Application of Dimensional Analysis for Prediction of NO_x Removal

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***Abstract*- This paper proposes a mathematical modelling by using dimensional analysis in investigating the relation of the electrical and physical parameters on the removal of nitrogen oxides (NO_x) concentration, by means of a hybrid pulsed streamer discharge plasma. To verify the viability of the equation, an experiment using discharge reactor was conducted. A pair of perforated metal electrodes was used, with glass and alumina ceramic forming a dielectric barrier. Concentration of NO_x is measured and compared with theoretical calculations. It is verified that the predictions results is generally agree with the experimental findings.**

Keywords: Diesel exhaust, pulsed streamer discharge, NO_x removal, non-thermal plasma

I. INTRODUCTION

Development of transportation sector brings along the air pollution problems due to the incomplete combustion that is emitted from diesel engine exhaust such as NO_x, sulfur oxides (SO_x), carbon monoxide (CO), hydrocarbons (HC) and soot [1]. Control of these emissions becomes more important as it has reached a critical level over recent years. It is considered to be silent death to the human and environment.

The increasing concern about the environmental problems has determined the interest toward establishing new and more effective pollution control technologies [2]. Removal of pollutant gases from diesel engine exhaust has been a challenge to researchers, as many conventional methods such as catalysis, exhaust gas recirculation and other engine design modifications could not bring down the level of exhaust gases to mandatory limits put across various countries [3].

The electrical discharge plasma is an attractive technology being used mainly due to the low cost and high energy efficient exhaust gas treatment method [4]. In our studies, hybrid pulsed streamer discharge plasma has been used to treat exhaust gases as it is more reliable, compact, low maintenance and high efficiency on reduction of pollutant gases (NO_x, SO_x and CO) from diesel engine exhaust.

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TABLE 1
SYMBOLS AND DIMENSIONS OF EACH PHYSICAL QUANTITIES

Physical Quantities	Symbols	Dimensions
Gas flow rate	f_r	L^3T^{-1}
Initial concentration of NO_x	NO_x	$L^{-3}M$
Applied voltage	V	$L^2MT^{-3}A^{-1}$
Temperature	T	K
Relative permittivity	ε_r	$L^{-3}M^{-1}T^4A^2$
Gap spacing	d_g	L
Pressure	P	$L^{-1}MT^{-2}$
Frequency	f	L^3T^{-1}

Mathematical model for the discharge chamber is developed by using the dimensional analysis. Dimensional analysis is a useful tool for predicting the concentration of NO_x from a given reaction chamber [5, 6]. However, it is necessary to determine the significant electrical and physical parameters that influence the reduction of NO_x. The predicted results of NO_x concentration from dimensional analysis are compared with the experimental results.

II. DIMENSIONAL ANALYSIS

There is several numbers of parameters that have been identified to have significant influence on reduction of NO_x [1-4, 7-13]. The most significant parameters amongst these are the gas flow rate f_r, initial concentration of NO_x, applied voltage V, temperature T, relative permittivity ε_r, gap spacing d_g, pressure P and frequency f. Table 1 indicates the symbols and dimensions of each physical quantities where L, M, T, A and K are the fundamental dimensions quantities of length, mass, time, current and temperature respectively.

The relationship between the reduction of NO_x and those parameters can be presented by an equation of the form,

$$NO_x = (NO_x)(f_r, V, T, \varepsilon_r, d_g, P, f) \quad (1) \qquad (1)$$

When all the parameters are expressed in terms of the fundamental dimensions, the corresponding dimensional matrix is

	k_1 f_r	k_2 NO_x	k_3 V	k_4 T	k_5 ε_r	k_6 d_g	k_7 P	k_8 f
L	3	-3	2	0	-3	1	-1	3
M	0	1	1	0	-1	0	1	0
T	-1	0	-3	0	4	0	-2	-1
A	0	0	-1	0	2	0	0	0
K	0	0	0	1	0	0	0	0

where k_1, k_2, k_3, k_4, k_5, k_6, k_7 and k_8 are indices of the variables of (1).

