9. How to Study

by Budi Mulyono

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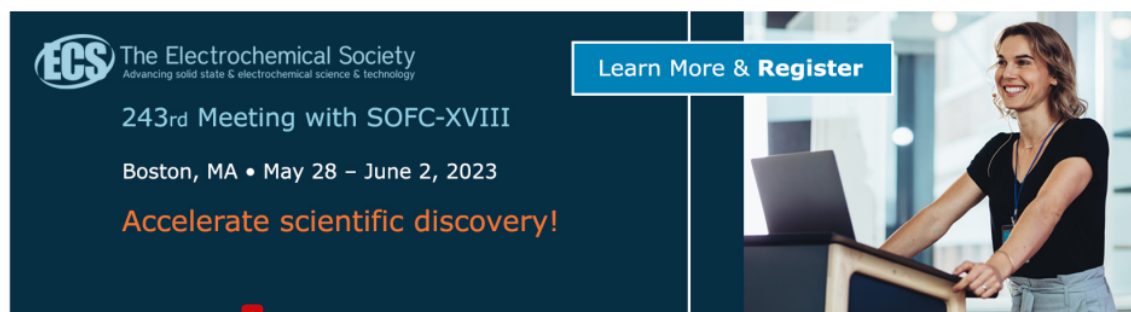
How to study limit through computer-aided learning with conceptual-conflict strategy

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How to study limit through computer-aided learning with conceptual-conflict strategy

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Abstract. This article talks about how to help students studying limit through computer-aided learning (CAL) with conceptual-conflict strategy (CCS). In their learning activities students use computer for graphing and calculating, and they are conditioned in learning atmosphere which could trigger conflict of conceptual about what they already knew and what they just know. In this case, the topic discussed is about definition of limit intuitively. As we know that students already studied limit at senior high school, but some students still have misconception about limit. They had their own perception about limit, for instance, limit is a value of function at a point, the value of limit is a result of substituting a value $x = c$ to a function $f(x)$, and limit is a boundary value of a function that there is no any other value of the function higher than the value of limit. The result of implementation of CAL with CCS indicates that students realized that they had some misconceptions about limit, and then they became more understand how to determine a value of limit by considering the right- and left-hand limits.

1. Introduction

Calculus is a compulsory course for mathematics-education students and in common calculus is divided into three sections which are calculus 1, calculus 2, and advance calculus. According to Purcell *et al* [1] calculus is the study of limits which means limit is the main topic in calculus. It tells that mathematics-education students should have good conceptual understanding and have no misconception about calculus. In fact, Mulyono & Hapizah [2] say that some mathematics-education students of a university in south Sumatera whom already studied calculus 1 still had problem in conceptual understanding and misconception about limit. It is supported by what Blaisdell [3] says that students whom take calculus course find difficult to learn limit. According to Kumsa *et al* [4] internationally both at junior high school and university levels many students had problems and difficulties to understand and to apply calculus, especially about concepts of limit. Liang [5] says that

mathematics students do not understand about concepts of limit then they cannot understand concepts of continuity, convergence, differential, and they are not ready to take other analytic courses.

This problem is very crucial for mathematics-education students because conceptual understanding about limit is the basic to study and to understand topics in calculus 2 and advance calculus. According to Liang [5] problems in learning calculus occur because teachers usually are more focused to calculation of limit, sometimes to graph illustration of limit, but rarely to theoretical aspect or definition of limit. Meanwhile Ugulu [6] says that misconceptions of limit occurred in students' mind are caused by inappropriate teaching techniques, and the misconceptions could stay for long time at least one or two years after the teaching, even though the students study topics related to each other. To overcome the problems mathematics teachers should design innovative-learning activities in which students could improve their conceptual understanding and fix their misconceptions about limit.

The need of innovative-learning design for teaching calculus is parallel to what Garner & Garner [7] say that one of major themes of calculus reform movement is the use of technology (computers and calculators). According to Stols [8] new technologies could help students to visualize concepts which are difficult to understand and could help students to construct or to develop an active problem-solving environment. Meanwhile Budi [9] says that learning activities in which computer use is embedded could help students reaching better achievement of they learned. According to Stoll [8] even though there are many advantages of using technologies, many teachers do not use technologies in their teaching. Meanwhile Mulyono & Hapizah [2] claim that their survey results inform that the use of computer was not optimized to help students studying concepts of limit, because the computer were used only for showing slides of topics taught.

