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Models of Internet Charging Scheme under Multiple QoS Networks

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***Abstract*:** Internet Service Providers (ISPs) nowadays deal with high demand to promote good quality information. However, the knowledge to develop new pricing scheme that serve both customers and supplier is known, but only a few pricing plans involve QoS networks. This paper will seek new proposed pricing plans are offered with QoS networks involved. We are going to solve multiple QoS Networks scheme as an optimization model by comparing two models in multiple QoS networks by taking into consideration decision whether to set up base price to be fixed to recover the cost or to be varied to compete in the market.

***Keywords*:** charging scheme, multiple QoS networks, profit maximization.

1. Introduction

Recent work on multiple QoS networks is due to [1]. She described the pricing scheme based auction to allocate QoS and maximize ISP's revenue. The auction pricing scheme is actually scalability, efficiency and fairness in sharing resources. The solution of the optimization problem goes from single bottleneck link in the network and then she generalized into multiple bottleneck links using heuristic method. In this paper, she used only single QoS parameter-bandwidth, while in networks, there are many parameters affect QoS that can be considered.

Although QoS mechanisms are available in some researches, there are few practical QoS network. Even recently a work in this QoS network [2], it only applies simple network involving one single route from source to destination.

Yang [1] and Yang et al. [3, 4] formulate pricing strategy for differentiated QoS networks. In their discussion, they focus on auction algorithm to find the optimal solution. We apply their mathematical formulation and combine it with mathematical formulation discussed by Byun and Chatterjee [2]. We apply their mathematical formulation and combine it with mathematical formulation discussed by Byun and Chatterjee [2](see in Puspita et al. [5, 6]).

Recent studies have been conducted to address problem of multiple service network. Sain and Herpers [7] discussed problem of pricing in multiple service networks. They solve the internet pricing by transforming the model into optimization model and solved using Cplex software. Also, Puspita et al. [8, 9] discussed the new approach and new improved model of [1, 2] and got better results in getting profit maximization of ISP.

Basically, we would like to modify the mathematical formulation of [2, 3] to be simpler formulation by taking into consideration the utility function, base price as fixed price or variable, quality premium, index performance, capacity and also bandwidth required. Next we consider the problem of internet charging scheme as Mixed Integer Nonlinear Programming (MINLP) to obtain optimal solution. In this part, we also would like to compare two models in which whether we fix decision variable of user admission to the class or not and also compare the original models [1] with our models to show better solution.

2. Previous Research on Pricing Scheme

Past researches mainly showed us the extra pricing occurs when there exists congestion that causes degradation of QoS. Three stages proposed by [10] consisted of no use of feedback and user adaptation, use of feedback of closed-loop and one kind of variation of closed loop forms.

Also, scheme named congestion avoidance was also proposed by [11] and scheme of smart market [12] and [13]. Karp [14] then discussed problem of congestion. Problem occurring when sending packet in a flow can be dropped if there exists congested flow. In order to reach destination, the packet should be transmitted again in other rate. But it is obvious that we do not know how much for the retransmission rate. How can go through? How can the source A, for instance, know and manage its flow over continuing certain time, meaning that time is divided into duration length of time like suggested [15] and [16].

Tuffin [17], Ros and Tuffin [18] and Odlyzko [19] also proposed Paris metro pricing scheme for charging the network. In this case, the different service class will have different price. The user has choice to choose channels to travel and price to pay. They strategy basically attempted to optimize the profits not just increasing the profits but rather more on controlling the congestion to gain maximum profit. They proposed scheme by using partition to show different class has different services. The drawback is still due to unknown idea whether this scheme is applicable for current network or not. Meanwhile, Altmann and Chu [20] offer new pricing plan that gives benefit to ISP and users. This plan is combination of flat rate and usage based pricing. In this plan, user will get benefit from unlimited access by choosing higher QoS and at the same time ISP is able to reduce its peak load. The drawback is still due to lack of information how that plans can be adopted into multiple route networks. For the next generation internet, the availability of fast transportation of data is required. The multicast communication can decrease due to limitation of bandwidth. So we need QoS specification and compute optimal routes to a multi-constrained problem, by using greedy algorithm such as meta-heuristics algorithm, like suggested in [21].

