series* **1480** (1)
- [13] Bangun P B J, Octarina S, Sepriliani S P, Hanum L, and Cahyono E S 2020 3-phase matheuristic model in two-dimensional cutting stock problem of triangular shape items *Science and Technology Indonesia* **5** (1) 1–5
- [14] Ma N, Liu Y, and Zhou Z 2019 Two heuristics for the capacitated multi-period cutting stock problem with pattern setup cost *Computational Operational Research* **109** 218–229



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