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Validate Proof of Information Service Financing Scheme Model by Using the Customer Self-selection Bundling Strategy Based on Quasi-linear Utility Function

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Abstract. This study aims to validate the customer self-selection model in the information service financing scheme based on the level of high end-low-end heterogeneous consumer satisfaction and high-low-end heterogeneous demand. The initial model was developed by utilizing an information service financing scheme with monitoring costs and marginal costs for perfect substitute and quasi-linear utility functions to obtain more optimal results. This research was completed by modeling the Mixed Integer Nonlinear Programming (MINLP) problem to determine the type of consumer. The data used is obtained from the local traffic server on the LPSE application which is divided into busy hours (09.00 AM-04.59 PM) and off-peak hours (05.00 PM-08.59 AM) Indonesian Time. This model is solved by using the propose lemmas to get the optimal solution. The results of the analysis that are obtained from the information service financing scheme based on the Quasi-Linear utility function will be compared with the analysis results from the information service financing scheme based on the Quasi-Linear utility function which utilizes the bundling scheme in order to obtain more optimal profits.

INTRODUCTION

This study aims to design a model of information service financing scheme by utilizing a bundling strategy [1,2] based on customer self-selection [3] which is called customized bundling strategy [4,5] for giving one price at a time when bundle is offered, combined with a market model for service providers [6,7] based on high-end-low-end heterogeneous consumers and high-demand-and low-demand heterogeneous consumers [8] with a quasi-linear utility function [9,10] at three financing schemes. The models will be validated by using local server data in Palembang and using lemma assistance to obtain optimal results. Previous research [9,11,12] also discussed internet financing using utility functions, namely the Cobb-Douglas [13] utility function, quasi-linear [14] utility function and perfect substitute [9] utility function where a utility function used is only the ISP's benchmark [15,16] in customer satisfaction. But that research is lack of information about involving customer self-selection. It only involved strategies setting up by service provider in joining networks and focusing only the advantages for service provider when each time the strategies deployed. In a new model for information service retrieval taking into account the appropriate function needs to be studied more deeply, so that by adopting the utility function it has been proven that it generates great advantages for ISPs to adopt the scheme at the available price. That disadvantage of past research creates research gap that need to be explored more possibility to concern with customer self-selection and consumer information [17,18].

This study discusses heterogeneous high-end-low-end and high-demand-low-end consumers. With high-end heterogeneous consumers, namely consumers with high enough needs and able to pay very high prices to obtain internet services of the best quality and very good network traffic, and conversely, low-end heterogeneous consumers, namely consumers who are able to pay relatively low prices but of sufficient quality to meet their needs. In fact, ISPs are faced with the problem of determining the right model to generate profits on the level of customer satisfaction, as well as attracting consumers' attention [19,20] which is quite high on interest in bundling packages and reducing unwanted losses. So that the ISP can pay attention to the utility function in order to get the maximum benefit for the provider internet service. This study discusses the utility function used to measure consumer satisfaction, namely the quasi-linear utility function. In fact, consumers with quasi-linear utility functions are monotonous and weakly convex but not convex integrated [9,21].

The quasi-linear utility function is one of the utility functions that is most widely applied to the problem of financing information services. This function is solved numerically by modeling the Mixed Integer Nonlinear Programming (MINLP) problem [7,22]. Although each consumer himself has a variety of different needs, desires and buying abilities, therefore there are consumers who are divided into groups based on consumer differences, namely homogeneous consumers, high end and low-end heterogeneous consumers, and high demand and heterogeneous consumers. low demand [23,24]. Based on the quasi-linear utility function, the validation of the model is conducted by completing the model using local server data during peak hours (09.00 AM-4.50 PM) and off-peak hours (05.00 PM– 08.59 AM) in Indonesian Time.

Then, the contribution of this research is to design the analytic model [25,26] solved differentially that will be fit into approach of customer self-selection. This new model will be validated by utilizing real data retrieved from local server data. The model designed through lemmas in different customers perspective, to seek for the ability that customers are able to themselves select the program in joining network.

METHOD

The steps taken in this study are as follows.

1. Validating the information service financing scheme model for heterogeneous customer problems based on a quasi-linear utility function with flat-fee, usage-based, and two-part tariff financing types. For flat-fee financing schemes, $Q_s = 0$, and Q is positive $Q_t = 0$ where Q_s and Q_t define the price in busy hour and non-busy hour, respectively. Q explain the price for subscribing the program.
 - a. For usage-based financing schemes, and positive and $Q = 0$, $Q_s > 0$, $Q_t > 0$
 - b. For a two-part tariff financing scheme, if, $Q > 0$, $Q_s > 0$, $Q_t > 0$.
2. Validating a quasi-linear utility function based on three types of financing schemes for heterogeneous consumer types: flat-fee, usage-based, and two-part tariffs, with the addition of marginal expenses and supervision costs.
3. Processing the data from a local server, namely traffic data on the ipse application.
4. Applying the optimal financing scheme on the data server local, namely traffic data.
5. Comparing models of financing schemes obtained from analysis in Step 5 so that the optimal financing scheme is obtained for each type of consumer, namely high-end and low-end heterogeneous consumers and high-demand and low-demand heterogeneous consumers.
6. Concluding and get the best information service financing solutions ad suggesting for further direction of the research.

