DEVELOPING STUDENTS' CONCEPTUAL UNDERSTANDING ON THE CONCEPT OF INTEGRAL THROUGH ITS HISTORICAL ASPECTS

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Extended Abstract

A good conceptual understanding is one type of understanding that student teachers must have because they will need it whenever they teach their students in school. However, many courses in college are taught in a very formal way that does not give students enough time to construct their conceptual understanding. Likewise, calculus courses in Mathematics Department of the State University of Surabaya are mostly taught in that way. As a result, many students are only skillful in calculating integral of a function but mostly do not really know the reason why is that so.

In this study, using Realistic Mathematics Education (RME) as the domain spesific theory, we focus on the introductory part of the concept of integral which consists of topics, namely: antiderivative, indefinite and definite integral. In order to help students to build their conceptual understanding on the concept of integral, we design learning trajectories following design research approach. The design consists of three meetings with three-cycle learning trajectories. The design starts with checking students' previous knowledge about differential using functions in physics, such as acceleration, velocity, distance traveled, and initial value problems. The second meeting facilitates students to construct their conceptual understanding on the Fundamental Theorem of Calculus by using the Exhaustion Method (the Method of Indivisible) and Cavalieri's Principle. The last meeting is used to formalize students' understanding. Using their knowledge from the second meeting, the students are introduced to Riemann sum using sigma notation and limit. In every meeting we check their conceptual understanding through presentation and discussion. Finally, we wrap up the learning together with the students to summarize what they have just studied by asking some conceptual questions, such as: what relation can you figure out between a function and its definite integral?

The results of this study is divided into three parts. First, in the first meeting that consists of four activities we try to contextualize the learning by giving motivational problems related to acceleration, velocity and distance travelled by a moving object (in this case, hot air baloon and free fall object) and applying initial value problems (IVP). We expect that they will use the concept of antiderivative, but it turns out that students directly use their knowledge on integration due to their knowledge in highschool. They do not really know why is that so. Moreover, they confuse about the initial condition given. It seems

that they are not familiar with the IVP. This activity is time consuming. Some students do not use integral concept, but recall their knowledge in highschool physics about motion and simply use the related formula instead. Based on those results, we revise the motivational problems such that it will not be too long and conceptually heavy, but rather to help students grasp the concept behind the idea.

In the second meeting that consists of three activities, we further study about students' understanding on the concept of definite integral through the method of indivisible and try to formulate the Fundamental Theorem of Calculus. Starting with the homework, an IVP associated with its graph of solutions, students are expected to determine the formula of the function passing through a certain point by solving the IVP. Then, using the graphs of the function and its antiderivative taken from the homework problem, students are asked to observed the relation between the area of the region below the function and above the x-axis on a certain interval and the values of its antiderivative function at the boundary the interval. This activity is intended to help students to explore the idea behind the concept of definite integral, not to find the solution. The following activities will use this experience and the indivisible method and Cavalieri' principle to find the area of a region that is bounded by a curve of a function and the x-axis. The activities really give students joy and insight toward the area of such region, even though it is still time consuming. Therefore, some revisions are needed for efficiency, for example change the orientation of ellipse figure when applying Cavalieri's principle and stressing that the Cavalieri' principle can only be applied to a certain ellipse.

In the third meeting, we bring the students into a more complex situation, i.e. asking students to calculate the area of a region which is bounded by a curve and x-axis. We expect that students use their experience of solving quadrature problems previously. Some problems arise when students try to solve the quadrature problem given. There are some confusions when students try to approximate the area of the region. They have difficulty to determine whether using lower or upper sum. Next activity is to construct Riemann Sum and combine it with indivisible method to to draw the Fundamental Theorem of Calculus. In this case, students need to be careful when formulating the Riemann sum, particularly when generalizing it involving the same size partition. Some notes for lecturer are added are to avoid those obstacles. The lecturer has to give demontrations about how to construct/draw lower or upper sum and about how to formulate the Riemann sum that will be used to derive the Fundamental Theorem of Calculus.

After those three meetings including their refinements, we conclude that the use of ivp helps students to figure out the relation between a function and its antiderivative. This will support students when constructing the Fundamental Theorem of Calculus. Moreover, the use of indivisible method helps students to have a good understanding toward the idea behind the definite integral concept. In addition, interviews with students give result that they response positively toward the teaching and learning

process have been done. They admit that the teaching and learning process is interesting and challenging, also really helps them to have a deeper understanding of the concepts compared to what they had obtained in highschool or other courses.