

## AN ANALYSIS OF STUDENT LEARNING WITH DIGITAL TOOLS FOR ALGEBRA FROM AN INSTRUMENTATION THEORY VIEW

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### **Abstract**

*In this paper, we describe the relationship between the use of digital tools and student conceptual understanding by interpreting students' behavior from an instrumentation theory perspective. The theoretical framework includes student difficulties in initial algebra learning and the instrumentation theory. The data included video registrations of a group of three seventh-grade students (12-13 year-old) using the Cover-up and the Balance Strategy applets for solving equations in one variable. The results revealed that the instrumentation theory provides an explicit relationship between techniques and schemes while solving the problems using digital tools. The techniques can be interpreted as observable procedural skills while using the digital tools and the schemes as conceptual understanding of the students, including their difficulties while solving the problems.*

**Keywords:** algebra education, applets, digital technology, equations in one variable, instrumentation theory.

### **INTRODUCTION**

Nowadays, Information and Communication Technology (ICT) plays an important role not only in daily life, but also in education, mathematics education, and algebra education in particular. Digital tools for learning mathematics, for instance, become more widespread and are used more frequently in the teaching and learning processes (e.g., Drijvers, & Barzel, 2012). The interest and the optimism in the potential of the digital tools for mathematics learning have grown and influenced educational stakeholders, such as mathematics educators, educational technology researchers, and teachers.

Review studies in mathematics education reveal that the use of ICT affects positively on mathematics achievement (Li & Ma, 2010) and on students' attitude towards mathematics (Barkatsas, Kasimatis, & Gialamas, 2009). Also, the use of ICT can attract students doing mathematical exploration and investigation (Ghosh, 2012). In algebra education, ICT use contributes significantly to student conceptual understanding and skills (Rakes, Valentine, McGatha, & Ronau, 2010). For example, the use of digital tools in algebra education can contribute to students' development of both symbol sense and procedural skills (Bokhove, & Drijvers, 2010), and may foster the development of the notion of the function concept (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012).

Concerning the above results, and in relation to the role of the use of technology toward student conceptual understanding and skills, several important pedagogical questions may rise: While using technology, digital tools in particular, what do students actually learn? How does technology influence students' conceptual understanding and procedural skills? What is the relationship between the use of technological tools and students' conceptual understanding and skills? (e.g., Drijvers, & Barzel, 2012). In this paper, we would like to investigate the subtle relationship between the use of digital tools, the applets for algebra education in particular, and student conceptual understanding. Also, we would like to see this relationship by considering its impact on student written work in the paper-and-pencil environment.

To address this issue, the theoretical framework of this paper includes student difficulties in algebra learning and the instrumentation theory. Next, the research question and method are presented and elaborated. Then, in the results and discussion section we describe an analysis of two episodes of student learning with digital tools – the Cover-up and the Balance Strategy applets – for algebra. Conclusions section finishes the paper.

## **THEORETICAL FRAMEWORK**

### **Difficulties in Initial Algebra**

Student conceptual understanding and skills in algebra can be analyzed in terms of difficulties emerged in the learning and teaching processes. Based on the literature review and explorative interview study, Jupri, Drijvers and Van den Heuvel-Panhuizen (2014) have identified five categories of difficulties in initial algebra. First, difficulties in applying arithmetical operations and properties in numerical and algebraic expressions include adding or subtracting like terms (e.g., Herscovics, & Linchevski, 1994; Linchevski, 1995); applying associative, commutative, distributive, and inverses properties; and applying priority rules of arithmetical operations (e.g., Booth, 1988; Warren, 2003). Second, difficulties in dealing with the notion of variable concern understanding it as a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Herscovics, & Linchevski, 1994). Third, the difficulties in understanding algebraic expressions include the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions (Arcavi, 1994; Thomas, & Tall, 1991). Fourth, the difficulties in understanding the different meanings of the equal sign include difficulties to comprehend it as a sign that invites a calculation in arithmetic, and as a sign of equivalence in algebra (Herscovics, & Linchevski, 1994; Kieran, 1981). And the fifth, the difficulties in mathematization concern the difficulty in transforming the problem situation to the world of mathematics and vice versa, as well as in reorganizing the symbolic world of mathematics (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

### **Instrumentation Theory**

To investigate the relationship between using a digital tool for algebra and student conceptual understanding and skills, we use the instrumentation theory. In this theory,

the following terms play a key role: artefact, tool, technique, scheme, and instrument (e.g., Drijvers, Godino, Font, & Trouche, 2013; Trouche, & Drijvers, 2010).