Puspita et al.[5, 6] also discussed internet charging scheme under multiple QoS networks by comparing two models that involve base price as a fixed and variable set up by ISP. The model created by setting up base price as fixed price will yield higher optimal solution if ISP intended to recover the cost. But if ISP would like to compete in market, then the choice of model involving base price as variable price would be the best option to choose.

3. Research Method

We attempt to apply optimization techniques in solving the problem in this paper. Like in [7], we also consider the optimization problem as MINLP that can be solved by using optimization tools. We transform the problem of pricing the internet in multi QoS networks into optimization model and attempt to solve it to get optimal solution. This solution will help us interpreting the current issues involving pricing, network share, base price, quality premium and also QoS level. Figure 1 below shows us the research model proposed in single link multi QoS networks.

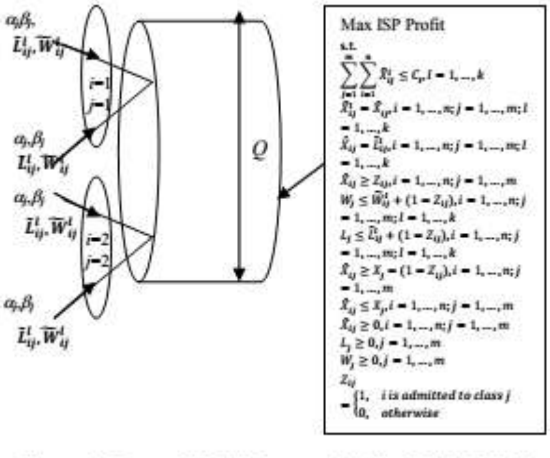

Figure 1. Research Model Proposed in Single Link Multiple QoS Networks

4. Results and Discussions

The idea basically generates from [1, 2, 3, 4] for single QoS network and also we also use utility function adopted by [1, 3, 4].

4.1. Assumptions

Assume that there is only one single network from source to destination since concentrate on service pricing scheme. Assume that the routing schemes are already set up by the ISP. As [1] pointed out, we have 2 parts of utility function namely, base cost which does not depend on resource consumption and cost which depends on resource consumption. The utility function has characteristics as marginal profit as function of bandwidth decreasing with increasing bandwidth. The Objective of ISP is to obtain maximized revenue subject to constraints based on system' available resources.

4.2. Mathematical Formulations

Model 1 original

We have parameters as follows:

α_j : Base price for class j
Q : Total bandwidth
V_i : Minimum bandwidth required by user i

Decision variables are as follows:

Z_{ij} = $\begin{cases} 1, & \text{if user i is admitted to class } j \\ 0, & \text{otherwise} \end{cases}$
$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j
L_{m_j} : Minimum bandwidth for class j
W_j : Price sensitivity for class j
X_j : Bandwidth assigned to each individual user in class j
$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

Model 1 original will be

$$Max\ Profit = \sum_j \sum_i (\alpha_j . Z_{ij}) + w_j log \frac{\tilde{X}_{ij}}{L_{m_j}} \quad (1)$$

Subject to

$$(\sum_j \sum_i X_{ij}) \leq Q \quad (2)$$
$$\tilde{X}_{ij} \geq L_{m_j} - (1 - Z_{ij}),\ i=1, ..., n; j=1,..., m \quad (3)$$
$$W_j \leq \widetilde{W}_{ij} + (1 - Z_{ij}),\ i=1, ..., n; j=1,..., m \quad (4)$$
$$\tilde{X}_{ij} \geq V_i - (1 - Z_{ij}),\ i=1, ..., n; j=1,..., m \quad (5)$$
$$\tilde{X}_{ij} \geq X_j - (1 - Z_{ij}),\ i=1, ..., n; j=1,..., m \quad (6)$$
$$\tilde{X}_{ij} \geq Z_{ij},\ i=1, ..., n; j=1,..., m \quad (7)$$
$$\tilde{X}_{ij} \geq 0,\ i=1, ..., n; j=1,..., m \quad (8)$$
$$L_{m_j} \geq 0,\ j=1, ..., m \quad (9)$$
$$W_j \geq 0,\ j=1, ..., m \quad (10)$$
$$\tilde{X}_{ij} \leq X_j,\ i=1, ..., n; j=1,..., m \quad (11)$$
$$Z_{ij} = \begin{cases} 1, & if\ user\ i\ is\ admitted\ to\ class\ j \\ 0, & otherwise \end{cases} \quad (12)$$
$$0 \leq \widetilde{W}_{ij} \leq c,\ i=1, ..., n; j=1,..., m \quad (13)$$

Where c is predetermined value of upper bound price sensitivity for user i at class j respectively. Also, the value of d is as the upper bound of quality index.

