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Local Instructional Theory in Algebra Topic using Commognitive Framework to Support Students' Critical Reasoning

Faculty of Teacher Training and Education
Universitas Sriwijaya

Local Instructional Theory in Algebra Topic using Commogni tive Framework to Support Student's Critical Reasoning

LOCAL INSTRUCTIONAL THEORY IN ALGEBRA TOPIC USING COMMOGNITIVE FRAMEWORK TO SUPPORT STUDENT'S CRITICAL REASONING

TEORI LOKAL INSTRUKSIONAL PADA TOPIK ALJABAR DENGAN COMMOGNITIVE FRAMEWORK UNTUK MENDUKUNG PENALARAN KRITIS SISWA

(dengan ringkasan dalam Bahasa Indonesia)

Disertasi:

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Local Instructional Theory in Algebra Topic using Commognitive Framework to Support Student's Critical Reasoning

DISSERTATION

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Preface

This study highlights the role of communication in teaching and learning using the commognitive framework. Back when I was a student in FISME, Utrecht University. I studied the assignments about how to elaborate on an interesting topic in teaching and learning. Therefore, I ended up describing *"How to Speak Mathematics"*. The complete title of my short essay was *"Encouraging Students to 'Speak' in Sharing Knowledge: Helping Students Develop Their Understanding"*. The paper explained the importance of communicating ideas while learning in the classroom and aimed to answer the question: *Does the discussion talk in a group of students help them understand knowledge?*

In learning mathematics, it is commonly believed that it is better for students to talk less, since mathematical problem-solving often demands focused and reflective thinking. However, this traditional belief neglects the powerful role that structured mathematical discourse can play in enhancing understanding. Engaging students in verbalizing their thought processes allows them to clarify ideas, uncover misunderstandings, and deepen their conceptual grasp. Mathematical communication does not merely involve giving correct answers; rather, it entails explaining the reasoning behind solutions, justifying steps, and responding to others' viewpoints.

Students' reasoning can be significantly improved through their engagement in mathematical talk, where they must articulate, defend, and sometimes revise their understanding in light of peer feedback. For instance, when students explain why a particular graph represents a function, or justify their method of solving an equation, they are not only demonstrating knowledge but actively constructing it. Through such dialogues, students internalize mathematical structures and develop more sophisticated ways of thinking. This aligns closely with the commognitive framework, which posits that thinking and communication are inseparable in mathematics learning—students learn mathematics by participating in its discourse.

From the perspective of the teacher, fostering communication can be achieved in two main ways. First, through orchestrated whole-class discussions, where the teacher facilitates the sharing and comparing of diverse solutions, encouraging students to reason publicly. Second, by forming small groups that allow students to engage in more intimate and collaborative discussions, where they exchange ideas, challenge each other, and build collective understanding. In both formats, the focus is on nurturing a community of inquiry in which students are expected to speak, listen, and respond with mathematical precision and reasoning.

At that time, based on my short reading session, I reviewed a study from Mercer (1996), which documented the discourse of students aged 9–10 years during a 90-minute session on a computer-based problem. Each pair of students was recorded while working together, observing the problem, discussing possible strategies, and clarifying their decisions. The analysis of their conversations revealed how different types of talk—such as exploratory talk, where students justify and reason through ideas—can substantially enhance problem-solving performance and conceptual understanding.

This evidence supports the notion that communication is not a peripheral activity in mathematics but a core part of learning itself. When students are encouraged to "speak mathematics," they are invited into the practices of mathematical reasoning. They become active participants in constructing meaning, rather than passive recipients of procedures. Thus, promoting mathematical communication—whether through guided questioning, structured group work, or whole-class discourse—is essential for developing students' reasoning abilities and fostering a deeper, more connected understanding of algebra.