An artefact is an object, a thing, either tangible or not. If an artefact is used to carry out a specific task, it is called a tool. In this study reported in this paper, the artefacts include the Cover-up applet and the Balance Strategy applet. If the Cover-up applet, for instance, is used for solving an equation, this applet can be seen as a tool, a digital tool for solving the equation (Trouche, 2004).

In order to be able to use a tool for solving a specific task, we need to apply a specific *technique*. According to Artigue (2002) a technique is a manner of solving a task using an artefact. As such, the technique can be observed from the user's behavior while using the artefact for solving the task. The technique itself is based on the cognitive foundation, which is called a scheme. Vergnaud (1996) defines a scheme as an invariant organization of behavior for a given class of situations. The technique and the scheme share similar characteristics, but the main difference is that the scheme is invisible and the technique is observable. In other words, a technique is the observable manifestation of the invisible scheme (Drijvers, Godino, Font, & Trouche, 2013).

Based on the above description, an *instrument* is defined as a mixed entity of scheme, technique, artefact and task (Trouche, & Drijvers, 2010; Trouche, 2004). The process of the user developing an instrument, consisting of cognitive schemes and observable techniques for using a specific artefact for a specific class of tasks is called *instrumental genesis*. This combination of technical and conceptual aspects within the instrumentation scheme makes the theory of instrumentation is promising for investigating the relationship among digital tools use, student conceptual understanding, and written paper-and-pencil work (Drijvers, 2003).

## RESEARCH QUESTION

To address the issue phrased in the Introduction, that is, to describe the subtle and complex relationship between the use of digital tools, the Cover-up and the Balance Strategy applets for algebra in particular, and student conceptual understanding and skills, we formulate the following research question:

*Which relationships between techniques and understanding do students develop while using the Cover-up and the Balance Strategy applets for solving equations?*

The problems addressed in the present study include equations in one variable of the form  $f(x) = c$ , which is the main topic of the algebra lesson with the Cover-up applet; and linear equations in one variable of the form  $f(x) = g(x)$ , the main topic of the algebra lesson with the Balance Strategy applet.

## METHOD

The study reported in this paper is a case study which was a part of a larger experimental study (Jupri, Drijvers, & Van den Heuvel-Panhuizen, submitted). The case study was based on two lessons, the algebra learning with the Cover-up applet and with the Balance

Strategy applet, on (linear) equations in one variable which is a part of the grade VII Indonesian mathematics curriculum (Depdiknas, 2007). The learning arrangements included the activities with the two applets which are embedded within the Digital Mathematics Environment (DME)<sup>1</sup>. The DME is a web-based environment providing: (i) interactive digital tools for mathematics learning; (ii) a design of open online tasks; (iii) immediate feedback for the tasks; (iv) access at any time and place, as long as technological infrastructure is met; and (v) a storage for student work (Boon, 2006; Drijvers, Boon, Doorman, Bokhove, & Tacoma, 2013).

Each lesson lasted for 80 minutes, and consisted of the following three parts. First, a paper-and-pencil activity was done to introduce the concept of equation through posing problems and class discussion. Second, a whole class demonstration how to work with an applet was carried out and was followed by a group-based digital activity done by students under the teacher guidance. Third and final, an individual paper-and-pencil test was carried out.

The data reported in this paper included video registrations of one group of three male students (12-13 year-old), its corresponding student digital work, and written work. By applying the instrumentation theory lens and student difficulties in algebra as described in the Theoretical Framework section, an integrative qualitative analysis on these data, with the help of Atlas.ti software, was carried out to investigate the relationship between the use of the applets and the targeted conceptual understanding.

## RESULTS AND DISCUSSION

This section presents the observation of two episodes of a group work of the three students: one episode from the Cover-up activity and another one from the Balance Strategy activity. For each episode, we first provide an analysis and its corresponding commentaries of a task carried out by the students in our observation. Next, we summarize the relationship between techniques and its corresponding schemes while solving the task. Then, we discuss student difficulties and understanding from the observation, including written student work from the individual paper-and-pencil test. The techniques can be interpreted as procedural skills and schemes as conceptual understanding of the students.