Not only the use of technologies, mathematics teachers should also consider using an appropriate strategy to help students improving conceptual understanding and fixing misconceptions about limit. Based on some literature reviews a teaching strategy which could be used to help students fixing their misconception is conceptual conflict strategy. According to Liang [5] conceptual conflict is a conflict between new concepts and previous concepts which students have, and learning process which connects previous knowledge to new knowledge is often related to conceptual change. Liang [5] describes conceptual conflict strategy into three steps. The first step is to let students show their misconceptions explicitly, which is the step of diagnosis of misconceptions. The second step is to let students argue and debate their opinion and previous conceptual understanding, which is the step of clarification of student's conceptions. The third step is to let a teacher create a confrontation towards student's misconceptions, which is the step of confrontation of student's misconceptions. The fourth step is to let a teacher helps students to accept the new knowledge and change their misconceptions, which is the step of accommodation.

Based on the above description, it is interesting to design learning activities in which the use of computers and conceptual conflict strategy are combined. The design of learning activities should help students to improve their conceptual understanding and to help them fix their misconception about limit.

2. Methods

There are some steps to design computer-aided learning with conceptual conflict strategy for learning limit. In February 2017 the researcher held a preliminary research in order to identify what student's problems in learning limit are. After the identification of student's problem in learning limit, the researcher reviewed some literatures related to computer-aided learning and conceptual conflict strategy, then developing a draft of learning activities by combining computer-aided learning and conceptual conflict strategy. Afterwards the researcher asked experts to give some comments on it. The design of learning activities was revised based on the expert's comments. In September-October

2017 the researcher tried it out to 16 mathematics-education students of a university in South Sumatera. After that the design of learning activities was revised based on findings of the try out. In August-October 2018 the research tried the revised design out to 12 mathematics-education students of a university in South Sumatera. After the last try out the researcher revised the design of learning activities and finally the last revised version of learning activities is produced.

3. Result and Discussion

3.1. The learning activities

There are four designs of learning activities produced, but here the researcher only discusses one of them. The learning activities discussed here talk about how to help students to understand about right- and left-hand limit and help students to fix their misconceptions about determining a limit value of a function. The researcher designs the learning activities in five steps, which are step of diagnosis of misconceptions, step of clarification of misconceptions, step of confrontation, step of accommodation, and step of evaluation.

3.2. Description of the learning activities

Students in this learning activities can use computers or laptops, and they are conditioned in a situation which triggers a conceptual conflict between their previous conceptual understanding to new concepts about how to determine a value of $\lim$. Commonly student's mistake in determining a value of $\lim_{x \to c} f(x)$ is that students simply substitute a value of variable $x = c$ into a function $f(x)$ without checking its right- and left-hand limit. This learning activities is designed to help students to understand how to determine a limit value appropriately and to fix their misconception about the value of limit.

3.3. The aim of learning activities

To fix student's misconceptions about how to determine a limit value of a function and to improve student's conceptual understanding about right- and left-hand

3.4. The six steps of learning activities

The first Step: Diagnosis of misconceptions

The teacher asks a question as follows: "*Does $\lim_{x \to 1} \frac{1}{x-1}$ exist? Please give your reason.*"

The prediction of student's mistake answers at this step are below:

1. $\lim_{x \to 1} \frac{1}{x-1}$ exists and its limit value is ∞, because the result of substitution $x = 1$ into $\lim_{x \to 1} \frac{1}{x-1}$ is $\frac{1}{0} = \infty$.

2. $\lim_{x \to 1} \frac{1}{x-1}$ does not exist, because the result of substitution $x = 1$ into $\lim_{x \to 1} \frac{1}{x-1}$ is $\frac{1}{0}$ = undefined.

In the class the researcher found a type of student's mistake answers as follows.

$$\lim_{x \to 1} \frac{1}{x-1} = \frac{1}{1-1} = \frac{1}{0} \ (undefined). \ So, \ its \ limit \ does \ not \ exist.$$

It shows that there were some students had misconception about how to determine a limit value of a function. The student simply substituted $x = 1$ into $\frac{1}{x-1}$.