In real life, I assure you, there is no such thing as algebra. " —

– Fran Lebowitz, *Goodreads Quotes*

Author, Weni Dwi Pratiwi

1

Introduction

1.1 Research Context

The importance of algebra within the context of mathematics education has been a pressing subject for educators and researchers alike. This literature review synthesizes current research findings and theoretical frameworks related to mathematical communication skills, particularly in the realm of algebra, underscoring how these skills are crucial not only for understanding algebraic concepts but also for enhancing overall mathematical proficiency. Algebra is a pivotal aspect of mathematics education, serving as a foundational tool for students' cognitive and communicative development. Understanding the intricacies of learning algebra requires a multi-faceted approach, particularly emphasizing mathematical communication skills. Two topics of algebra discussed in this study is relations and functions and System of Linear Equations of two variables. For the concept of System Linear Equations of Two Variables (SLETV), there are some misconception for instance students may not distinguish when to use substitution vs. elimination, or may switch between methods incorrectly, reflecting a lack of generalized strategy, they also missapply arithmetic rules: Mistakes in combining like terms or maintaining equivalence (e.g., adding to only one side of an equation) suggest gaps in schema formation, and believe that the solution to a system is always where the graphs intersect, but failing to realize what parallel (no intersection) or coincident lines (infinite solutions) mean. Igor' Konan equation) suggest gaps in schema formation, and believe that the solution to a system is always where the graphs intersect, but failing to realize what parallel (no intersection) or lines (infinite solutions) mean. Igor' Kontorovich (2021) suggested that mathematics classroom should improve the learning for this topic to include some of this following intervention: (1) present fully worked examples, gradually remove steps so students must generalize the procedures themselves, (2) Introduce systems with varied structure (e.g., coefficients, solution types) to develop schema for generalization, not rote memorization, (3) Comparison Tasks: Having students compare two different solution methods or systems to build understanding of underlying general principles, (4) Sequencing by Complexity: Begin with systems sharing similar structures, progressively introduce more complexity so students can generalize strategies effectively, (5) Encouraging connections between algebraic, graphical, numerical, and verbal representations to foster transfer and deeper understanding, and (6) Incorporate and focused re-teaching on . The concept of functions discussed in this study is the concept of functions taught in the 8th grade of junior high school. This section discusses the history of the development of the concept of functions from a . The concept of functions is taught not only at the school level but also in college mathematics. 'Recognizing functional relationships between quantities' is 'recognizing functional relationships between quantities', , , and understanding relationships between two variables in , which is seen as supporting students' reasoning in focusing their understanding on and interaction. More specifically, the framework explains that the essential elements of the concept of functions are the domain, where the input is located, and the codomain, where the output is selected from an input, and the process for obtaining the output from the input itself. The concept of functions is a for learning other concepts such as limits, derivatives, and integrals, which are important concepts in calculus. Functions belong to the realm of algebra.

In elementary education, algebra is preceded by arithmetic, which helps students understand algebra better. Warren and T. Cooper, 2005 argues that arithmetic should precede algebra because it provides the foundation for algebra. This leads to a separation of topics in the curriculum, with arithmetic being taught years before algebra. This means that the primary focus in is on operations involving specific numbers (arithmetic) before operations with general quantities, variables, and functions (algebra). Traditionally, students are taught to count discrete objects Dougherty et al., 2015, and as they move through different number systems, algorithms and routines typically change. The choice to focus on the concept of functions was made for two main reasons. First, as mentioned earlier, the concept of functions is important for modern mathematics, unlike, for example, the , which is a subject with extensive discussion and requires a long discussion time. The concept of functions has a brief discussion but is then widely used, making it very interesting to study from a teaching perspective. Second, this concept has been extensively researched in mathematics education (see, for example, Cho, Norris, and Moore-Russo, 2017;**Kontorovich2021PreuniversityDiscourses**;Fonger, Ellis, and Dogan, 2020;Frank and Patrick W. Thompson, 2021 and many others, but mostly from the perspective of student learning. This research provides a background for examining teachers' teaching practices. One aspect of the function concept domain related to this research is algebra and graphical representation. These two have different symbolizations that are articulated in such a way as to ultimately describe and define the concept of a function. Functions and graphs cannot be taught as separate concepts; these two concepts are like a communicative system between two things, and they are also the construction and organization of mathematical ideas (Leinhardt, Stein, and Zaslavsky, 1990). Patrick W Thompson and Carlson (2017) in his study explains that the concept of functions is very problematic because each mathematician has a different concept of functions. This is what then leads to various in understanding functions, not only among high school students but also among college students. One aspect of the domain of function concepts related to this research is algebra and graphical representation. These two aspects have different that are articulated in such a way as to ultimately describe and define the concept of functions. Functions and graphs cannot be taught as separate concepts; these two concepts are like a system between two things, and they are also the construction and organization of mathematical ideas Leinhardt, Stein, and Zaslavsky, 1990. (Berg and Didactician, 2009) describes the results of his study on teachers' conceptions of functions. In his interviews with 152 teachers, the concept of a function was more commonly understood as an equation or formula. One teacher said, "a function is really an equation." In addition to this misconception, several other common misunderstandings in interpreting functions include: (1) students think that the range must be mapped back to the domain with a single value, (2) students misunderstand that the rules set in the domain will affect the relationship between two variables, (3) students rely too much on specific indicators such as the use of the vertical line test to determine whether a relationship is a function or not. This is because in algebra classes, many teachers still teach by merely explaining the material in the textbook and providing examples through bare mathematics tasks, while students only pay attention and take notes Jupri, 2015. Additionally, arrow diagrams, , and are used only as methods, not for understanding. Ball, Ladel, and Siller, 2018.