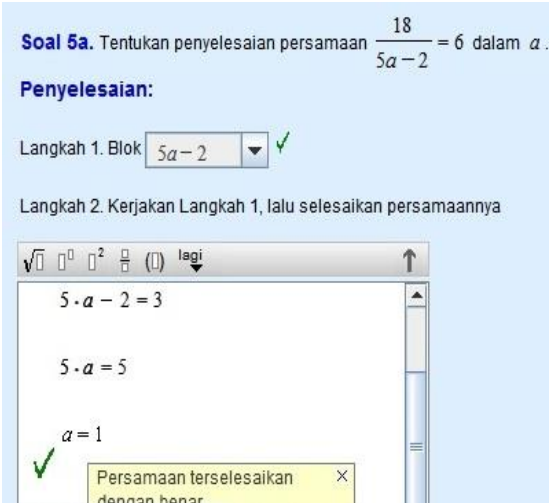
### Relationship between Techniques and Understanding in the Cover-Up Activity

Table 1 presents an observation of the group's work on a task from the Cover-up activity. The left column provides a description of the observation, and the right column provides commentaries which are based on the theoretical lens described in Section 2.

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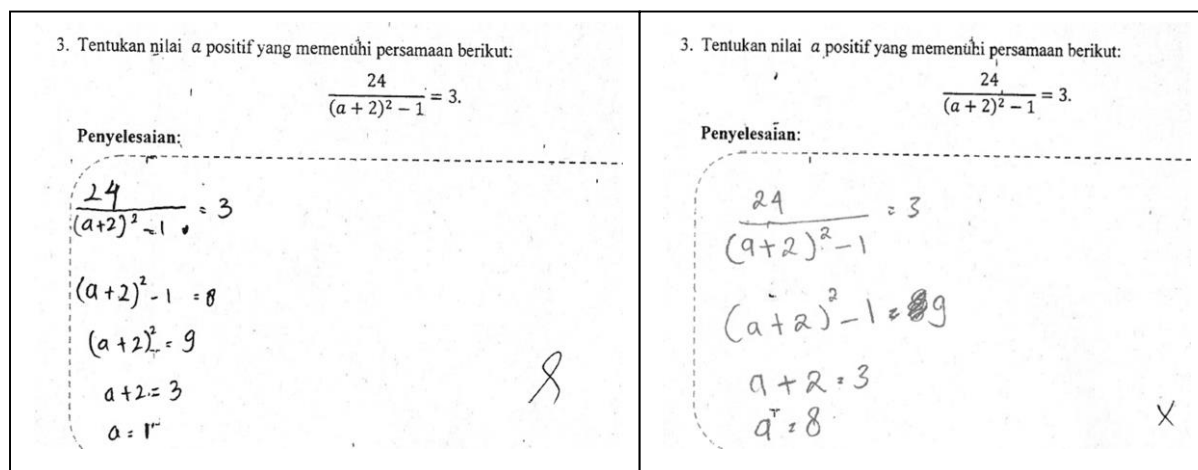
<sup>1</sup> The Cover-up and the Balance Strategy applets, embedded in the DME, are developed by Peter Boon, Freudenthal Institute, Utrecht University, the Netherlands

Table 1.  
A case observation of the group's work for a task taken from the Cover-up activity