The second step: Clarification of misconceptions

The teacher asks students whom have the mistake answers for $\lim_{x \to 1} \frac{1}{x-1}$ to explain their reason of their answers, and the teacher as the moderator asks other students to response and to discuss the explanation. In this step the teacher does not give any correction to every student's comment. The

prediction at this step is that some students tends to do a common mistake in determining $\lim\limits_{x\to 1}\dfrac{1}{x-1}$ which is just simply substituting the value of $x = 1$ to $\dfrac{1}{x-1}$.

In the class the researcher saw that some students still believed in the way to solve $\lim\limits_{x\to 1}\dfrac{1}{x-1}$ by substituting the value of $x = 1$ to $\dfrac{1}{x-1}$ as what they usually used it at junior high school. From the discussion the researcher also found that some students thought that to solve $\lim\limits_{x\to c} f(x)$ is the same to find the value of $f(x)$ at $x = c$. It clarifies that students still had problems in determining a limit value of a function.

The third step: Confrontation

The teacher asks students to answer all questions related to $f(x) = \dfrac{1}{x-1}$ as follows.

Please pay attention to the graph of $f(x) = \dfrac{1}{x-1}$ and its relation to the change of value $f(x)$ at table 1.

  a) Draw graph of $f(x) = \dfrac{1}{x-1}$. (Use Geogebra)
  b) Complete Table 1. (Use Ms Excel)
  c) Determine the value of $f(x)$ for $x = 1$.
  d) Is $f(x)$ defined at $x = 1$?
  e) What happen to values of $f(x)$ when x is close to 1?
  f) Is $f(x)$ close to a certain number when x is close to 1 from the left side?
  g) Is $f(x)$ close to a certain number when x is close to 1 from the right side?

Table 1. The values of $f(x)$

x	$f(x) = \dfrac{1}{x-1}$
1,1	......
1,01	......
1,001	......
1,0001	......
$\vdots$	$\vdots$
1	?
$\vdots$	$\vdots$
0,9999	......
0,999	......
0,99	......
0,9	......

The prediction at this step is that students see a contradiction between their previous conceive and what they just find about how to determine the value of $\lim\limits_{x\to 1}\dfrac{1}{x-1}$.

In the class the researcher saw that students did not have problem to complete the table 1 and they could draw the graph of $f(x) = \dfrac{1}{x-1}$.. Based on their own answers, students could see that the values of $f(x)$ when x is close to 1 do not tend to a certain number. It is because they saw that the values of $f(x)$ become smaller ($-\infty$) when x is close to 1 from the left side, and the values of $f(x)$ become greater ($+\infty$) when x is close to 1 from the right side. So, students realized that they could not say that the answer of $\lim\limits_{x\to 1}\dfrac{1}{x-1}$ is ∞ like their previous answer. It shows that there was a contradiction that students found between their previous conceive and what they just discovered about how to determine the value of $\lim\limits_{x\to 1}\dfrac{1}{x-1}$. The researcher at this point believe that students were in a conceptual-conflict

condition which they became dissatisfied with their previous conceive about how to determine the value of $\lim\limits_{x\to 1}\frac{1}{x-1}$.

The fourth Step: Accommodation

At this step the teacher is more active to explain definition of right- and left-hand limits and its symbols. After that the teacher gives some questions to make students more understand about right- and left-hand limits, and how to claim whether a function has limit or not at a point. The teacher should remind students to consider the left- and the right-hand limits of a function at a point before they claim whether the limit of a function exists or not at a point. The prediction at this step is that students more understand about right- and left-hand limits and are more deliberate to claim whether a function has limit or not at a point.

In the class the teacher asked students to read carefully the definition right- and left-hand limits as follows.

> *To say that* $\lim\limits_{x\to c^+} f(x) = L$ *means that when x is near but to the right of c then $f(x)$ is near L. Similarly, to say that* $\lim\limits_{x\to c^-} f(x) = L$ *means that when x is near but to the left of c then $f(x)$ is near L.*

After that the teacher asked student to answer questions below.

1. Let $g(x) = \begin{cases} \sqrt{x-1} & \text{if } x \geq 1 \\ 1-x & \text{if } x < 1 \end{cases}$

   a) What is the value of limit $g(x)$ when $x \to 1^+$?
   b) What is the value of limit $g(x)$ when $x \to 1^-$?
   c) Does $g(x)$ have limit when $x \to 1$? Please give your reason.