According to the Buckingham's theorem [14];

No of dimensionless variables = (No of variables) – (No of dimensions)

Therefore, the number of dimensionless variables is three since the numbers of variables are eight and there are five dimensions. The five dimensions leading to five equations that can be written as;

$$3k_1 - 3k_2 + 2k_3 - 3k_5 + k_6 - k_7 + 3k_8 = 0 \qquad (2)$$

$$k_2 + k_3 - k_5 + k_7 = 0 \qquad (3)$$

$$-k_1 - 3k_3 + 4k_5 - 2k_7 - k_8 = 0 \qquad (4)$$

$$-k_3 + 2k_5 = 0 \qquad (5)$$

$$k_4 = 0 \qquad (6)$$

In order to get k_4, k_5, k_6, k_7 and k_8 in terms of k_1, k_2 and k_3, three dimensionless variables written as π_1, π_2 and π_3 are introduced.

	k_1 f_r	k_2 NO_x	k_3 V	k_4 T	k_5 ε_r	k_6 d_g	k_7 P	k_8 f
π_1	1	0	0	0	0	0	0	-1
π_2	0	1	0	0	0	-4	-1	2
π_3	0	0	1	0	0.5	-1	-0.5	0

(7)

The steps to obtain the value for k_4, k_5, k_6, k_7 and k_8 are shown as follows;

For π_1; $k_1 = 1$ $k_2 = k_3 = 0$

Substituting k_3 into (5) results in $k_5 = 0$. Thus, from (3), $k_7 = 0$ and $k_8 = -1$ from (4). By replacing all known k_n values in (2), k_6 is found to be zero. The steps are similar for π_2 and π_3.

Therefore, the set of dimensionless products resulting from (7) is as follows:

$$\pi_1 = \frac{f_r}{f} \qquad (8)$$

$$\pi_2 = \frac{NO_x \cdot f}{P \cdot d_g^4} \qquad (9)$$

$$\pi_3 = \frac{V \cdot \varepsilon_r^{0.5}}{d_g \cdot P^{0.5}} \qquad (10)$$

According to the Buckingham's theorem [14-15], the correlation between the dimensionless products is stated as follows:

$$\pi_2 = \Psi(\pi_1, \pi_3) \qquad (11)$$

By considering the monomial form [5-6], the dimensional less parameter leads to the relationship

$$\pi_2 = D_c \, \pi_1^{\varepsilon 1} \, \pi_3^{\varepsilon 2} \qquad (12)$$

where D_c is a dimensional constant. Substituting (8) to (10) into (12) enables the NO_x concentration to be expressed as

$$NO_x = D_c \left[\frac{P \cdot d_g^4}{f}\right]\left[\frac{f_r}{f}\right]^{\varepsilon 1} \left[\frac{V \cdot \varepsilon_r^{0.5}}{d_g P^{0.5}}\right]^{\varepsilon 2} \qquad (13)$$

D_c, ε_1 and ε_2 are all constants to be determined. Based on the evidence in the literature, reduction of NO_x is proportional to f_r, d_g and P and inversely proportional to f, V and ε_r. Thus, $\varepsilon_1 = 1$ and $\varepsilon_2 = -1$ and (13) can be simplified to

$$NO_x = D_c \frac{P^{1.5} d_g^5 f_r}{V \cdot f^2 \varepsilon_r^{0.5}} \qquad (14)$$

By assuming that P, d_g, f_r and ε_r would always be constant for every experiments, thus a new dimensionless constant (D_c') could be found. Hence, the final general equation for NO_x concentration is

$$NO_x = D_c' \times \frac{1}{V \cdot f^2} \qquad (15)$$

By applying regression techniques in IBM SPSS Statistics, the correlation between NO_x concentration with frequency and applied voltage can be determined. Hence, the following equation is obtained (3) (2

$$NOx = 2.19E^2 - 0.48(V.f) - 1.1E^{-14}(V.f)^2 - 2.55E^{-19}(V.f)^3 \qquad (16)$$