Observation	Commentary
<p>Task 5a. Solve the following equation for <math>a</math>:</p> $\frac{18}{5a - 2} = 6$ 	<p>Task 5a is a problem addressed in the Cover-up activity. The figure below the task shows the student digital work stored in the DME.</p>
<p>Students in the group read the task out loud. They are thinking and reading the given first step, and choosing a part of the equation that should be covered firstly.</p>	<p>By reading the task, students are expected to realize that the equation is in the form of <math>f(a) = c</math>, and to realize that <math>a = \langle \text{numerical value} \rangle</math> as the solution. In addition, they are expected to be able to see the structure of the algebraic expression in the left hand side as a division between 18 and <math>5a - 2</math>. In this way, the students would understand that they should choose <math>5a - 2</math> as the expression to be covered in the first step.</p>
<p>Student 1: So, the equation means that 18 is divided by "something" equals 6.</p> <p>Student 2: Three, three, three, ... [He then tries to highlight <math>5a - 2</math> with the mouse. However, he could not highlight <math>5a - 2</math> as the first part of the equation to be covered. Rather than covering <math>5a - 2</math>, the Student 2 highlights the whole equation.]</p>	<p>After successfully doing the given first step, Student 1 interprets correctly the left part of the equation as "18 divided by something" equals 6. This means he understands the structure of the equation.</p> <p>Student 2 knows that the value of <math>5a - 2 = 3</math>.</p>
<p>Student 1: Is 5 subtracted by 2 equals 3 (?)</p> <p>Student 3: Why is it difficult to be covered? [He tries to highlight <math>5a - 2</math>. Next, After Student3 has been successfully covering <math>5a - 2</math> and assigning its value 3, i.e., by highlighting <math>5a - 2</math> with the mouse, and typing 3, he forgets to press enter.]</p>	<p>Even if Student1 knows the first part to cover is <math>5a - 2</math>, he seems not to understand why the Student2 assigned 3 as the value for <math>5a - 2</math>. Student 1 guesses that 3 comes from <math>5 - 2 = 3</math>.</p>
<p>Student 2: Enter!</p>	<p>This misunderstanding reveals a student difficulty in understanding <math>5a - 2</math> as an</p>

[Student 3 presses enter and it is correct!]	algebraic expression, and an expected answer obstacle in particular.
Student 1: $a$ [should be covered from the equation $5a - 2 = 3$ .]  Student 2: No! Next to be covered is $5a$ , $5a$ , $5a$ , ... Student 3: Yes, $5a$ . Student 1: Its value is 3 Student 3: 1 Student 1 & 2: No, no, it is 5. [Laughing. Student3 highlights $5a$ from the equation $5a - 2 = 3$ , the applet automatically produces $5a = \dots$ in the line below. He then types 5 and presses enter. It is correct!]	Student 1 seems not to understand that the easier part to cover for the equation $5a - 2 = 3$ is $5a$ than $a$ directly. Student2 suggests that $5a$ is the easier part to cover and to assign a value.  Student1 and Student3 could not assign a proper value for $5a$ . This because they do not understand the meaning of the equation $5a - 2 = 3$ as "something is subtracted by 2 equals 3". This suggests a difficulty in understanding a variable.
Students 1 & 2: Now $a$ , $a$ , $a$ [should be covered from the equation $5a = 5$ .]  Student 3: 5 times something ... Student 2: 5 times something equals 5 Students 1 & 3: [So the value of $a$ is] 1.  [Student 3 highlights $a$ with the mouse, types 1 as a numerical value for it, and presses enter. ]  Student 3: Correct! Yes! [The applet provides a final feedback "The equation is solved correctly!" and a green tick mark!]	After getting $5a = 5$ from the previous step, the students seem to understand how to proceed: choosing $a$ , highlighting it, typing 1 as its value, and pressing enter!

From the above observation, first we summarize the relationship between techniques used by the students while using the Cover-up applet for solving an equation and the corresponding conceptual understanding. Based on the instrumentation theory, we view that the conceptual understanding can be described as schemes that students used for solving the equation and the techniques represent procedural skills for solving the equation. In our view, recognizing the equation  $\frac{18}{5a-2} = 3$  as the form  $f(a) = c$  and its structure as a division between 18 and  $5a - 2$ , as well as recognizing  $a = <$  numerical value  $>$  are the schemes that are invisible, in the sense that there are no observable specific techniques. Identifying, for instance,  $5a - 2$  as the sub-expression to be covered at the start of the cover-up strategy is a scheme that corresponds to a technique of highlighting the identified sub-expression using the mouse. Finally, assigning a numerical value to the covered sub-expression is the scheme for the technique of typing a numerical value and pressing enter.



