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# An Application of GAMS to Solve Robust Capacitated Vehicle Routing Problem with Time

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**Abstract.** This paper addresses the robust capacitated vehicle routing problem with time windows by considering operational costs, for instance, vehicle maintenance costs, fuel costs and costs for the crew. Decision making from the robust capacitated vehicle routing problem with the time windows model under the uncertainty of the number of customer demands and travel time The robust capacitated vehicle routing problem with the time windows model is simulated on the problem of distributing gallon water to minimize operational costs and optimization of distribution routes. This paper uses linear programming in General Algebra Modeling Software (GAMS) to solve optimization problems. The results obtained are then compared with the results of optimization with the Clarke and Wright algorithm. This algorithm is a heuristic method used to solve the vehicle route problem. The results obtained with the GAMS solution are more optimal than the Clarke and Wright algorithm, resulting in lower operating costs. The Clarke and Wright algorithm is a solution to the optimization model with a heuristic approach. The optimal cost of the gallon water distribution problem solved by the Clarke and Wright algorithm is less than using the GAMS programming language.



## INTRODUCTION

In 1959, Dantzig and Ramser first introduced the Truck Dispatching Problem, a model of how a homogeneous vehicle could serve oil demand from several stations from several centres and minimize travel distances. Furthermore, in 1964 Clarke and Wright generalized this problem into a linear optimization problem commonly encountered in the logistics and transportation domain [1]. This linear optimization problem is known as the vehicle routing problem (VRP). Capacitated vehicle routing problem (CVRP) is a vehicle route problem that considers vehicle capacity and is an integral part of logistics distribution [2,3]. Vehicle routing problem with time windows (VRPTW) is a type of VRP that views travel times as uncertain [4]. This type of VRP that looks at customer demand, travel time, and service time is known as robust optimization, which is used to overcome the random nature of several parameters [5–7].

This study will discuss the robust capacitated vehicle routing problem with time windows (RCVPTW). The RCVRPTW optimization model is solved by General Algebraic Modeling Software (GAMS). GAMS is a potent optimization tool used to formulate, solve and analyze optimization problems. CPLEX is one of the solvers in GAMS to solve linear programming. The RCVRPTW optimization model is also solved by the Clarke and Wright algorithm, a heuristic method [8,9]. The Clarke and Wright algorithm is one method that can be used to solve VRP [10]. The Clarke and Wright algorithm has the concept of savings framed by calculating savings from two customers into the same route [11].

The RCVRPTW model is simulated on the problem of the gallon water distribution route. Water companies serve several customers, and each customer has an uncertain number of requests. The distribution of gallon water has uncertainty in the number of requests to be classified as robust optimization. The drinking water company

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distributes gallons of water every day during regular working hours. This regular working time can be viewed as a time window. The number of uncertain requests affects the length of service time and travel time, resulting in delays. Drinking water companies also consider the operational costs per day in distributing gallons of water. Therefore, in this study, the optimal cost of the gallon water distribution problem will be sought using the RCVRPTW model and solved using the GAMS programming language and the Clarke and Wright algorithm. The calculations use GAMS 25.1.1 and the Clarke and Wright algorithm to determine the optimal operating costs, then compared and analyzed.

## METHOD

In this study, we use GAMS and the Clarke and Wright algorithm to determine the optimal operating costs of the RCVRPTW model. The basic structure of GAMS coded mathematical model consists of sets, variables, data, equations, model and the output. The RCVRPTW model is solved in the GAMS programming language with the CPLEX solver. Then, the RCVRPTW model was also solved by the Clarke and Wright algorithm. The steps involved in the Clarke & Wright algorithm is as follows [11]:

1. Determine the distance matrix from the warehouse to various depots, respectively.
2. Compute the savings matrix
$$s_{ij} = d_{io} + d_{0j} - d_{ij} \tag{1}$$
for every pair $(i, j)$ of demand points.
3. Rank and list the savings $s_{ij}$ values in the descending order of their magnitude.
4. Assigning customers to the vehicles. Based on three steps
   - A new route is initiated, including both $i$ and $j$ if neither has previously been assigned to a route.
   - Precisely one of the two ($i$ or $j$) has already been included in an existing route, and that point is not interior to that route, in which case the link $(i, j)$ is added to the same route.
5. Process the algorithm accordingly as mentioned till the savings list exhausts. If not, return to step 3, processing the next entry in the list; otherwise, stop.