III. COMPARISON OF THE MATHEMATICAL MODEL WITH EXPERIMENTAL RESULTS

A. *Experimental Setup*

An experimental hybrid pulsed streamer discharge chamber is constructed in order to verify the validity of the mathematical model obtained in (15). Fig. 1 show the discharge chamber which was constructed by perforated metal as electrode and glass or alumina ceramic as dielectric barrier with relative permittivity, ε_r is 7 and 10 respectively. The DC pulse high voltage is injected at the electrode and the chamber's body which was made by stainless steel acts as ground.

The high voltage supply uses a two stage silicon power amplifier using a 2N222A and a 2N3055 bipolar transistor. This design is used as it will survive the incredible high voltage transients. The circuit utilize two NE555 electronic timers which the first NE555 timer, U1 determines the frequency of the pulse and the second timer, U2 determines the pulse duration.

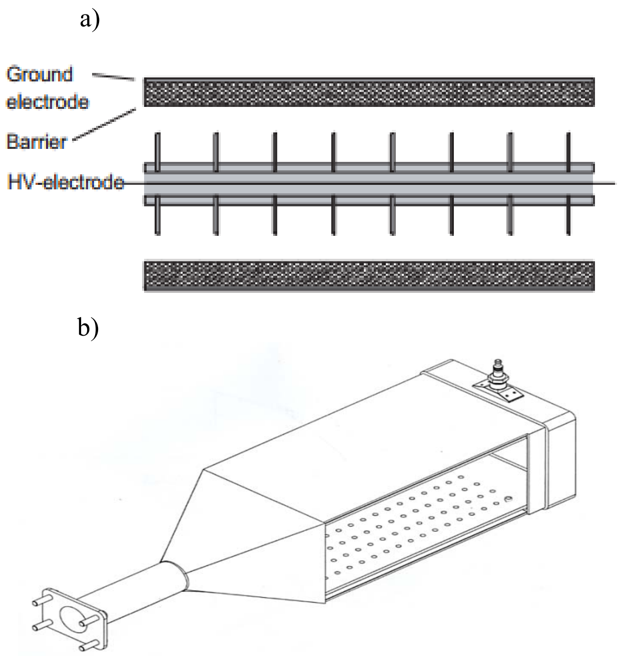

Fig. 1. Reactor configuration for a) dielectric barrier discharge b) hybrid pulsed streamer discharge

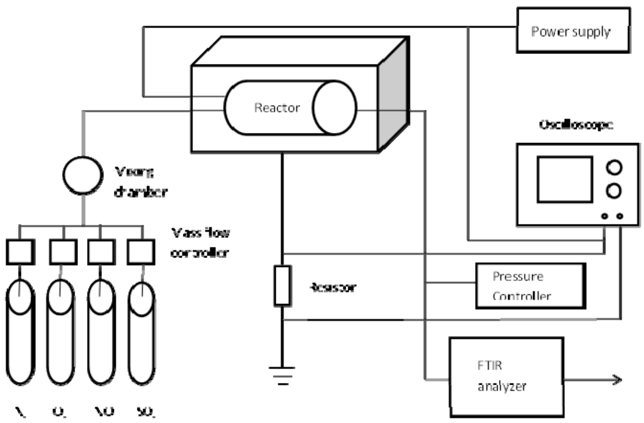

Fig. 2. The schematic diagram of the experimental setup

B. *Result and Discussion*

The results obtained using (16) is compared with the experimental results reported in the literature to verify the validity of the analysis. The effect of changing the frequencies on the NO_x concentration is observed for a number of different applied voltages.