**Figure 1.** Written work by Student 2 (left) and Student 1 (right) after engaging in the Cover-up activity

Based on the conversations during the group work, we conjecture that Student 2 seemed to have a better understanding than the other two students in the group. We identified that Students 1 and 3 encountered difficulties in understanding algebraic expression (an expected answer obstacle), such as when viewing  $5a - 2$  as  $5 - 2 = 3$ , and in understanding the concept of variable as an unknown. The different mastery of understanding of the students is manifest in their written work, solving a similar task as in the digital group work, from the individual paper-and-pencil test after engaging the Cover-up activity as shown in Figure 1. The left part shows a correct Student 2's work and the right part displays an incorrect Student 1's work, which is similar to Student 3's work.

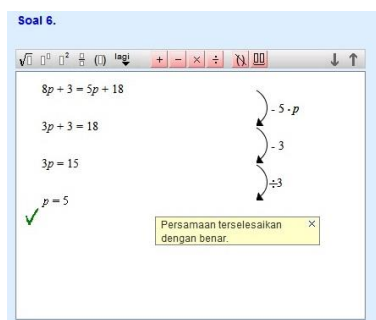
**Relationship between Techniques and Understanding in the Balance Strategy Activity**

Table 2 presents an observation of the group's work on a task from the Balance Strategy activity. The left column provides a description of the observation, and the right column provides commentaries which are based on the theoretical lens described in Section 2.

Table 2.

*A case observation of the group's work for a task taken from the Balance Strategy activity*

Observation	Commentary
Task 6. Solve the following equation for $p$ : $8p + 3 = 5p + 18$	Task 6 is a problem addressed in the Balance Strategy activity. The figure below the task shows the student digital work stored in the DME.



<p>Students read the task together aloud.</p>	<p>By reading the task, students are expected to realize that the equation is in the form of <math>f(p) = g(p)</math>, and to realize that <math>p = &lt; \text{numerical value} &gt;</math> is the solution.</p>
<p>Student 1: [It must firstly be] subtracted by 3, subtracted by 3.          Student 2: No, it is subtracted by <math>5p</math>.          Student 1: Okay, it is directly [subtracted by] <math>5p</math>.          Student 2: [He clicks the subtraction button from the applet toolbar and types <math>5p</math>, and presses enter. It is correct!]          Observer: Please you type the result in the next line!          Students 1 &amp; 3: <math>3p + 3 = 18</math>          Student 3: [He types <math>3p + 3 = 18</math> on the solution window, and presses enter. Correct!]</p>	<p>Student 1 and Student 2 have different ideas about the first step to do for solving the equation. The Student 1 suggests to subtract 3 from both sides of the equation, while Student 2 suggests to subtract <math>5p</math> from both sides. Both ideas are correct.           Student 1 follows Student 2's idea probably because he realizes that either his or his friend idea is correct.</p>
<p>Observer: What is the next [step]?          Students 1 &amp; 2: Subtracted by 3.           Student 1: <math>3p = 18</math>, eh no, <math>3p = 15</math>          Student 3: [Types <math>3p = 15</math>.]</p>	<p>Both Students 1 and 2 agree to subtract 3 from both sides of the equation <math>3p + 3 = 18</math>.           Initially, Student 1 forgets to subtract 3 from the right side, but then he realizes that he should do so.</p>
<p>Student 1: Divide!          Student 1: Divide, divide....          Student 2: Divide by 3.          Student 1: <math>3p</math>?          Student 2: No, not <math>3p</math>.          Student 3: [He clicks the division button, types 3, and presses enter.].          Student 2: <math>p = 5</math>.          Student 3: [Types <math>p = 5</math>, and presses enter. It is correct as indicated by the final feedback, "The equation is solved correctly!]</p>	<p>Even if Student 1 seems to know that the next step to do for solving <math>3p = 15</math> is by doing a division to both sides of the equation, he is still not sure whether to divide by 3 or by <math>3p</math>. Student2 suggests Student1 that the equation must be divided by 3 and not by <math>3p</math>.</p>

The relationship between techniques used by the students while using the Balance Strategy applet for solving an equation and the corresponding conceptual understanding is the following. In our view, recognizing the equation  $8p + 3 = 5p + 18$  as the form  $f(p) = g(p)$  and realizing  $p = < \text{a numerical value} >$  as the solution are the invisible schemes which students should have in order to be able solve the equation. Having ideas to choose an appropriate operation, with corresponding a number or an expression, and to carry out it to both sides of the equation are the schemes that corresponds to techniques of choosing and clicking an appropriate operation button, typing a number or an expression, and pressing enter. Finally, providing a result of the operation carried out to both sides of the equation is the scheme that corresponds to the technique of typing a



new simpler equation and of pressing enter to check whether the new equation is correct or not.