## RESULT AND DISCUSSION

A company sometimes encounters common problems in logistics distribution, such as an uncertain number of requests for each customer and regular working time limits. The robust capacitated vehicle routing problem with the time windows model can be implemented in the problem of distributing gallon water in the city of Prabumulih. The vehicles used to distribute gallons of water to each customer are homogeneous, and it is assumed that the vehicle capacity is 150 gallons. The gallon water distributor serves several customers, and the regular working time is assumed to be from 08.00 WIB to 17.00 WIB. Let G = (V, E) define a directed complete graph with the set V representing the set of all vertices with $V = \{1,2,3,..,n\}$ and the set E representing the arc from point i to point j with $E = \{(i,j)|i \in V, j \in V \text{ with } i \neq j\}$. It is defined that the set T represents the time of each customer with $T = \{[a_i, b_i]|i = 1,2,3,\dots,n\}$ where $a_i$ represents the earliest arrival time at customer i and $b_i$ represents the last arrival time at customers i. The vehicle speed is assumed to be 40 km/h, and the distance matrix is symmetrical.

$$\left[r_{ij}\right] = \begin{bmatrix} 0 & 1.6 & 3.8 & 6.9 & 13 & 2.8 & 11 & 1.8 \\ 1.6 & 0 & 2.6 & 8.1 & 14 & 1.6 & 12 & 0.8 \\ 3.8 & 2.6 & 0 & 11 & 17 & 3.6 & 14 & 2 \\ 6.9 & 8.1 & 11 & 0 & 9.2 & 7.7 & 6.6 & 9.2 \\ 13 & 14 & 17 & 9.2 & 0 & 15 & 6.3 & 15 \\ 2.8 & 1.6 & 3.6 & 7.7 & 15 & 0 & 11 & 2.3 \\ 11 & 12 & 14 & 6.6 & 6.3 & 11 & 0 & 13 \\ 1.8 & 0.8 & 2 & 9.2 & 15 & 2.3 & 13 & 0 \end{bmatrix} \tag{2}$$

and

$$\left[tv_{ij}\right]=\begin{bmatrix} 0 & 4 & 10 & 12 & 19 & 7 & 15 & 5 \\ 4 & 0 & 7 & 14 & 21 & 4 & 17 & 2 \\ 10 & 7 & 0 & 21 & 27 & 9 & 24 & 6 \\ 12 & 14 & 21 & 0 & 15 & 14 & 12 & 17 \\ 19 & 21 & 27 & 15 & 0 & 22 & 10 & 23 \\ 7 & 4 & 9 & 14 & 22 & 0 & 15 & 5 \\ 15 & 17 & 24 & 12 & 10 & 15 & 0 & 20 \\ 5 & 2 & 6 & 17 & 23 & 5 & 20 & 0 \end{bmatrix} \quad (3)$$

**TABLE 1.** Service times and demands

| Node $i$ | $ts_i$ (Minutes) | $q_i$ (Gallon) |
|---|---|---|
| 1 | - | - |
| 2 | 59 | 65 |
| 3 | 26 | 23 |
| 4 | 37 | 31 |
| 5 | 18 | 16 |
| 6 | 29 | 31 |
| 7 | 5 | 5 |
| 8 | 10 | 9 |

**TABLE 2.** Parameters

| Notation | Explanation |
|---|---|
| $n$ | Number of customers |
| $c_1$ | Vehicle maintenance costs |
| $c_2$ | Travel costs |
| $c_3$ | Cost for crew |
| $K$ | Vehicle capacity |
| $T$ | Distribution time to customers each work area |
| $r_{ij}$ | Distance between customer $i$ to customer $j$ ( $i, j = 1, \ldots, n$ , $i \neq j$) |
| $tv_{ij}$ | Average travel time between customer $i$ to customer $j$ |
| $ts_i$ | Service time at customer $i$ |
| $d$ | Number of demand $i$ |
| $[p_i;q]$ | Time windows |

**TABLE 3.** Variables

| Notation | Explanation |
|---|---|
| $z$ | Cost of optimization |
| $x_{ij}$ | The vehicle travels from node $i$ to node $j$ |
| $m_i$ | Vehicle arrival time when serving customers $i$ |
| $l_i$ | Vehicle loud when serving customers $i$ |

$$x_{ij}=\begin{cases}1; \text{if there is a vehicle traveling from } i \text{ node } j \\ 0; \text{the others}\end{cases} \quad (4)$$

**TABLE 4.** Operating costs

| Operating costs (per day) | Value |
|---|---|
| Vehicle maintenance costs | 6,944 |
| Travel costs | 33,3333 |

| Operating costs (per day) | Value |
|---|---|
| Cost for crew | 135,000 |

The mathematical model of robust capacitated vehicle routing problem with time windows on gallon water distribution in a sub route in the work area is built as follows:

Objective function:

Minimize

$$z = 47913.6x_{14} + 90272x_{15} + 76384\, x_{17} + 931500x_{14} + 1755000x_{15} + 1485000x_{17} \\ +229997.7x_{41} + 306663.6\, x_{45} + 219997.8x_{47} + 433329x_{51} + 306663.6x_{54} + \\ 209997.9x_{57} + 366663\, x_{71} + 219997.8\, x_{74} + 209997.9\, x_{75} \quad (5)$$

Subject to:

$$x_{14} + x_{15} + x_{17} = 1;\ x_{14} + x_{54} + x_{74} = 1; x_{17} + x_{47} + x_{57} = 1; \\ x_{41} + x_{45} + x_{47} = 1;\ x_{51} + x_{54} + x_{57} = 1; x_{71} + x_{74} + x_{75} = 1 \quad (6)$$

$$x_{14} + x_{54} + x_{74} - x_{41} - x_{45} - x_{47} = 0;\ x_{15} + x_{45} + x_{75} - x_{51} - x_{54} - x_{57} = 0; \\ x_{17} + x_{47} + x_{57} - x_{71} - x_{74} - x_{75} = 0 \quad (7)$$

$$-m_1 + m_4 + 7.97x_{41} \leq 7; -m_4 + m_5 + 7.85\, x_{45} \leq 7; -m_5 + m_7 + 7.5\, x_{75} \leq 7;\ 8 \\ \leq m_1 \leq 8{,}5; \\ 11 \leq m_4 \leq 12;\ 13 \leq m_5 \leq 14;\ 15 \leq m_7 \leq 16 \quad (8)$$

$$-l_4 + l_1 + 181\, x_{14} \leq 150; -l_5 + l_4 + 166\, x_{45} \leq 150; -l_7 + l_5 + 155\, x_{57} \leq 150; \\ 0 \leq l_1 \leq 150;\ 31 \leq l_4 \leq 150;\ 16 \leq l_5 \leq 150;\ 5 \leq l_7 \leq 150 \quad (9)$$

The RCVRPTW model expressed from the objective function (4), constraints (5)-(8) can be expressed in the GAMS modeling language. This is given a GAMS model.

```
*/Transportation problem*/
*/Variable definition*/
VARIABLES
z;
POSITIVE VARIABLES
x14,x15,x17,x41,x45,x47,x51,x54,x57,x71,x74,x75
y1,y4,y5,y7
t1,t4,t5,t7;
*/Necessary equations*/
EQUATIONS
OF Objective function
Const1 the vehicle travels exactly once from node 1
Const2 the vehicle travels exactly once from node 1
Const3 the vehicle travels exactly once from node 1
Const4 the vehicle travels exactly once from node 4
Const5 the vehicle travels exactly once from node 5
Const6 the vehicle travels exactly once from node 7
Const7 the vehicle starting from origin node
Const8 the vehicle starting from origin node
Const9 the vehicle starting from origin node
Const10 arival time limits
Const11 arival time limits
Const12 arival time limits
Const13 vehicle capacity limit
Const14 vehicle capacity limit
```

```
Const15 vehicle capacity limit
Const16 arrival time limits from 1
Const17 arrival time limits from 1
Const18 arrival time limits from 4
Const19 arrival time limits from 4
Const20 arrival time limits from 5
Const21 arrival time limits from 5
Const22 arrival time limits from 7
Const23 arrival time limits from 7
Const24 vehicle capacity limits from 1
Const25 vehicle capacity limits from 1
Const26 vehicle capacity limits from 4
Const27 vehicle capacity limits from 4
Const28 vehicle capacity limits from 5
Const29 vehicle capacity limits from 5
Const30 vehicle capacity limits from 7
Const31 vehicle capacity limits from 7;

OF..
z=E=47913.6*x14+90272*x15+76384*x17+931500*x14+175500*x15+1485000*x17+229997.7*x41+306663*x45
+219997.8*x47+433329*x51+306663*x54+209997.9*x57+366663*x71+219997.8*x74+209997.9*x75;
Const1.. x14+x15+x17 =E= 1;
Const2.. x14+x54+x74 =E= 1;
Const3.. x17+x47+x57 =E= 1;
Const4.. x41+x45+x47 =E= 1;
Const5.. x51+x54+x57 =E= 1;
Const6.. x71+x74+x75 =E= 1;
Const7.. x14+x54+x74-x41-x45-x47 =E= 0;
Const8.. x15+x45+x75-x51-x54-x57 =E= 0;
Const9.. x17+x47+x57-x71-x74-x75 =E= 0;
Const10.. t4-t1+7.97*x41 =L= 7;
Const11.. t5-t4+7.85*x54 =L= 7;
Const12.. t7-t5+.5*x75 =L= 7;
Const13.. y1-y4+181*x14 =L= 150;
Const14.. y4-y5+166*x45 =L= 150;
Const15.. y5-y7+155*x57 =L= 150;
Const16.. t1 =G= 8;
Const17.. t1 =L= 8.5;
Const18.. t4 =G= 8.5;
Const19.. t4 =L= 11;
Const20.. t5 =G= 12;
Const21.. t5 =L= 14;
Const22.. t7 =G= 15;
Const23.. t7 =L= 16;
Const24.. y1 =G= 0;
Const25.. y1 =L= 150;
Const26.. y4 =G= 31;
Const27.. y4 =L= 150;
Const28.. y5 =G= 16;
Const29.. y5 =L= 150;
Const30.. y7 =G= 5;
Const31.. y7 =L= 150;
*/Solving the model*/
MODEL TP/all/;
SOLVE TP USING mip MINIMIZING z;
```