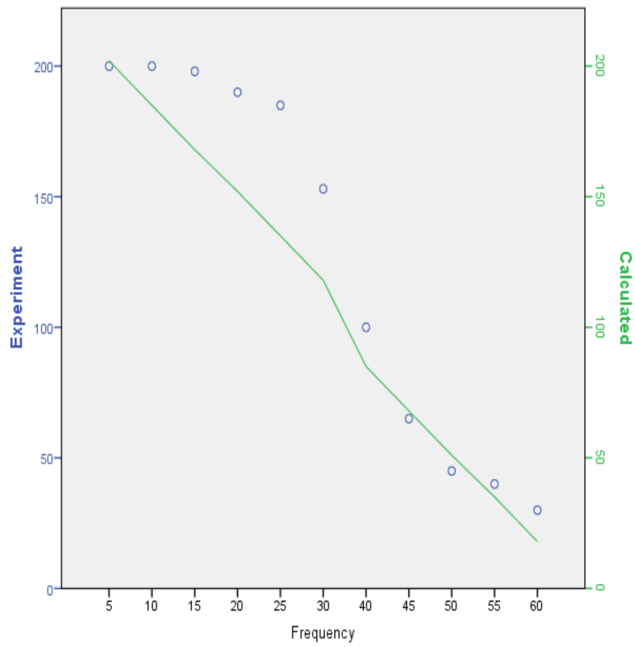

Fig. 3. NO_x concentration (ppm) versus frequency (kHz) at applied voltage of 7.0kV

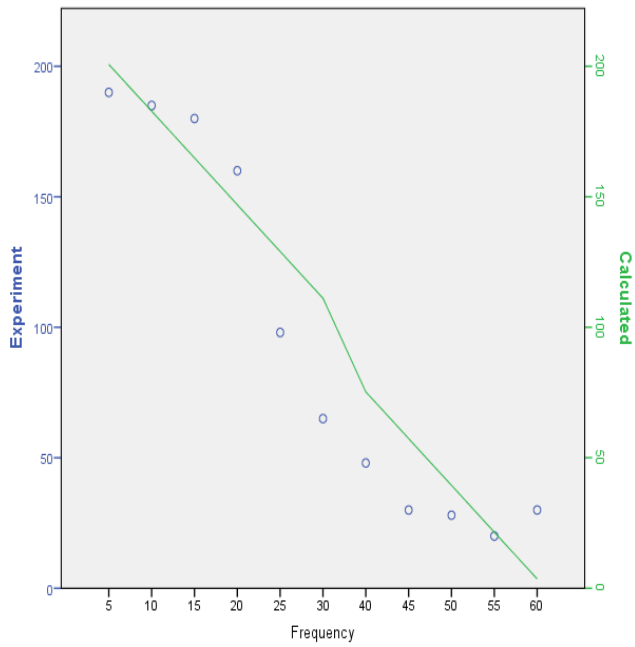

Fig. 4. NO_x concentration (ppm) versus frequency (kHz) at applied voltage of 7.5kV

Fig. 3-5 shows the graph of NO_x concentration versus frequency for various applied voltages. As can be perceived from the comparison made, there is a good agreement between the prediction calculation and the experimental data. It can be seen that the NO_x concentration decreases with increasing frequency at a given applied voltage.

However, it has to be noted that NO_x concentration attained an optimum value at a frequency of 50 kHz. Increasing the frequency above this value will lead to an increase in the NO_x concentration. As the frequency increase, the average electron kinetic energy is high. Thus, the rate for electron-impact dissociation of N_2 is higher than