In fact, it would be better if functions and graphs were taught together, as functions and graphs cannot be taught as separate concepts. These two concepts are like a between two things, and they are also the construction and organization of .

1.2 A Commognitive Framework to Support Cognitive Process in Learning Topic Algebra

Sfard (2020) defined commognitive as a notion of the approach to learning grounded in the assumption that thinking can be regarded as the individualized form of communication. She further explained that thinking are acts of informing, arguing, asking, and answering ourselves. To think mathematically means participating in mathematical discourse. Learning mathematics then becomes the process of individualizing . Consequently, learning mathematics is understood as the process of individualizing mathematical discourse, where learners gradually internalize the ways of speaking, reasoning, and symbolizing that are characteristic of the mathematical community. Sfard (2007) further noted that meaningful learning often occurs when learners encounter commognitive conflict, a situation where their current discourse clashes with a new or unfamiliar one. This conflict prompts a transformation in the way learners think and communicate mathematically, allowing for deeper understanding. Under this commognitive perspective, cognitive processes, including conceptual understanding, reasoning, problem solving, and decision making, are shaped by how individuals communicate about mathematics. Cognitive processes encompass various elements including conceptual understanding, problem-solving, and reasoning, all of which are integral to effective learning and intellectual engagement. Conceptual understanding refers to the deep comprehension of facts and relationships in a given domain, which enables learners to apply knowledge flexibly in problem-solving scenarios. As highlighted by Kurniadi et al., cognitive processes include skills such as reading, , and mathematical communication, enhancing one's capability to manipulate information and apply it to diverse contexts (Kurniadi et al., 2021). This transformation of knowledge through various cognitive activities fosters and supports the development of scientific thought, as noted by Korobova et al., emphasizing the role of research activities in nurturing students' independence and cognitive innovation (et al., 2018). The quality and structure of thus directly influence how students understand concepts, justify procedures, approach problems, and make decisions. Figure below illustrates how communication supports cognitive processes from a commognitive perspective.

This study concentrates only on three cognitive processes: conceptual understanding, problem-solving, and critical thinking. The decision-making process is addressed thereafter alongside an alternative theory-based intervention.

Commognitive approach also has an important role in encouraging a relationship between students' algebraic thinking abilities and oral mathematical communication abilities. The learning provided by researchers using instruments that trigger for the emergence of indicators that will be measured from these two abilities. The results of worksheets work by students in groups apart from generating algebraic ideas in solving problems, students are also required to work together, discuss and collaborate in solving problems with their group friends. This allows students to combine skills along with good oral communication in solving problems related to algebra. Therefore, the relationship that occurs between students' algebrai thinking abilities and students' oral mathematical communication abilities, viewed from a problem based learning perspective, can produce results in the form of a strong relationship. This strong positive relationship between algebraic thinking abilities and students' oral abilities also cannot be separated from the role of a teacher in designing learning activities that can integrate oral mathematical communication as a tool to help students understand algebraic concepts and their applications in everyday life. . Indeed, in practice, researchers realize that students who