RESULT AND DISCUSSION

The analysis of utility functions based on quasi-linearity has financing schemes for consumers who are heterogeneous (high-end and low-end) as well as consumers who are heterogeneous (high-end and low-end) (high-demand and low-demand). There are three types of financing systems in this study: flat-fee, usage-based, and two-part tariff. With marginal cost and control cost, this model is employed and applied. For IC (Incentive compatibility) we will take:

$$IC = x^a(y^a) - q(y^a) \geq x^a(y^a) - q(y^a) \text{ with } \forall_{a,b} \neq a$$

So we can take it as an example of the type of consumer \hat{a} .

Utility Functions Based on Quasi-Linear

The general form of a quasi-linear utility function is as follows:

$$U(S, T) = iS + g(T); g(T) = T^j$$

Quasi-Linear Utility Functions in High-End and Low-End Heterogeneous Consumers

For example, there are high-end heterogeneous consumers (upper class) with ($i = 1$) and low-end consumers (lower class) with ($j = 2$).

$$\hat{\alpha}_1 + \hat{\alpha}_2 = 1, \hat{\alpha} = 1, 2.$$

For financing schemes on flat-fee

In Optimization of Consumer Problems:

$$\text{Max}_{S_a, T_a, U_a} Um = i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a - (S_a + T_a) x \quad (1)$$

with constraints:

$$\begin{aligned} S_a &\leq \bar{S} U_a \\ T_a &\leq \bar{T} U_a \\ i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a - (S_a + T_a) x &\geq 0 \\ U_a &= 0 \text{ or } 1 \end{aligned} \quad (2)$$

Manufacturer Problem Optimization:

$$\begin{aligned} \text{Max}_{Q_s, Q_t} \hat{\alpha}_1 (Q_s S_1^* + Q_t T_1^* + Q U_1^*) + \hat{\alpha}_2 (Q_s S_2^* + Q_t T_2^* + Q T_2^*) \\ \text{with}(S_a^*, T_a^*, U_a^*) = \text{argmax } i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a \end{aligned}$$

with constraints:

$$\begin{aligned} S_a &\leq \bar{S} U_a \\ T_a &\leq \bar{T} U_a \\ i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a &\geq 0 \\ U_a &= 0 \text{ atau } 1 \end{aligned}$$

For usage-based and two-part tariff financing schemes

Consumer Problem Optimization:

$$\text{Max}_{S_a, T_a, U_a} Um = i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a - (x + r) S_a - (x + r) T_a \quad (3)$$

with constraints:

$$\begin{aligned} S_a &\leq \bar{S} U_a \\ T_a &\leq \bar{T} U_a \\ i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a - (S_a + T_a) x &\geq 0 \\ U_a &= 0 \text{ or } 1 \end{aligned} \quad (4)$$

Manufacturer Problem Optimization:

$$\begin{aligned} \text{Max}_{Q_s, Q_t} \hat{\alpha}_1 (Q_s S_1^* + Q_t T_1^* + Q U_1^*) + \hat{\alpha}_2 (Q_s S_2^* + Q_t T_2^* + Q T_2^*) \\ \text{with}(S_a^*, T_a^*, U_a^*) = \text{argmax } i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a \end{aligned}$$

with constraints:

$$\begin{aligned} S_a &\leq \bar{S} U_a \\ T_a &\leq \bar{T} U_a \\ i_a S_a + g(T_a) - Q_s S_a - Q_t T_a - Q U_a &\geq 0 \\ U_a &= 0 \text{ or } 1 \end{aligned}$$

Case 1. If the ISP charges a flat rate for service, then $Q_s = 0, Q_t = 0, Q > 0$ where the price that will be used by the ISP will not affect the usage time of busy or non-busy hours, so consumers will choose the maximum consumption level of $S_1 = \bar{S}, S_2 = \bar{S}, T_1 = \bar{T}, \text{ dan } T_2 = \bar{T}$. Then, for each high-end consumer will be charged no more than $i_1 \bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$ and a low-end consumer no more than $i_2 \bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$. Problem 1 is a flat-fee financing scheme so that it will be equalized for both types with heterogeneous consumers. If it is determined as $i_1 > i_2$, then the cost determination for high-end consumers will follow the cost price of low-end consumers. So that, $i_1 (\hat{\alpha}_1) < i_2 (\hat{\alpha}_1 + \hat{\alpha}_2) \Leftrightarrow i_1 < \frac{i_2 (\hat{\alpha}_1 + \hat{\alpha}_2)}{\hat{\alpha}_1}$.

So if consumers are charged a fee of $i_1 \bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$, only high-end consumers can participate in the service.

If consumers are charged a fee of $i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$, then both types of consumers can participate in the service, namely high-end consumers and low-end consumers. In order to maximize profits, the ISP charges a fee of $i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$. In this Problem for optimization of the producer problem will be

$$\begin{aligned} \text{Max}_Q \hat{\alpha}_1(QU_1^*) + \hat{\alpha}_2(QU_2^*) &= \hat{\alpha}_1\{i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x\} + \hat{\alpha}_2\{i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x\} \\ &= (\hat{\alpha}_1 + \hat{\alpha}_2)[i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x] \end{aligned}$$

The maximum profit earned by the manufacturer is:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) [i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x]$$

The following lemma is derived from this problem.