Model 1 modified

Parameters

α_j : Base price for class j
β_j : Quality premium of class j that has I_j service performance
I_j : Quality index of class j
Q : Total bandwidth

V_i : Minimum bandwidth required by user i

Decision variables

Z_{ij} = $\begin{cases} 1, \text{if user i is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

Our Model 1 modified is as follow.

$$Max\ Profit = \sum_j \sum_i ((\alpha_j . Z_{ij} + \beta_j . I_j) + w_j log \frac{\tilde{x}_{ij}}{L_{m_j}}) \qquad (14)$$

Subject to

Constraint (2)-(13) and additional constraints as follow.

$$\alpha_j + \beta_j . I_j \geq \alpha_{j-1} + \beta_{j-1} . I_{j-1}, j > 1 \qquad (15)$$

$$0 \leq I_j \leq d, j=1, \ldots, m \qquad (16)$$

Model 2 original

Parameters

Q : Total bandwidth

V_i : Minimum bandwidth required by user i

Decision variables

Z_{ij} = $\begin{cases} 1, \text{if user i is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

α_j : Base price for class j

Next, Model 2 original is below.

$$Max\ Profit = \sum_j \sum_i (\alpha_j + w_j log \frac{\tilde{x}_{ij}}{L_{m_j}}) Z_{ij} \qquad (17)$$

Subject to

Constraint (2)-(13) and additional constraints as follow.

$$a \leq \alpha_j \leq b, j=1, \ldots, m \qquad (18)$$

$$\alpha_j \geq \alpha_{j-1}, j > 1 \qquad (19)$$

Model 2 modified

Parameters

Q : Total bandwidth

V_i : Minimum bandwidth required by user i

Decision variables

Z_{ij} = $\begin{cases} 1, \text{if user i is admitted to class } j \\ 0, \quad \text{otherwise} \end{cases}$

$\tilde{X}_{ij}$: Final bandwidth obtained by user i for class j

L_{m_j} : Minimum bandwidth for class j

W_j : Price sensitivity for class j

X_j : Bandwidth assigned to each individual user in class j

$\widetilde{W}_{ij}$: Price sensitivity for user i in class j

I_j : Quality index of class j

α_j : Base price for class j

β_j : Quality premium of class j that has I_j service performance

We also have the Model 2 modified that is

$$Max\ Profit = \sum_j \sum_i ((\alpha_j + \beta_j . I_j) + w_j log \frac{\tilde{x}_{ij}}{L_{m_j}}) Z_{ij} \qquad (20)$$

Subject to

Constraint (2)-(13), (15)-(16) and (18)-(19).

Where a and b are predetermined value of lower bound and upper bound base price respectively.

Model explanations are as follows.

- Objective function (1) basically states that ISP wants to maximize its profit by maximizing its utility function with base price α_j to be fixed to recover cost.
- Constraint (2) tells us that total final bandwidth of all users cannot exceed the total bandwidth available.
- Constraint (3) states that bandwidth obtained by user i should exceed minimum bandwidth for class j if user i is admitted to class j or otherwise.
- Constraint (4) tells us about price sensitivity for class j should be less than the price sensitivity for user i in class j if user i is admitted to class j.
- Constraint (5) gives the information about bandwidth obtained by client i for class j should exceed minimum bandwidth required by user i if user i is admitted to class j.
- Constraint (6) tells us that bandwidth obtained by user i in class j should exceed bandwidth assigned to each individual user in class j if user i is admitted to class j.
- Constraint (7) shows that bandwidth obtained by user i in class j should be greater than the availability of user i in class j and should be nonnegative (8).
- Nonnegativity requirement occurs in price sensitivity (10) and minimum bandwidth for class j in (9).
- Constraint (11) shows that bandwidth obtained by user i in class j should not exceed bandwidth assigned to each individual user in class j.
- Constraint (12) tells us the value of whether the user i is admitted to class j or not.
- Constraint (13) states the price sensitivity of user i in class j lies between range of 0 and predetermined value (c) of price sensitivity for user i.
- Objective function (14) basically states that ISP wants to maximize its profit by maximizing its utility function with base price α_j to be fixed to recover cost and quality premium also to be fixed to enable user to choose the class based on their budget and preferences with chosen QoS level.
- Constraint (15) shows that the summation of price and quality premium to yield perfect service for j class should exceed the one in (j–1) class with j>1.