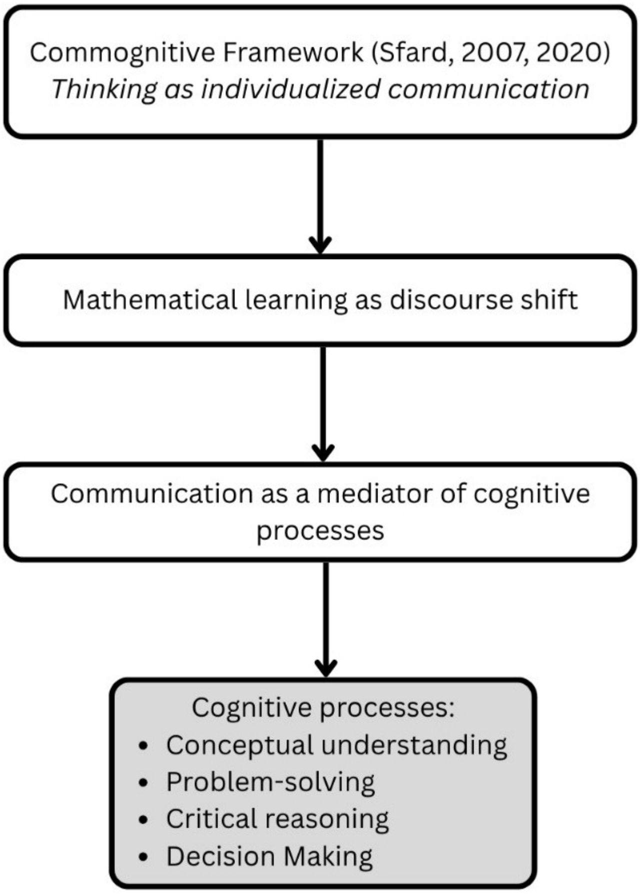

Figure 1.1: Cognitive Processes

come from different backgrounds have different ways of learning and understanding concepts. It can be seen from the data obtained that a small number of students with relatively good algebraic thinking skills still have oral mathematical communication skills that do not accompany their algebraic thinking skills, and vice versa, a small number of students who have good oral mathematical communication skills are not accompanied by sufficient algebraic thinking skills (Blanton et al., 2015).
However, the role of oral mathematical communication here is not as a way for students to learn, but as a translation tool for students to understand mathematical language which has its own uniqueness[1]. This mathematical language sometimes becomes a barrier for students in understanding abstract mathematical concepts, in this case algebra. Having good oral mathematical communication skills is certainly an advantage for students in understanding algebra, and vice versa, students who have good skills are likely to also have good oral mathematical communication skills.
Also, this is one of the factors that can show the reason why the level of relationship that occurs between students' algebraic thinking abilities and students' oral mathematical communication skills through the problem based learning model falls into the category of a strong positive relationship. The relationship between algebraic thinking abilities and students' oral mathematical communication abilities through the problem based learning model also cannot be separated from how big the contribution of one ability is to the development of other abilities. Researchers also calculated the coefficient of determination value through a regression equation from the data obtained. The coefficient of determination value obtained is . This value shows that the formation of oral mathematical communication skills is as much influenced by the ability to think algebraically. Of the several factors that influence the formation of oral mathematical communication skills, more than half of the process is contributed by algebraic thinking skills (Patrick Byers, 2016). This fairly high score is also

in line with how algebraic thinking skills used to apply algebra itself can encourage students to understand mathematical language more formally and accurately. Students' ability to communicate is also trained through abstract symbols which students must be able to represent both in writing and visually. However, in practice, of course this value does not merely make algebraic thinking abilities the only way to develop students' mathematical communication abilities, or in other words, there are still other factors that also contribute to oral mathematical communication abilities (Suwardi, 2022)(Moh Zayyadi et al., 2019). This is also in line with the fact in the data that there are still students who do not have a balanced score between these two abilities.