Lemma 1:

If the ISP uses a flat-fee financing scheme, the prices that will be charged are as follows $i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x$ with the maximum profit obtained is:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) [i_2\bar{S} + g(\bar{T}) - (\bar{S} + \bar{T})x]$$

Case 2. If the ISP uses a usage-based financing scheme then $Q_x > 0, Q_y > 0, Q = 0$ is set, and so the optimization problem for high-end heterogeneous consumers:

The consumer problem optimization function becomes:

$$\text{Max}_{S,T,U} Um = i_1S_1 + g(T_1) - Q_sS_1 - Q_tT_1 - (x+r)S_1 - (x+r)T_1 \quad (5)$$

In order to maximize the Objective Function (4.5) on high-end heterogeneous consumers, differentiation is made against and with the following conditions and $S_1T_1 \frac{\partial Um}{\partial S_1} = 0 \frac{\partial Um}{\partial T_1} = 0$

$$\Leftrightarrow \frac{\partial(i_1S_1 + g(T_1) - Q_sS_1 - Q_tT_1 - (x+r)S_1 - (x+r)T_1)}{\partial S_1} = 0$$

$$\Leftrightarrow i_1 - (x+r) = Q_s \quad (6)$$

$$\Leftrightarrow S_1^* = \bar{S}$$

and

$$\Leftrightarrow \frac{\partial(i_1S_1 + g(T_1) - Q_sS_1 - Q_tT_1 - (x+r)S_1 - (x+r)T_1)}{\partial T_1} = 0$$

$$\Leftrightarrow g'(T_1) - (x+r) = Q_t \quad (7)$$

$$\Leftrightarrow T_1^* = \bar{T}$$

Next, optimization problem for low-end heterogeneous consumers:

Functions on consumer problem optimization:

$$\text{Max}_{S,T,U} Um = i_2S_2 + g(T_2) - Q_sS_2 - Q_tT_2 - (x+r)S_2 - (x+r)T_2 \quad (8)$$

In order to maximize the Objective Function (8) on low-end heterogeneous consumers, differentiation is made against and with the following conditions:

$$\text{and } \frac{\partial Um}{\partial S_2} = 0 \frac{\partial Um}{\partial T_2} = 0$$

$$\Leftrightarrow \frac{\partial(i_2S_2 + g(T_2) - Q_sS_2 - Q_tT_2 - (x+r)S_2 - (x+r)T_2)}{\partial S_2} = 0$$

$$\Leftrightarrow i_2 - (x+r) = Q_s \quad (9)$$

$$\Leftrightarrow S_2^* = \bar{S}$$

and

$$\Leftrightarrow \frac{\partial(i_2S_2 + g(T_2) - Q_sS_2 - Q_tT_2 - (x+r)S_2 - (x+r)T_2)}{\partial T_2}$$

$$\Leftrightarrow g'(T_2) - (x+r) = Q_t \quad (10)$$

$$\Leftrightarrow T_2^* = \bar{T}$$

Manufacturer problem optimization:

$$\begin{aligned} \text{Max}_{Q_s, Q_t} \hat{\alpha}_1(Q_sS_1^* + Q_tT_1^*) + \hat{\alpha}_2(Q_sS_2^* + Q_tT_2^*) &= \hat{\alpha}_1[Q_s(\bar{S}) + Q_t(\bar{T})] + \hat{\alpha}_2[Q_s(\bar{S}) + Q_t(\bar{T})] \\ &= \hat{\alpha}_1[(i_1 - (x+r))\bar{S} + (g'(\bar{T}) - (x+r))\bar{T}] + \hat{\alpha}_2[(i_2 - (x+r))\bar{S} + (g'(\bar{T}) - (x+r))\bar{T}] \\ &= \hat{\alpha}_1[i_1\bar{S} - (x+r)\bar{S} + \bar{T}g'(\bar{T}) - (x+r)\bar{T}] + \hat{\alpha}_2[i_2\bar{S} - (x+r)\bar{S} + \bar{T}g'(\bar{T}) - (x+r)\bar{T}] \\ &= \hat{\alpha}_1[i_1\bar{S} + \bar{T}g'(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}] + \hat{\alpha}_2[i_2\bar{S} + \bar{T}g'(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}] \end{aligned}$$

If it is used to solve the problem of maximizing functionality during rush hour, the ISP must minimize the price Q_s , and so the best price Q_s cannot be larger than $i_1 - (x + r)$. If the ISP sets the price below $i_2 - (x + r)$ then the profit is not optimal. If it is used to solve the problem during non-peak hours, the best pricing is $Q_t \leq \bar{T}g'(\bar{T}_1) - (x + r)$. On the other hand, if the ISP sets the price below, then the profit will not be optimal when $T_1^* \leq \bar{T}$ and $T_2^* \leq \bar{T}$. Therefore, the best price Q_t is $g'(T_2) - (x + r) \leq Q_t \leq g'(T_1) - (x + r)$. If prices are in this interval, demand from high-end consumers will remain at and, while demand from low-end consumers continues to increase as prices fall. Thus, the optimal price given for peak hours is

$$Q_s = i_2 - (x + r)$$

and the optimal price in off-peak hours is

$$Q_t = g'(\bar{T}) - (x + r)$$

so the maximum profit is:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) (i_2 \bar{S} + \bar{T}g'(\bar{T}) - (x + r)\bar{S} - (x + r)\bar{T})$$

Based on this Problem, the following lemma is obtained.