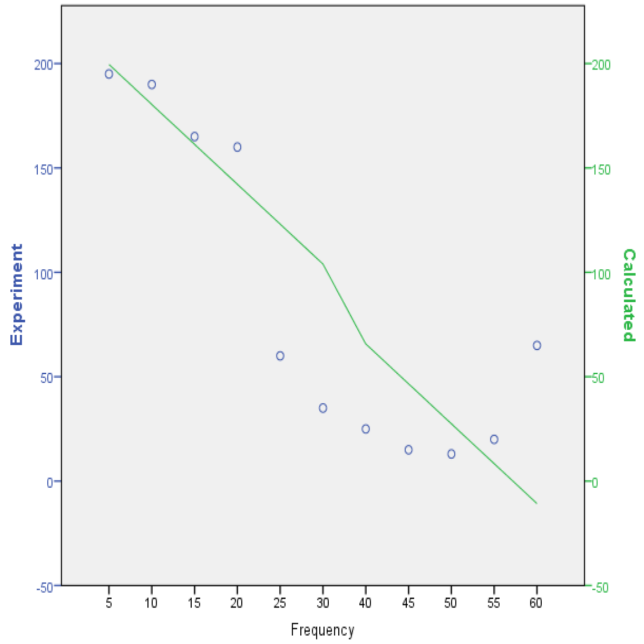

Fig. 5. NO_x concentration (ppm) versus frequency (kHz) at applied voltage of 8.0kV

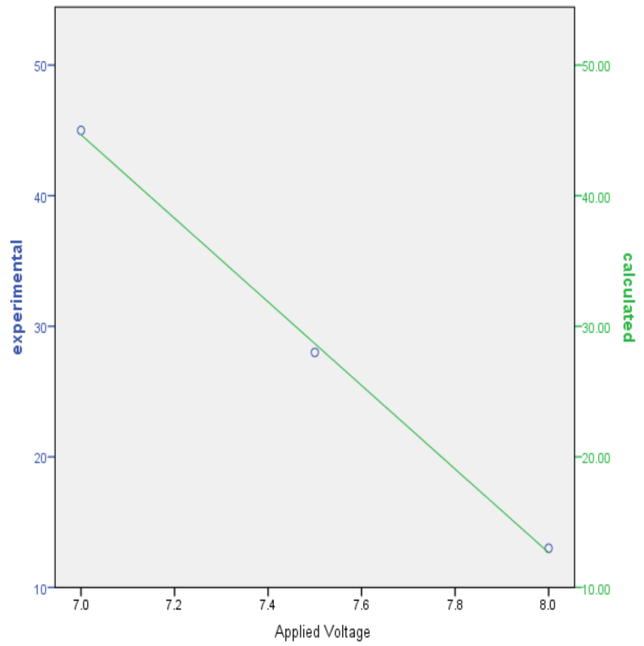

Fig. 6. Results of 50 kHz frequency and the NO_x concentration (ppm)

O_2 [16]. The dissociation of N_2 produced a large number of ground-state N (^4S) and excited nitrogen atoms, N (^2D):

$$e + N_2 \longrightarrow e + N(^4S) + N(^4S, {}^2D)$$

The existence of N (^4S) would effectively contribute to the removal of NO. However, in the presence of O_2, the excited nitrogen atoms, N (^2D) can lead to undesired reactions that would produce NO rather than removal of NO.

Therefore, an increment in NO_x concentration above the frequency of 50 kHz is due to the counterbalance reaction that took place in the system.

$$N(^4S) + NO \longrightarrow N_2 + O$$

$$N(^2D) + O_2 \longrightarrow NO + O$$

Fig. 6 illustrates graph of NO_x concentration versus applied voltage at a specific frequency mapped from the data obtained from Fig. 3-5. In order to observe the correlation between the applied voltage and the NO_x concentration, these two parameters are analysed at a specific frequency for every experiments, for example, 50 kHz [17]. Good agreement is again evident, with results confirming that with increasing applied voltage the concentration of NO_x will decrease.

IV. CONCLUSION

A mathematical model by using dimensional analysis has been applied to calculate the removal rate of NO_x concentration and it has been proven that the predicted equation is in good agreement with experimental results. A hybrid pulsed streamer discharge plasma chamber was constructed in order to verify the validity of the predicted calculations. The removal of NO_x increases with increasing frequency and applied voltage. At an applied voltage of 8.0 kV, the removal of NO_x attained an optimum value which is 93.5 % at a frequency of 50 kHz.

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