1.2.1 Research Questions

We now specify the mathematical topic, the research objectives, and the related research questions in order to examine the issues mentioned. We must further limit ourselves since secondary school algebra is too wide to examine within the parameters of this study. These poor results, however, might be the consequence of students' struggles when they first begin learning algebra, which in Indonesia begins in grade VII (12–13 years old). As a result, we decide to look into how pupils learn algebra and how they express it while doing so. Linear equations in two variables and the ideas of relation and function—which already incorporate fundamental algebraic principles like the concepts of variable and algebraic equivalency—are major topics at the start of algebra. As a result, this topic will be covered in the algebra curriculum by addressing the two general concerns that were elaborated in previous section: the why of low algebraic student performance and the how of enhancing algebraic student performance.

Table 1.1: Research Aims and Research Questions

Research Aims	**Research Questions**
To explain students' difficulties in learning algebra and its cognitive processes (conceptual understanding, problem solving, and critical reasoning)	What are students difficulties in learning algebra and what are the in understanding algebra (conceptual understanding, problem solving, and)?
To develop worksheet to support student conceptual understanding about algebra	How to develop worksheet to support students' conceptual understanding?
To describe statistically the relationship between algebraic thinking and communication skill (oral and written)	How is the relationship between algebraic thinking and communication skill (oral and writte)?
To investigate students' written mathematical communication skills and algebraic thinking using commognitive framework	How is the written mathematical communication skills and algebraic thinking using commognitive framework?
To describe students' oral communication skill in understanding algebra	How is the students' oral communication skill in understanding algebra?
To describe students' critical reasoning in understanding topic relation and function	How is the students' critical reasoning in understanding topic relation and function?
To develop Local Instruction Theory in Algebra Topic using Commognitive Framework to support critical reasoning	How to develop Local Instruction Theory in Algebra Topic using Commognitive Framework to support critical reasoning?

1.2.2 Research Impacts

1. It can serve as a tip for educators to enhance their instructional approach for algebraic topics.

2. It can serve as a reference for innovation in junior high school mathematics education and assist teachers in doing classroom-related research.

3. Enhances the breadth and diversity of learning designs that relate directly to the theme of community functions.

4. For advanced scholars, it can function as a reflective and evaluative resource to engage in discourse around this topic with improved methodologies and frameworks.

1.3 Structure of The Dissertation

This dissertation consists of seven chapters. This first chapter provides an overview of the study. Chapters 2-7 contain articles that have been submitted to, or published in different research journals in the field of mathematics education. The table summarizes the dissertation's structure and shows the relation between chapters and publications.

Table 1.2: Dissertation Outline

Chapter and Title	Publication
Chapter 1: Introduction	
Chapter 2: Developing Problem-based Learning Worksheet of Relations and Functions Topic to Support Students' Conceptual Understanding	Pratiwi, W.D.,Zulkardi., Putri, R. I. I., & Hiltrimartin, C. Development of Worksheet Relations and Functions Porblem - Based Learning for Supporting Ability Solving Problem. Lentera Sriwijaya: Jurnal Ilmiah Pendidikan Matematika, 6(2), 14-25. http://doi.org/10.36706/jls.v6i2.15.
Chapter 3: Developing Worksheet of Topic Relations and Functions to Support Problem Solving Skills	Pratiwi, W. D., Zulkardi., Putri, R. I. I., & Hiltrimartin, C. Developing problem-based learning worksheet of relations and functions topic to support students' conceptual understanding. Jurnal Gantang, 9(2), 255 – 264. https://doi.org/10.31629/jg.v9i2.6958
Chapter 4: Communication Skill and Algebraic Thinking	Pratiwi, W. D., Zulkardi, Z., Putri, R. I. I., & Hiltrimartin, C. (2025). Students' Communication Skill and Algebraic Thinking through Commognitive Framework in Algebra Learning. Mathematics Education Journal, 19(3), 413–436. https://doi.org/10.22342/mej.v19i3.pp413-436.
Chapter 5: Student's Oral Communication Skill in Understanding Algebra	Pratiwi, W. D., Zulkardi., Putri, R. I. I., & Hiltrimartin, C. Student's Oral Communication Skill in Understanding Algebra . Mosharafa (Submitted)