Lemma 2:

If the ISP uses usage-based pricing, then the optimal price given for peak hours is $Q_s = i_2 - (x + r)$ and the optimal price for non-peak hours is $Q_t = g'(\bar{T}) - (x + r)$ and the maximum profit will be

$$(\hat{\alpha}_1 + \hat{\alpha}_2) (i_2 \bar{S} + \bar{T}g'(\bar{T}) - (x + r)\bar{S} - (x + r)\bar{T}).$$

Case 3. If the ISP uses a two-part tariff financing scheme, then $Q_s > 0, Q_t > 0, Q > 0$. There are costs incurred if consumers choose to join the service as well as prices charged during peak and off-peak hours. First-order conditions for the optimization equation on the problem of high-end consumers and low-end consumers.

Eq. (6) and (9) are high-end and low-end demand curves in peak hours, Eq. (7) and (10) are high-end and low-end consumer demand curves in off-peak hours. If $i_1 > i_2$ is set, it can be assumed that

$$i_1(\hat{\alpha}_1) < i_2(\hat{\alpha}_1 + \hat{\alpha}_2) \Leftrightarrow i_1 < \frac{i_2(\hat{\alpha}_1 + \hat{\alpha}_2)}{\hat{\alpha}_1}$$

This means that if the consumer is charged a fee of

$$Q_s = i_1 - (x + r)$$

and

$$Q_t = g(T_1) - (x + r)$$

and

$$Q = i_1 S - i_1 \bar{S} + g(T) - \bar{T}g'(\bar{T}) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r) - S(x + r)T$$

then only high-end consumers can take part in this service. If the consumer is charged a fee of

$$Q_s = i_2 - (x + r)$$

and

$$Q_t = g'(\bar{T}) - (x + r)$$

and

$$Q = i_2 S - i_2 \bar{S} + g(T) - \bar{T}g'(T) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T$$

As a result, both high-end and low-end consumers can benefit from the service. ISPs can also choose to reduce expenses because many consumers find membership fees to be a barrier. In order to attract more customers, ISPs might offer pricing that are lower.

$$Q_s = i_2 - (x + r), Q_T = g'(T_2) - (x + r),$$

and minimize subscription fees

$$Q = i_2 S - i_2 \bar{S} + g(T) - \bar{T}g'(T) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T.$$

Optimization of the problem on the manufacturer to be:

$$\begin{aligned} & \text{Max}_{Q_s, Q_t} \hat{\alpha}_1 (Q_s S_1^* + Q_t T_1^* + Q U_1^*) + \hat{\alpha}_2 (Q_s S_2^* + Q_t T_2^* + Q U_2^*) \\ & = \hat{\alpha}_1 [(i_2 - (x + r))\bar{S} + (g'(\bar{T}) - (x + r)\bar{T}) + (i_2 S - i_2 \bar{S} + g(T) - \bar{S}g'(\bar{T}) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T)] \\ & + \hat{\alpha}_2 [(i_2 - (x + r))\bar{S} + (g'(\bar{T}) - (x + r)\bar{T}) + (i_2 S - i_2 \bar{S} + g(T) - \bar{T}g'(\bar{T}) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T)] \\ & = \hat{\alpha}_1 [i_2 \bar{S} - (x + r)\bar{S} + g'(\bar{T})\bar{T} - (x + r)\bar{T}] \\ & + \{ (i_2 S - i_2 \bar{S} + g(T) - \bar{T}f'(\bar{T}) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T) \} \\ & + \hat{\alpha}_2 [(i_2 - (x + r))\bar{S} + g'(\bar{T})\bar{T} - (x + r)\bar{T}] \\ & + \{ (i_2 S - i_2 \bar{S} + g(T) - \bar{T}g'(\bar{T}) + (x + r)\bar{S} + (x + r)\bar{T} - (x + r)S - (x + r)T) \} \end{aligned}$$

$$\begin{aligned}
&= \hat{\alpha}_1 [i_2 \bar{S} + g(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}] \\
&+ \hat{\alpha}_2 [(i_2 + g(\bar{T}) - (x+r))\bar{S} - (x+r)\bar{T}] \\
&= (\hat{\alpha}_1 + \hat{\alpha}_2) [i_2 \bar{S} + g(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}]
\end{aligned}$$

Thus the maximum profit to be achieved is:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) [i_2 \bar{S} + g(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}]$$

Based on this Case 3, the following lemma is obtained.