However, there were some students at the first time confused to read the function because $g(x)$ has two mathematics expressions. The researcher saw that the students did not find difficult to answer all the questions, after the students were explained a bit how to read $g(x)$. They used Ms Excel to calculate the value of limit $g(x)$ when $x \to 1^+$ and when $x \to 1^-$, like Table 2 as follows.

Table 2. The value of $g(x)$

x	$g(x)$	x	$g(x)$
0,9	0,1000000	1,1	0,3162278
0,99	0,0100000	1,01	0,1000000
0,999	0,0010000	1,001	0,0316228
0,9999	0,0001000	1,0001	0,0100000
0,99999	0,0000100	1,00001	0,0031623
0,999999	0,0000010	1,000001	0,0010000
0,9999999	0,0000001	1,0000001	0,0003162
⋮	⋮	⋮	⋮
1	0	1	0

From Table 2 students could see that the right-hand limit of $g(x)$ is 0 and the left-hand limit of $g(x)$ is 0 as well. They learned that the value of $\lim\limits_{x\to 1} g(x)$ is 0, not because of the result of substitution $x = 1$ into $g(x)$.

The students also could see clearly Figure 1 that the values of $g(x)$ is close to 0 when x is close to 1 both from right and left sides.
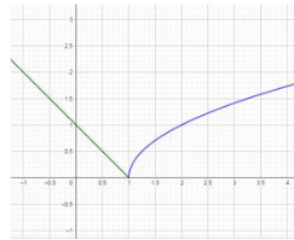
Figure 1. The graph of $g(x)$

After that, the teacher explained and emphasized the limit theorem as follows

$$\lim_{x \to c} f(x) = L \ if \ only \ if \ \lim_{x \to c^-} f(x) = L \ and \ \lim_{x \to c^+} f(x) = L.$$

The students then were asked to study Figure 2 to understand the limit theorem above. They were asked to give reasons for every limit value at Figure 2. The researcher saw that the students did not find difficult to give the reasons based on the limit theorem above. For example, they said that $\lim_{x \to 1} f(x)$ does not exist because the right-hand limit is not the same as the left-hand limit when x is close to 1, which are $\lim_{x \to 1^+} f(x) = 1.7$ and $\lim_{x \to 1^-} f(x) = 1.5$.
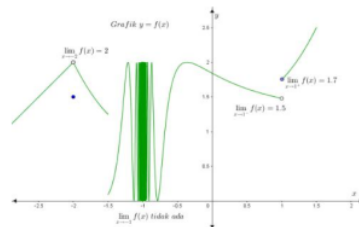

Figure 2. Left- and right-hand limits

They also realized that even though the value of $f(x)$ at $x = 1$ is defined which is $f(1) = 1.7$, they did not say that the value of $\lim_{x \to 1} f(x) = f(1)$.

At this state, the researcher could say that students learned and understood how to determine limit of a function by considering the right- and the left-hand limits.
To make students more understand how to determine a limit value of a function, the teacher asked students to answer some questions of practice 1 below.
Please look at carefully Figure 3, then answer all questions as follows.
a. *What is the value of* $\lim_{x \to -3} f(x)$ *?*
b. *What is the value of* $f(-3)$ *?*
c. *What is the value of* $f(-1)$ *?*
d. *What is the value of* $\lim_{x \to -1} f(x)$ *?*
e. *What is the value of* $\lim_{x \to -1^-} f(x)$ *?*
f. *What is the value of* $\lim_{x \to -1^+} f(x)$ *?*
g. *What is the value of* $f(1)$ *?*
h. *What is the value of* $\lim_{x \to 1} f(x)$ *?*
i. *What is the value of* $\lim_{x \to 1^-} f(x)$ *?*
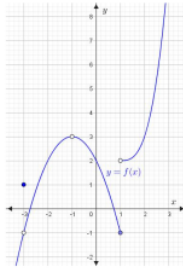j. *What is the value of* $\lim_{x \to 1^+} f(x)$ *?*

Figure 3. a graph for practice 1

In the class the researcher saw that some students took time a bit longer than the others to answer all the questions, but they did not find difficult to answers all the questions.

The fifth step: Conclusion

At this step the teacher asks students to make some conclusion of what they just learn about how to determine limit of a function and about the right- and left-hand limits. The prediction at this step is that students make conclusions about how to determine limit of a function and about the right- and left-hand limits in the form of their own words.