- Constraint (16) shows that the range of index quality should lie between 0 and 1 with predetermined d value set up by ISP.
- Objective function (17) basically states that ISP wants to maximize its profit by maximizing its utility function with base price α_j to be variable to enable ISP to have market competition if there are chances.
- Constraint (18) tells us the range of base price (a and b) is lower bound and upper bound of predetermined base price, respectively.
- Constraint (19) shows that base price for j class is more than base price or j-1 class with j>1.
- Objective function (20) basically states that ISP wants to maximize its profit by maximizing its utility function with base price α_j to be variable to enable ISP to have market competition if there are chances and quality premium to be fixed to enable user to choose the class based on their budget and preferences with chosen QoS level.

4.3.Solutions in Multiple Classes

We begin with introducing two classes and two users, so j=2 and i = 2. The results by LINGO 13.0 are presented in Table 1.

In Table 1, we can see the solver status of our model. Model class for each model is Mixed Integer Nonlinear Programming (MINLP), with status of current solutions are all local optimal. Infeasibility is 0 for each model except there exist a very small infeasibility in model original with 3.9 x 10^{-14}. Solver type is Branch and Bound solver and active field lists that 2 of subproblems remaining to be evaluated for each model until that number go to 0. Highest Generated Memory Used (GMU) for memory allocation in model 1 modified is 42K and in model 2 modified which is 44K. Elapsed Runtime (ER) for each model is 0 sec time to generate and solve those models. Best objective for model 1 and model 2 are model 1 modified and model 2 modified. Those models obtain highest value of 254.985 and 171.1, respectively. The theoretical bound on the objective is also the same value like best objective for each model. Steps = 0 shows us that there are 0 of branches in the branch and bound tree. The solver run the model until the active field turns to 0 as stated in the Table 1. Lastly, it takes 35 iterations to solve model 1 and it takes 14 iterations to solve model 2.

In Table 2, user 1 is allowed to take class 1 and class 2 since final price sensitivity for user I in class j is at least greater than or equal to price sensitivity for class 1 or class 2. That is why only user 1 is admitted to either class 1 or 2. Bandwidth obtained by user 1 in class 1 is 25 bps. It happens also for user 2 in class 1 or 2.

Bandwidth obtained by user 1 in class 1 is at least than or equal to bandwidth for class 1. Bandwidth obtained by user 1 in class 2 also occur the same condition like in class 1.

We can see from the objective value [1] in each model that the modified model yields better solution compared to original model proposed by [1].

Next result deals with i=3 and j =3 that is shown in Table 3. User 1 and user 2 are allowed to take class 1, 2 and 3 since their final price sensitivity is at least greater than or equal to price sensitivity of those classes. That is the reason why user 1 and 2 are admitted to class 1, 2 and 3.

User 3 is not permitted to use any class in each model. Bandwidth obtained for each user in each class is 11.1 kbps. Also, the bandwidth obtained for user i in class j is at least less than or equal to bandwidth for class j.

We also can see that the modified model yields slightly better result than original model proposed by [1]. Whether if we fix or vary the base price.