Chapter 6: Student's Critical Reasoning on Relation and Function	Pratiwi, W. D., Zulkardi., Putri, R. I. I., & Hiltrimartin, C. Critical reasoning ability of junior high school students on relation and function materials with connecting, organizing, reflecting, extending (CORE) models. AIP Conf. Proc. 3052, 020028 (2024). https://doi.org/10.1063/5.0201050
Chapter 7: Local Instructional Theory in Algebra Topic using Commognitive Framework to Support Student's Critical Reasoning	Pratiwi, W. D., Zulkardi., Putri, R. I. I., & Hiltrimartin, C. Local Instructional Theory in Algebra Topic using Commognitive Framework to Support Student's Critical Reasoning. Mathematics Teaching Research Journal (Submitted).
Chapter 8	Conclusion.

1.4 Description of Each Chapter

Chapter 1 describe the overview of all chapter and general information of what-why-how questions about the research in this dissertation.

Chapter 2 describe the qualitative research using a design research method to develop a valid and practical worksheet on relations and functions, based on problem-based learning (PBL), to enhance students' conceptual understanding. Conducted in an eighth-grade class at SMP Negeri 8 Palembang, the study focused on six students and involved two main phases: a preliminary stage and a prototyping stage (with five sub-stages). Data were collected through walkthroughs, tests, interviews, and questionnaires. The results showed that the worksheet was both valid and practical, with an average validation score of 84.03%, and effectively supported students' understanding of PBL-based relational and functional concepts.

Chapter 3 reports a design research aimed to develop valid and practical problem-based learning (PBL) worksheets on to support students' problem-solving abilities. The study focused on eighth-grade students at an SMP in Palembang, addressing the importance of the topic and students' low problem-solving skills. It involved multiple phases, including a preliminary stage and a formative evaluation stage, which comprised expert review, self-evaluation, one-on-one, small-group sessions, and field testing. The results showed that the developed worksheets were valid and practical for enhancing students' understanding of relations and functions through PBL.

Chapter 4 reports a study investigating students' mathematical communication skills and their connection to early using the commognitive framework, which includes word use, visual mediators, narratives, and routines. Employing a mixed-methods descriptive design, the research involved 29 eighth-grade students and used written tests for data collection. Results showed varying levels of communication skills, with most students falling in the fair to good range. Students with stronger communication skills demonstrated better expression of algebraic ideas. The study highlights the usefulness of the commognitive framework in enhancing conceptual understanding and supporting effective mathematics instruction.

Chapter 5 describes a qualitative descriptive study aimed at examining the oral mathematical communication skills of eighth-grade students at SMP 24 Palembang using commognitive-based learning. Due to students' speech difficulties, the —focusing on word use, visual mediators, narratives, and routines—was applied through LKPD (student worksheets) to support learning. Data were collected through interviews, observations, and written tests. The results revealed that students displayed varying levels of oral communication skills, categorized as low, medium, and high.

Chapter 6 reports a qualitative descriptive study aimed at examining students' critical reasoning abilities in learning relations and functions. Conducted with eighth-grade students at SMP Negeri 8 Palembang, the research focused on six students of varying ability levels. The study followed three stages: preparation, teaching implementation, and data analysis. Learning activities were supported by student worksheets (LKPD) and aligned with the school's system. Results showed that students most frequently demonstrated critical reasoning indicators related to interpreting, analyzing situations, and applying concepts, while decision-making and explanation skills appeared less frequently.

Chapter 7 elaborates the design research study to contribute to local in algebra topic using commognitive framework, the activities use the context of bulding cities and road with specific rules to discover the concept of fuctions. The research consists of phases of , namely: (1) preparation phase, (2) Design Experiment (pilot experiment and teaching experiment), (3) restrospective analysis. The lesson was designed to create communication path using commognitive framework to describe critical reasoning as part of cognitive processes in understanding algebra.

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