In the class the researcher saw that students made some conclusions by their own words about how to determine limit of a function and about the right- and left-hand limits in the form of their own words. Some students' conclusions are written below:

o   *The value of limit both from the right side and left side must close to the same value. If one of the right- and left-hand limits is not the same as the other, then the limit does not exist.*

o   *When x is close to c both from right and left sides, the values of $f(x)$ do not close to the same certain number then $\lim_{x \to c} f(x)$ does not exist.*

It shows that the students already understood to consider the-right and left-hand limits before they a value of limit $f(x)$ when x close to a certain number.

The sixth step: Evaluation

In order to see students' achievement of the learning process above, the teacher asks students to answer all the questions below.

A. *Please complete the sentences below to make it correct.*

1. $\lim_{x \to c^+} f(x) = L$ *means that .... gets close to .... when .... gets sufficiently close to .... from ....*

2. *If* $\lim_{x \to c^-} f(x) = M$ *and* $\lim_{x \to c^+} f(x) = M$, *then ....*

B. *Please write your answers correctly to all the questions below.*

3. *Does* $f(x) = \sqrt[3]{x-1}$ *have the right-hand limit which is the same as the left-hand limit when* $\to 1$? *Please explain it.*

4. *Please look at Figure 4 carefully and answer all the questions below.*

a) *What is the value of* $\lim_{x \to -3} f(x)$?   f) *What is the value of* $\lim_{x \to -1^+} f(x)$?

b) *What is the value of* $f(-3)$?   g) *What is the value of* $f(1)$?

c) *What is the value of* $f(-1)$?   h) *What is the value of* $\lim_{x \to 1} f(x)$?

d) *What is the value of* $\lim_{x \to -1} f(x)$?   i) *What is the value of* $\lim_{x \to 1^-} f(x)$?

e) *What is the value of* $\lim_{x \to -1^-} f(x)$?   j) *What is the value of* $\lim_{x \to 1^+} f(x)$?
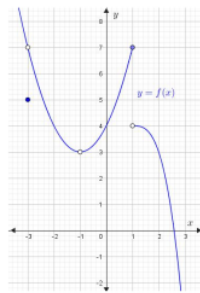
Figure 4. A graph for question 4

The teacher reviews and grades students' answers outside the class. The prediction at this step is that students could answer all the questions correctly.

Table 3. The score of students' answers

Student	Question													Point	Score
	1A	2A	3B	4Ba	4Bb	4Bc	4Bd	4Be	4Bf	4Bg	4Bh	4Bi	4Bj		
01	1	1	0,7	1	1	1	1	1	1	1	1	1	1	12,7	97,69
02	1	1	0,8	1	1	1	1	1	1	1	0	0	1	10,8	83,08
03	1	1	0,8	1	1	1	1	1	1	1	0	0	1	10,8	83,08
04	0	1	0,8	1	1	1	1	1	1	1	1	1	1	11,8	90,77
05	1	1	0,8	1	1	1	1	1	1	1	1	1	1	12,8	98,46
06	1	1	0,8	1	1	1	1	1	1	1	1	1	1	12,8	98,46
07	1	1	0,2	1	1	1	1	1	1	1	1	1	1	12,2	93,85
08	1	1	0,95	1	1	1	1	1	1	1	1	1	1	12,95	99,62
09	1	1	0,5	1	1	1	1	1	1	1	1	1	1	12,5	96,15
10	0,5	1	0	1	1	1	1	1	1	1	1	1	1	11,5	88,46
11	1	1	0,5	1	1	1	1	1	1	1	1	1	1	12,5	96,15
12	1	1	0,9	1	1	1	1	1	1	1	1	1	1	12,9	99,23

Table 3 shows the score distribution of students' answers. Over all the result of students' answers is quite good because the lower score is 83.08, and there are only 4 students (33.33%) whom got scores under the score mean of all students (93.75). The mistake that students made when answering question 3B is only incorrect calculation, but they showed that they considered the right- and left-hand limits to answer question 3B.

2 Conclusion

The result of implementation of CAL with CCS indicates that students realized that they had some misconceptions about limit, and then they became more understand how to determine a value of limit by considering the right- and left-hand limits. Mathematics teachers could help their students to overcome problems in learning mathematics by design learning activities based on students' problems.

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