DEVELOPING THE 5TH GRADE STUDENTS' UNDERSTANDING OF THE CONCEPT OF MEAN

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Abstract

One of the first statistical measures that students encounter in school is the arithmetic mean, sometimes known as a mean or an average. Many studies have made an effort to promote students' understanding of the concept of mean. The present study also focused on developing students' understanding of the concept of mean based on Realistic Mathematics Education (RME) also known as PendidikanMatematikaRealistik Indonesia (PMRI). The goal is to contribute to a local instructional theory in learning the concept of mean. The research question of this study is how the measuring contexts can support students developing their understanding in learning the concept of mean. We used design research as the methodology to develop students' understanding in learning the concept of mean. Three activities on measuring were designed; (1) repeated measurement context; (2) prediction on glider experiment; and (3) prediction on glider experiment on the bar chart. The subjects were one class of 5th grade; consists of 29 students and one teacher in SD InpresGalanganKapal II Makassar. The data were collected through video-recordings of lessons and students' written works. The results show that the measuring activities were meaningful contexts for students to develop their initial understanding of the concept of mean.

Keywords: The Concept ofmean, Measuring activities, Realistic Mathematics Education (RME), PendidikanMatematikaRealistik Indonesia (PMRI).

INTRODUCTION

The mean in this study refers to the arithmetic mean, one of the measures of central tendency in statistics together with the mode, median, and midrange. Almost all countries introduce the mean starting from primary school. In Indonesia, for instance, the new curriculum 2013 mandates the schools to introduce the concept of mean from 5th grade (the previous curriculum started from 6th) (Kemmendiknas, 2013). It is important to understand the concept of mean because it is not only a mathematical school topic, but it is also frequently used in everyday life, for example, the average of the velocity of a car, the average of the students' scores in a classroom, and the average of people's incomes in a country.However, psychologists, educators, and statisticians all experience that many students, even in college do not understand many of the basic statistical concepts they have studied. Some studies also describe the difficulties regarding the concept of mean (Strauss &Bichler, 1988; Gal et al., 1989; Zazkis, 2013; Hardiman et al., 1984).

Most students understand the mean as an "add-them-all-up-and-divide" algorithm (Zazkis, 2013). Moreover, many elementary and middle school mathematics textbooks have defined the mean as the way it is computed (Bremigan, 2003). It is also supported by the exercises and the examples elaborated which do not allow students to develop their understanding of the concept of mean. Most of them are procedural problems where the students only use the formula when the data are given. Unfortunately, in Indonesia, most teachers teach the concept of the mean in the traditional way, focusing on the computation but not the understanding of the concept of mean. They tend to follow the definition and the problems provided in the textbooks without elaborated more on developing students' understanding of the concept.

Therefore, we need to support students to learn not only how how to calculate the mean but also to develop their understanding of the concept of the mean itself. However, hardly any Indonesian study with this concept of mean has been carried out neither in the theory nor practical studies. Based on those issues, it is important to design meaningful contexts and activities in order to support the developing of students' understanding about the concept of the mean. Therefore, the present study concerns on developing students' understanding in learning the concept of mean. In this study, we focus on measuring activities. The research question of this study is: *how can the context support 5th grade students to develop their understanding in learning the concept of mean?*

THEORETICAL FRAMEWORK

The mean is the most common measure of central tendency that is used in many studies. In the school, most of the textbooks define the mean as the way it is computed, add the data and divide it by the number of data. However, it is not a simple mathematical entity. The mean is not as simple as the algorithm. It is interrelated with the concepts of center and spread. The interrelation has been described by Strauss & Bichler (1988). It proposed seven properties of the arithmetic mean. These properties show clearly that the mean has a strong relation with other statistical measures. In addition, instead of all seven properties, the learning design of this study will focus on four properties :

a the average is located between the extreme values;
b the average is influenced by values other than the average;
c the average does not necessarily equal one of the values that was summed;
d the average value is representative of the values that were averaged.

The complexity of the concept mean also is illustrated in a study by Mokross & Russel (1995). They found five predominant approaches used by students; (a) average as the mode, (b) average as an algorithm, (c) average as reasonable, (d) average as a midpoint, and (e) average as a mathematical point of balance. The students who used the first two approaches did not recognize the notion of representation, while the three other approaches were considered to imply the concept of mean as the representation of a data set. This present study considers (a), (c), and (d) as the conjectures students may use during the lesson. Since the focus of the study was young students that have never been taught about the mean, (b) and (e) are excluded.

In addition, the interpretation of the mean is not easy. Konold & Pollatsek (2004) illustrates four interpretations of the mean : (a) data reduction, (b) fair share, (c) typical value, and (d) signal in noise. Table 1 is taken from Konold & Pollastek's article which provides the example context for the four interpretations. From the four interpretations, the main focuses of this study are the typical value and the signal in noise. Furthermore, an interesting article from Bakker & Gravemeijer (2006) provided an historical phenomenology of the mean. One way of using the mean in ancient times is as an estimation, which is quite similar to the interpretation of the mean as the signal in noise.