Lemma 3 :

If the ISP uses a two-part tariff, Q_s and Q_t optimal will be

$$Q_s = i_2 - (x+r)$$

And

$$Q_t = g'(\bar{T}) - (x+r)$$

and

$$Q = i_2 X - i_2 \bar{X} + g(T) - \bar{T} g'(\bar{T}) + (x+r)\bar{S} + (x+r)\bar{T} - (x+r)S - (x+r)T$$

with the maximum profit obtained, namely:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) [i_2 \bar{S} + g(\bar{T}) - (x+r)\bar{S} - (x+r)\bar{T}]$$

Quasi-Linear Utility Functions in High-Demand Heterogeneous Consumers and Low-Demand Heterogeneous Consumers

Consumers with high use levels (type 1) with maximum consumption levels \bar{S}_1 and \bar{T}_1 and consumers with low usage levels (type 2) with maximum consumption levels \bar{S}_2 and \bar{T}_2 are presumed to be two types of consumers. With $i_1 = i_2 = i, j_1 = j_2 = j$ there are two types of consumers: $\hat{\alpha}_1$ (type 1) and $\hat{\alpha}_2$ (type 2) Calculation of the maximum profit on each of the ISP's financing schemes.

Case 4. If the ISP uses a financing scheme on a flat-fee then $Q_s = 0, Q_t = 0, Q > 0$ is set which means, if a consumer (a high-demand or low-demand consumer) chooses to join the service then that consumer fully utilizes the service by choosing a consumption level $S_1 = \bar{S}_1, T_1 = \bar{T}_1$ or $S_2 = \bar{S}_2, T_2 = \bar{T}_2$ with a maximum rate $i\bar{S}_1 + g(\bar{T}_1) - (\bar{S}_1 + \bar{T}_1)x$ or $i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x$ (for high-demand and low-demand consumers). Thus, the ISP can provide a price for each high-demand consumer no more than $i\bar{S}_1 + g(\bar{T}_1) - (\bar{S}_1 + \bar{T}_1)x$ and no more than $i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x$ a low-demand consumer. If the ISP cannot distinguish between high-demand consumers and low-demand consumers, the ISP must provide the same price for both consumers, so that the ISP can set prices $i\bar{S}_1 + g(\bar{T}_1) - (\bar{S}_1 + \bar{T}_1)x$ by only serving high-level consumers or setting prices $i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x$ by serving both types of consumers, namely high-demand and low-demand consumers. If it is assumed that

$$\hat{\alpha}_1 [i\bar{S}_1 + g(\bar{T}_1) - (\bar{S}_1 + \bar{T}_1)x] < (\hat{\alpha}_1 + \hat{\alpha}_2) [i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x]$$

the best price set by the ISP for the service is

$$i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x$$

and serve both types to these consumers with the maximum profit obtained is $(\hat{\alpha}_1 + \hat{\alpha}_2) [i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x]$

Based on this Problem, the following lemma is obtained.

Lemma 4:

If the ISP uses a flat-fee financing scheme, the prices charged will be $Q = i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x$ with the maximum profit achieved are $(\hat{\alpha}_1 + \hat{\alpha}_2) [i\bar{S}_2 + g(\bar{T}_2) - (\bar{S}_2 + \bar{T}_2)x]$

Case 5. If the ISP uses a usage-based financing scheme, $Q_s > 0, Q_t > 0, Q = 0$ is set, and which means that the ISP provides different prices, namely prices during peak hours and off-peak hours. For optimization of high and low usage level consumer problems result in:

High-demand heterogeneous consumer optimization problem:

$$\text{Max}_{S,T,U} Um = iS_1 + g(T_1) - Q_s S_1 - Q_t T_1 - (x+r)S_1 - (x+r)T_1 \quad (11)$$

In order to maximize the Objective Function (11) on heterogeneous high-demand, it is necessary to differentiate towards S_1 and T_1 with the necessary and sufficient conditions i.e. $\frac{\partial Um}{\partial S_1} = 0, \frac{\partial Um}{\partial T_1} = 0$

$$\Leftrightarrow \frac{\partial (iS_1 + g(T_1) - Q_s S_1 - Q_t T_1 - (x+r)S_1 - (x+r)T_1)}{\partial S_1} = 0$$

$$\Leftrightarrow i - (x+r) = Q_s \quad (12)$$

$$\Leftrightarrow S_1^* = \bar{S}_1$$

and

$$\Leftrightarrow \frac{\partial(iS_1 + g(T_1) - Q_s S_1 - Q_t T_1 - (x+r)S_1 - (x+r)T_1)}{\partial T_1} = 0$$

$$\Leftrightarrow g'(T_1) - (x+r) = Q_t$$

$$\Leftrightarrow T_1^* = \bar{T}_1$$
(13)

For low-demand heterogeneous consumers:

$$\text{Max}_{S,T,U} Um = iS_2 + g(T_2) - Q_s S_2 - Q_t T_2 - (x+r)S_2 - (x+r)T_2$$
(14)

In order to maximize the optimization function of the low-demand heterogeneous consumer problem, differentiation is carried out against S_2 and T_2 and with the conditions

$$\frac{\partial Um}{\partial S_2} = 0 \text{ and } \frac{\partial Um}{\partial T_2} = 0$$

$$\Leftrightarrow \frac{\partial(iS_2 + g(T_2) - Q_s S_2 - Q_t T_2 - (x+r)S_2 - (x+r)T_2)}{\partial S_2} = 0$$

$$\Leftrightarrow i - (x+r) = Q_s$$
(15)

$$\Leftrightarrow S_2^* = \bar{S}_2$$

and

$$\Leftrightarrow \frac{\partial(iS_2 + g(T_2) - Q_s S_2 - Q_t T_2 - (x+r)S_2 - (x+r)T_2)}{\partial T_2} = 0$$

$$\Leftrightarrow g'(T_2) - (x+r) = Q_t \Leftrightarrow T_2^* = \bar{T}_2$$
(16)