Table 1. Solver status of Model 1 and Model 2

	Model 1		Model 2	
Solver status	**Original**	**Mod**	**Original**	**Mod**
Model Class	MINLP	MINLP	MINLP	MINLP
State	Local opt	Local opt	Local opt	Local opt
Objective	254.872	254.985	171.024	171.1
Infeasibility	3.9×10^{-14}	0	0	0
Iterations	35	35	14	14
Extended Solver state				
Solver type	B & B	B & B	B & B	B & B
Best obj	254.872	254.985	171.024	171.1
Obj. Bound	254.872	254.985	171.024	171.1
Steps	0	0	0	0
Active	0	0	0	0
Update Int	2	2	2	2
GMU(K)	39	42	41	44
ER(sec)	0	0	0	0

Table 2. Results for i=2 and j=2

	Model 1		Model 2	
	original	modified	original	modified
α_1(\$/bps)	0.2	0.2	0.3	0.3
α_2(\$/bps)	0.3	0.3	0.3	0.3
I_1	-	0.9	-	0.9
I_2	-	0.8	-	0.8
W_1	5	5	5	5
W_2	5	5	5	5
$\tilde{X}_{11}$ (bps)	25	25	25.5	25.5
$\tilde{X}_{21}$(bps)	25	25	24.5	24.5
$\tilde{X}_{12}$ (bps)	25	25	25.5	25.5
$\tilde{X}_{22}$(bps)	25	25	24.5	24.5
L_{m1}(bps)	0.01	0.01	0.01	0.01
L_{m2}(bps)	0.01	0.01	0.01	0.01
Z_{11}	1	1	1	1
Z_{12}	1	1	1	1
Z_{21}	0	0	0	0
Z_{22}	0	0	0	0

$\widetilde{W}_{11}$	5	5	5	5
$\widetilde{W}_{12}$	5	5	5	5
$\widetilde{W}_{21}$	4	4	4	4
$\widetilde{W}_{22}$	4	4	4	4

So, the choice of whether to fix or vary the base price depends on ISP. If the choice of recovering cost would be the main goal, then, ISP should choose to fix the base price. But, if ISP chooses to have competition in market, then the choice of varying the base price will be the best choice.

To sum up, the choice of model depends on ISP point of view but in all models, our proposed models yield slightly better optimal solution than model proposed by [1].

Table 3. Results for i=3 and j=3

	Model 1		Model 2	
	original	modified	original	modified
α_1($)	0.3	0.3	0.3	0.3
α_2($)	0.4	0.4	0.4	0.4
α_3($)	0.5	0.5	0.4	0.4
I_1	-	0.9	-	0.9
I_2	-	0.85	-	0.85
I_3	-	0.8	-	0.8
W_1	4	4	4	4
W_2	4	4	4	4
W_3	4	4	4	4
$\widetilde{X}_{11}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{21}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{31}$(bps)	11.1	11.1	10.4	10.4
$\widetilde{X}_{12}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{22}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{32}$(bps)	11.1	11.1	10.4	10.4
$\widetilde{X}_{13}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{23}$(bps)	11.1	11.1	11.4	11.4
$\widetilde{X}_{33}$(bps)	11.1	11.1	10.4	10.4
L_{a1}(bps)	0.01	0.01	0.01	0.01
L_{a2}(bps)	0.01	0.01	0.01	0.01
L_{a3}(bps)	0.01	0.01	0.01	0.01
Z_{11}	1	1	1	1
Z_{12}	1	1	1	1
Z_{13}	1	1	1	1
Z_{21}	1	1	1	1
Z_{22}	1	1	1	1
Z_{23}	1	1	1	1
Z_{31}	0	0	0	0
Z_{32}	0	0	0	0
Z_{33}	0	0	0	0
$\widetilde{W}_{11}$	5	5	5	5
$\widetilde{W}_{12}$	5	5	5	5
$\widetilde{W}_{13}$	5	5	5	5
$\widetilde{W}_{21}$	4	4	4	4
$\widetilde{W}_{22}$	4	4	4	4
$\widetilde{W}_{23}$	4	4	4	4
$\widetilde{W}_{31}$	3	3	3	3
$\widetilde{W}_{32}$	3	3	3	3
$\widetilde{W}_{33}$	3	3	3	3

5. Conclusion

The model represented shows the connection between bandwidth required, bandwidth obtained and QoS by giving the assumptions and data; we can find the optimal solution with profit maximization. ISP has choices to whether adopt modified model 1 or modified model 2 according their priorities. Our proposed models show slightly better result than model proposed by [1].

However, due to assumptions, we have limited the model into static optimal solution and cannot be dynamic solution where we should have various demands for capacity (peak and off-peak).

Further research should address issue with more generalization of users and classes applying the model.

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