DISPLAY
z.l,x14.l,x15.l,x17.l,x41.l,x45.l,x47.l,x51.l,x54.l,x57.l,x71.l,x74.l,x75.l,y1.l,y4.l,y5.l,y7.l,t1.l,t4.l,t5.l,t7.l;

Then, based on equation (1), the saving matrix form is obtained as follows:

$$\left[s_{ij}\right]=\begin{bmatrix} 0 & 2.8 & 0.4 & 0.6 & 2.8 & 0.6 & 2.6 \\ 2.8 & 0 & -0.3 & -0.2 & 3 & 0.8 & 3.6 \\ 0.4 & -0.3 & 0 & 10.7 & 2 & 11.3 & -0.5 \\ 0.6 & -0.2 & 10.7 & 0 & 0.8 & 17.7 & -0.2 \\ 2.8 & 3 & 2 & 0.8 & 0 & 2.8 & 2.3 \\ 0.6 & 0.8 & 11.3 & 17.7 & 2.8 & 0 & -0.2 \\ 2.6 & 3.6 & -0.5 & -0.2 & 2.3 & -0.2 & 0 \end{bmatrix} \tag{10}$$

For entries from the saving matrix that are negative are considered zero. The next step is to sort the saving matrix pairs (i, j) with the largest to the smallest value. The largest value, 17.7, is for entries (4, 6), and the smallest value, 0.4, is for entries (3,1). From arc (4, 6), the demand from customer 4 and customer 6 is calculated on the condition that it does not exceed the vehicle capacity. The calculation results of the RCVRPTW model were completed using GAMS software and also resolved with the Clarke and Wright algorithm as Table 5 and Table 6

**TABLE 5.** Route vehicle

| | **Route** | | |
|---|---|---|---|
| **GAMS** | 1-2-6-1 | 1-3-8-1 | 1-4-5-7-1 |
| **Clarke and Wright Algorithm** | 1-3-8-6-4-5-7-1 | 1-2-1 | - |

The RCVRPTW model completed using GAMS with the CPLEX solution obtained three sub-routes of the gallon water distribution vehicle as shown in Table 5. The RCVRPTW model completed by the Clarke and Wright algorithm resulted in two sub-routes of the gallon water distribution vehicle.

**TABLE 6.** Comparison of cost optimization with GAMS and the Clarke and Wright Algorithm

| | | | |
|---|---|---|---|
| **GAMS** | 373775.6 | 666052.6 | 1862738 |
| **Clarke and Wright Algorithm** | 1949373 | 280443.2 | - |

As seen in Table 6, the total optimal cost obtained by the Clarke and Wright algorithm is smaller than that obtained by GAMS. The optimal cost for the gallon water distribution problem is obtained by using GAMS and the Clarke and Wright algorithm approach based on the RCVRPTW model considering the limitations of vehicle capacity, service time and travel time.

## CONCLUSION

As seen in Table 6 that the optimal cost obtained by GAMS is more significant than that obtained by the Clarke and Wright algorithm. This paper presents a robust capacitated vehicle routing problem with the time windows model to optimize operational costs based on gallon water distribution problems. GAMS complete the calculation of this model. The optimal total value of operational costs is IDR. 2902566. The robust capacitated vehicle routing problem with the time windows model satisfies the feasible area if the number of requests from customers does not exceed the vehicle capacity and meets the extreme areas of the time. The results of the calculations are compared with the results of calculations with the Clarke and Wright algorithm; the total value of the optimal cost is IDR. 6983621. The results show that GAMS can effectively solve optimization problems in the robust capacitated vehicle routing problem with the time windows model.

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