ISP optimization problems will be:

$$\text{Max}_{Q_s, Q_t} \hat{\alpha}_1(Q_s S_1^* + Q_t T_1^*) + \hat{\alpha}_2(Q_s S_2^* + Q_t T_2^*) = \hat{\alpha}_1[Q_s(\bar{S}_1) + Q_t(\bar{T}_1)] + \hat{\alpha}_2[Q_s(\bar{S}_2) + Q_t(\bar{T}_2)]$$

From Eq.(12)-(15) it is known that during Q_s and Q_t decreases, S_1^* , S_2^* , T_1^* and T_2^* increases. Since S_1 , S_2 , T_1 and T_2 are limited to \bar{S}_1 , \bar{S}_2 , \bar{T}_1 , and \bar{T}_2 to maximize this equation, the ISP must minimize Q_s, Q_t and so that Q_s the best price is $Q_s \leq i - (x+r)$

and

$$g'(\bar{T}_2) - (x+r) \leq Q_t \leq g'(\bar{T}_1) - (x+r).$$

On the other hand, if the ISP sets the price $Q_s < i - (x+r)$ then the profit is not optimal because $S_1^* \leq \bar{S}_1, S_2^* \leq \bar{S}_2$. Therefore, the price

$$Q_s = i - (x+r)$$

and

$$Q_t = g'(\bar{T}_2) - (x+r).$$

When the price is in the interval, the demand for low-demand consumers remains constant at \bar{S}_2 while demand from high-demand consumers continues to increase as long as prices fall, so that the maximum profit will be achieved:

$$\begin{aligned} & \hat{\alpha}_1[Q_s(\bar{S}_1) + Q_t(\bar{T}_1)] + \hat{\alpha}_2[Q_s(\bar{S}_2) + Q_t(\bar{T}_2)] \\ &= \hat{\alpha}_1[(i - (x+r))\bar{S}_1 + (g'(\bar{T}_2) - (x+r))\bar{T}_1] + \hat{\alpha}_2[(i - (x+r))\bar{S}_2 + (g'(\bar{T}_2) - (x+r))\bar{T}_2] \\ &= \hat{\alpha}_1[i\bar{S}_1 - (x+r)\bar{S}_1 + \bar{T}_1 g'(\bar{T}_1) - (x+r)\bar{T}_1] + \hat{\alpha}_2[i\bar{S}_2 - (x+r)\bar{S}_2 + \bar{T}_2 g'(\bar{T}_2) - (x+r)\bar{T}_2] \\ &= \hat{\alpha}_1[i\bar{S}_1 + \bar{T}_1 g'(\bar{T}_1) - (x+r)\bar{S}_1 - (x+r)\bar{T}_1] + \hat{\alpha}_2[i\bar{S}_2 + \bar{T}_2 g'(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2] \end{aligned}$$

Based on this Problem, the following lemma is obtained.

Lemma 5 :

If the ISP uses a usage-based financing scheme, then the optimal price during peak hours is $Q_s = i - (x+r)$ and the optimal price during off-peak hours is $Q_t = g'(\bar{T}_2) - (x+r)$ and the maximum profit is

$$(\hat{\alpha}_1 + \hat{\alpha}_2) [i\bar{S}_2 + \bar{T}_2 g'(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2]$$

Case 6. If the ISP uses a two-part tariff financing scheme, then $Q_s > 0, Q_t > 0, Q > 0$, which means there is a subscription fee if the consumer chooses the service and the price charged during peak hours and off-peak hours. The first-order conditions for optimizing high-demand and low-demand consumer problems use Eq. (12- (16).

The cost of the best service program Q is that it can be assigned to the consumer at a fee charged to low-demand consumers. If the service fee is more than the set price, it will cause the ISP to lose all high-demand consumers, which means it is assumed that it is more profitable to use and $.Q_s = i - (x+r)Q_t = g'(T_2) - (x+r)$

By using Constraint (4), it is obtained as follows:

$$\begin{aligned}
& iS_2 + g(T_2) - Q_s S_2 - Q_t T_2 - QU_2 - (x+r)S_2 - (X+r)T_2 \geq 0 \\
& \Leftrightarrow iS_2 + g(T_2) - (i - (x+r))S_2 - (g'(T_2) - (x+r))T_2 - Q - (x+r)S_2 - (x+r)T_2 \geq 0 \\
& \Leftrightarrow iS_2 + g(T_2) - iS_2 + (x+r)S_2 - g'(T_2)T_2 + (x+r)T_2 - Q - (x+r)S_2 - (x+r)T_2 \geq 0 \\
& \Leftrightarrow Q \leq iS_2 + g(T_2) - iS_2 - g'(T_2)T_2 + (x+r)S_2 + (x+r)T_2 - (x+r)S_2 - (x+r)T_2 \\
& \text{because } S_1^* \leq \bar{S}_1, S_2^* \leq \bar{S}_2, T_1^* \leq \bar{T}_1, T_2^* \leq \bar{T}_2, \text{ then we get:}
\end{aligned}$$

$$Q_s = i - (x+r), Q_t = g'(\bar{T}_2) - (x+r) \text{ and } Q \leq g(\bar{T}_2) - g'(\bar{T}_2)\bar{T}_2$$