Main possible difficulties that might emerge while solving the equation  $f(p) = g(p)$  include, for instance, choosing an appropriate operation to do to both sides of the equation, doing the same operations to both sides of the equation—such as forgetting to subtract 3 from both sides of the equation  $3p + 3 = 18$  as the case in our observation, and arithmetical calculation mistakes. Of course there are other possible difficulties, such as applying a distributive property if the equation contains an algebraic expression with a bracket.

From the conversations, similar to the case of the Cover-up activity, we conjecture that Student 2 seemed to have a better understanding than the other two students in the group (at least better than Student 1, as Student 3 did not speak to much in the conversation). For instance, Student 1 made mistakes when simplifying the result of subtracting 3 from both sides of the equation  $3p + 3 = 18$ , in which he initially thought  $3p = 18$  as the result; also Student 1 seemed to not sure what to divide for the equation  $3p = 15$ , whether to divide by 3 or  $3p$  to obtain the solution. This conjecture is strengthened by observing written student work shown in Figure 2. The left part shows a correct Student 2's work and the right part displays an incorrect Student 1's work, which is similar to Student 3's work.

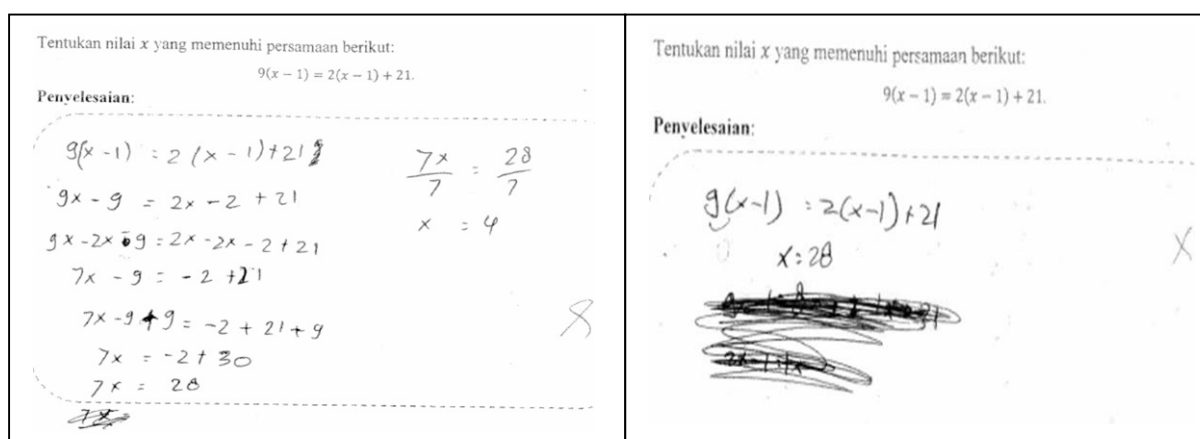


Figure 2. Written work by Student 2 (left) and Student 1 (right) after engaging in the Balance Strategy activity

### CONCLUSIONS

The relationship between techniques and understanding that students develop while using the Cover-up applet for solving equations includes: highlighting an expression, typing a numerical value for the highlighted part and pressing enter to check correspond to the ability to see an appropriate expression to be covered and to assign an appropriate numerical value for the covered expression. The main relationship between techniques and understanding that students develop while using the Balance Strategy applet for solving equations includes: choosing and clicking an appropriate operation button, typing a number or an expression, typing a new simpler equation, and pressing enter

correspond to finding an appropriate operation—with a corresponding number or an expression, and to providing the result of the operation carried out to both sides of the equation. To establish these relationships, students should perceive the interplay between the techniques that they use and their cognitive schemes.

We showed that the two theoretical lenses, student difficulties in algebra and the instrumentation theory, are fruitful to analyze student work with digital tools. The first lens is fruitful to better understand student conceptual understanding in terms of encountered difficulties while solving equations. The instrumentation theory shows how the conceptual understanding (in terms of the schemes) and procedural skills (in terms of techniques) are linked to each other.

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