Optimization of the producer problem becomes:

$$\begin{aligned}
& \text{Max}_{Q_s, Q_t} \hat{\alpha}_1(Q_s S_1^* + Q_t T_1^* + QU_1^*) + \hat{\alpha}_2(Q_s S_2^* + Q_t T_2^* + QU_2^*) \\
& = \hat{\alpha}_1((i - (x+r))S_1^* + (g'(\bar{T}_2) - (x+r))T_1^* + Q) + \hat{\alpha}_2((i - (x+r))S_2^* + (g'(\bar{T}_2) - (x+r))T_2^* + Q) \\
& = \hat{\alpha}_1[i\bar{S}_1 - (x+r)\bar{S}_1 + g'(\bar{T}_2)\bar{T}_1 - (x+r)\bar{T}_1 + \{g(\bar{T}_2) - g'(\bar{T}_2)\bar{T}_2\}] + \hat{\alpha}_2 [i\bar{S}_2 - (x+r)\bar{S}_2 + g'(\bar{T}_2)\bar{T}_2 - \\
& (x+r)\bar{T}_2 + \{g(\bar{T}_2) - g'(\bar{T}_2)\bar{T}_2\}] \\
& = \hat{\alpha}_1[i\bar{S}_1 + g'(\bar{T}_2)\bar{T}_1 + g(\bar{T}_2) - g'(\bar{T}_2)\bar{T}_2 - (x+r)\bar{S}_1 - (x+r)\bar{T}_1] + \hat{\alpha}_2 [i\bar{S}_2 + g(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2] \\
& = \hat{\alpha}_1[i\bar{S}_1 + g'(\bar{T}_2)(\bar{T}_1 - \bar{T}_2) + g(\bar{T}_2)] + \hat{\alpha}_2 [i\bar{S}_2 + g(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2]
\end{aligned}$$

If the ISP uses a two-part tariff financing scheme by setting Q_s , Q_t , and Q , then the ISP can set the optimal price at $Q_s = i - (x+r)$ and $Q_t = g'(\bar{T}_2) - (x+r)$. The subscription fee is equal to the consumer surplus of the low-level consumer and the maximum profit is: $Q_s > 0, Q_t > 0, Q > 0$

$$Q_s = i - (x+r), Q_t = g'(\bar{T}_2) - (x+r), Q > 0$$

$$\hat{\alpha}_1 [i\bar{S}_1 + g'(\bar{T}_2)(\bar{T}_1 - \bar{T}_2) + g(\bar{T}_2)] + \hat{\alpha}_2 [i\bar{S}_2 + g(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2]$$

Based on this case 6, the following lemma is obtained.

Lemma 6 :

If the ISP uses the price on a two-part tariff, then the optimal price is $Q_s = i - (x+r)$ and $Q_t = g'(\bar{T}_2) - (x+r)$ and $Q = g(\bar{T}_2) - g'(\bar{T}_2)\bar{T}_2$, the maximum profit achieved by the ISP is:

$$\hat{\alpha}_1 [i\bar{S}_1 + g'(\bar{T}_2)(\bar{T}_1 - \bar{T}_2) + g(\bar{T}_2)] + \hat{\alpha}_2 [i\bar{S}_2 + g(\bar{T}_2) - (x+r)\bar{S}_2 - (x+r)\bar{T}_2]$$

In the optimal financing scheme, it will be processed using traffic data on the ipse application obtained from local server observations in Palembang within 1 month (30 days), namely from April 19, 2021 to May 18, 2021 divided into peak hours (09.00 AM - 16.59 PM) and off-peak hours (17.00 PM - 08.59 AM), Indonesian Time.

Traffic Data Processing

Traffic data is used in the optimal model. The optimal model is processed using traffic data from the ipse application so as to obtain the optimal solution from the service financing scheme model. Using the data in Table 1, the parameter values and values from the ipse application data into the optimal model.

Table 1 Usage data during peak hours and off-peak hours

Consumption Rate	Service Usage (kbps)
$\bar{S} = \bar{S}_1$	0.0120275391
\bar{S}_2	0.0113103516
\bar{S}_{min}	0.0007411094
$\bar{T} = \bar{T}_1$	0.019234375
\bar{T}_2	0.016940023
\bar{T}_{min}	0.0034535156

where

1. $\bar{S} = \bar{S}_1$ is the maximum rate of data transfer during peak hours, measured in kilobytes per second.
2. \bar{S}_2 is the second highest rate of data transfer during peak hours, measured in kilobytes per second.
3. \bar{S}_{min} , in kilobytes per second, this is the lowest amount of usage during peak hours.
4. $\bar{T} = \bar{T}_1$ is the maximum consumption rate in kilobytes per second during off-peak hours.
5. \bar{T}_2 is the second highest amount of consumption in kilobytes per second during off-peak hours.
6. \bar{T}_{min} is the lowest amount of consumption in kilobytes per second during off-peak hours.

Data For Service Usage

The data can be entered into a quasi-linear function model based on flat-fee, usage-based, and two-part tariff financing

schemes using the values in Table 1. For heterogeneous consumers (high-end and low-end) in the quasi-linear utility function, usage-based financing schemes outperform flat-fee and two-part tariff financing schemes, whereas heterogeneous consumers (high-demand and low-end) demand in the two-part tariff financing scheme outperforms usage-based and flat-fee financing schemes.

Optimal Financing Scheme for Heterogeneous Consumers (High-End and Low-End)

Following that, we will go over how to calculate heterogeneous consumers for high-end and low-end consumers using the data in Table 2. The two-part tariff financing system outperforms the usage-based financing scheme and the flat-fee financing scheme in the quasi-linear utility function.

Table 2 Heterogeneous Consumers (High-End and Low-End) with Service Usage Data

Financing Scheme	Quasi-Linear Utility Functions
Flat-fee	$(\hat{\alpha}_1 + \hat{\alpha}_2)(0,0120275391i_2 + (0,019234375)^{j_2} - 0,312610391x)$
Usage-based	$(\hat{\alpha}_1 + \hat{\alpha}_2) \left(0,0120275391i_2 + j_2(0,019234375)^{j_2} - 0,312610391(x + r) \right)$
Two-part tariff	$(\hat{\alpha}_1 + \hat{\alpha}_2) \left(0,0120275391i_2 + (0,019234375)^{j_2} - 0,312610391(x + r) \right)$

Based on the quasi-linear utility function, more optimal benefits are obtained when the ISP chooses to use a usage-based financing scheme, namely:

$$(\hat{\alpha}_1 + \hat{\alpha}_2) \left(0,0120275391i_2 + j_2(0,019234375)^{j_2} - 0,312610391(x + r) \right).$$

The maximum profit obtained is the most optimal in the quasi-linear utility function, namely

$$(\hat{\alpha}_1 + \hat{\alpha}_2) \left(0,0120275391i_2 + j_2(0,019234375)^{j_2} - 0,312610391(x + r) \right).$$

Optimal Financing Scheme for Heterogeneous Consumers (High-Demand and Low-Demand)

Calculations will also be performed for heterogeneous consumers with high and low demand. In this situation, consumers who are in high demand and those who are in low demand, the usage of this service is divided into two, namely data taken from the first highest usage and data taken from the second highest usage as Table 3 shown.

Table 3 Heterogeneous Consumers (High-Demand and Low-Demand) with Service Usage Data

Financing Scheme	Quasi-Linear Utility Functions
Flat-fee	$(\hat{\alpha}_1 + \hat{\alpha}_2)[0,0113103516i + (0,016940023)^j - 0,0282503746x]$
Usage-based	$(\hat{\alpha}_1 + \hat{\alpha}_2) [228,516a + b(343,75)^b - 572,266(c + t)]$
Two-part tariff	$\hat{\alpha}_1[0,0113103516i + j(0,016940023)^{j-1}(0,002294352) + (0,016940023)^j] + \hat{\alpha}_2[0,0113103516i + (0,016940023)^b - 0,0282503746(x + r)]$

In Table 3, based on a quasi-linear utility function, the profits obtained are more optimal when ISPs choose to use a two-part tariff financing scheme, namely:

$$\hat{\alpha}_1[0,0113103516i + j(0,016940023)^{j-1}(0,002294352) + (0,016940023)^j] + \hat{\alpha}_2[0,0113103516i + (0,016940023)^b - 0,0282503746(x + r)]$$

The maximum profit at the most optimal is in the quasi-linear utility function, namely:

$$\hat{\alpha}_1[0,0113103516i + j(0,016940023)^{j-1}(0,002294352) + (0,016940023)^j] + \hat{\alpha}_2[0,0113103516i + (0,016940023)^b - 0,0282503746(x + r)]$$

Based on Tables 2 and 3, the data processing is carried out with the same method stage, only different values are \bar{S} and \bar{T} for each data. Based on these results, the quasi-linear utility function is more optimal. The results of this processing can be seen from the high level of service usage i and j the amount of marginal costs and minimum supervision costs. Marginal cost (x) is determined based on the level of production caused By multiplying the number of production units, the marginal cost decreases as the number of units produced increases. Depending on the sort of

usage-based financing scheme utilized to maximize profit at the best price for a diverse group of customers (high-end and low-end) or based on their willingness to pay high.

The end-user has ability to choose according to the preference decided. If customer decides to choose two-part tariff scheme, then ISP will achieve highest revenue than other schemes.

CONCLUSION

The concluding remarks achieved for this research are that the model produced in this study is the 6 lemmas chosen for customer based on their willingness to pay and their demands. Average total usage traffic per month is 6.3926561667 (kbps) for busy hours while average total usage traffic per month is 11.18770667 (kbps) for off-peak hours. The further study also seeks for possibility to have optimization problem of information service scheme based on the same utility function and customer self-selection to compare the effectiveness for each models designed analytically and as mathematical programming problem.

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