Table 1. The four interpretations of the mean

Interpretation/ Meaning	Example context
Data reduction	Ruth brought 5 pieces of candy, Yael brought 10 pieces, Nadav brought 20, and Ami brought 25. Can you tell me in one number how many pieces of candy each child brought? (From Strauss &Bichler, 1988)
Fair Share	Ruth brought 5 pieces of candy, Yael brought 10 pieces, Nadav brought 20, and Ami brought 25. The children who brought many gave some to those who brought few until everyone had the same number of candies. How many candies did each girl end up with? (Adapted from Strauss & Bichler, 1988)
Typical value	The numbers of comments made by eight students during a class period were 0, 5, 2, 22, 3, 2, 1, and 2. What was the typical number of comments made that day? (Adapted from Konold& Garfield, 1992)
Signal in noise	A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each student were 6.2, 6.0, 6.0, 15.3, 6.1, 6.3, 6.2, 6.15, 6.2. What would you give as the best estimate of the actual weight of this object? (Adapted from Konold& Garfield, 1992)

Many studies focus on how the mean is taught in school. Some studies made an effort to promote students' understanding of the mean. Some models and contexts were tested in those studies (Hardiman et al.,1984; Zaskis, 2013; Cortina 2002; Gal et al., 1990; Gal et al., 1989; Bremigan, 2003). Promoting a meaningful understanding also means that the problems or the contexts should be meaningful for students. It is in line with the idea of Realistic Mathematics Education. RME is a theory of mathematics education which believes that mathematics should be taught in a meaningful way to students. The idea of RME has largely been determined by Freudenthal's view of mathematics as a human activity. It must be related to the reality, close to the students' world, and relevant to the society (van den Heuvel-Panhuizen, 2001).

METHOD

The main goal of this study is to contribute to a local instructional theory for students in learning the concept of mean. To achieve this goal, an instructional sequence is designed as an innovation to improve mathematics education. Therefore, a design research is chosen as the methodology in this study. It provides a methodology to understand and to improve the educational practices through an iterative process (van den Akker, et. al., in press).

This study was conducted in the 5th grade of SD InpresGalanganKapal 2 (elementary school) in Makassar. We planned to have three cycles. However, this present paper focuses on the second cycle. The participant consists of 29 students and one teacher. The data were collected through video-recordings of lessons and students' written works.

RESULT AND DISCUSSION

Activity 1 : Repeated Measurement Context

This meeting the teacher told the story of one person that had five times different measurements in a month. The context was about a woman, namely Anita was taking five times height measurements in a month (police station, hospital, roller coaster, fitness center, and herself). The result showed the five different measurements (look figure 1). The students worked in group to help Anita decide her height. The aim of this activity is to see how students' strategies to choose one measurement as the representatives of five measurements.

As in our conjectures, the students chose one of the five measurements they thought more convincing. Most of the students took the highest measurements since they thought that the higher the better. When the teacher asked to consider all the measurements, the students still kept their own answer. This showed that the context did not work as we expect since the students did not consider other measurements to decide the representative measurement.

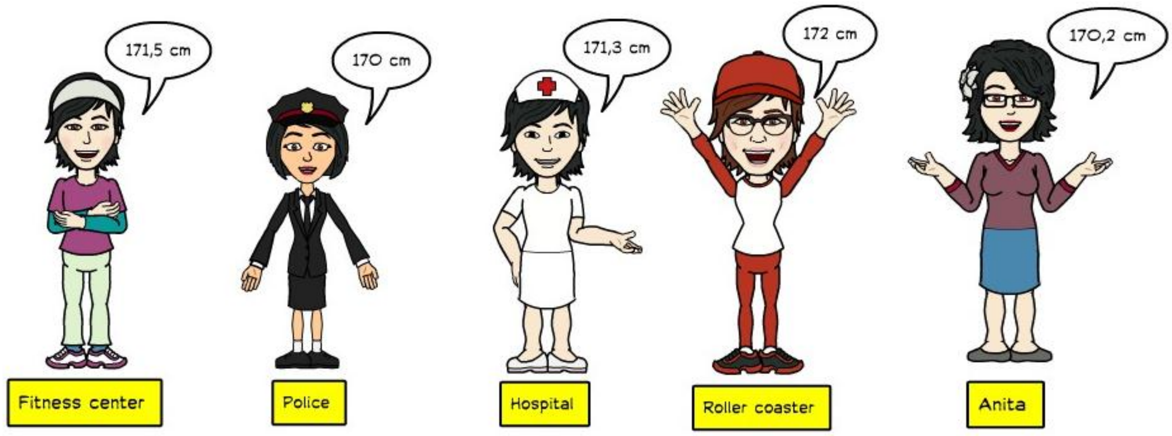

Figure 1. The different weights of apples

Activity 2 : Glider Experiment (Prediction)
At this meeting, the students had an experiment to make Hagravens' cylinder glider. This activity adapted by Ainley, J., Jarvis, T. and McKeon, F. (2011). They have thrown the glider five times and measured the distance of the glider. Lastly, they were asked to predict the next throw by considering the five data of measurement they had. The aim of this activity is to investigate how the students' strategy to predict the next throwing of the glider.

Figure 2. The students thrown the glider away and measures the distance

As the result, all groups showed different strategies in predicting the next throw. They used their own strategy to predict the next throw. The following transcript shows a discussion to decide the prediction. In the beginning, these two students used a scratch paper to find their strategy.

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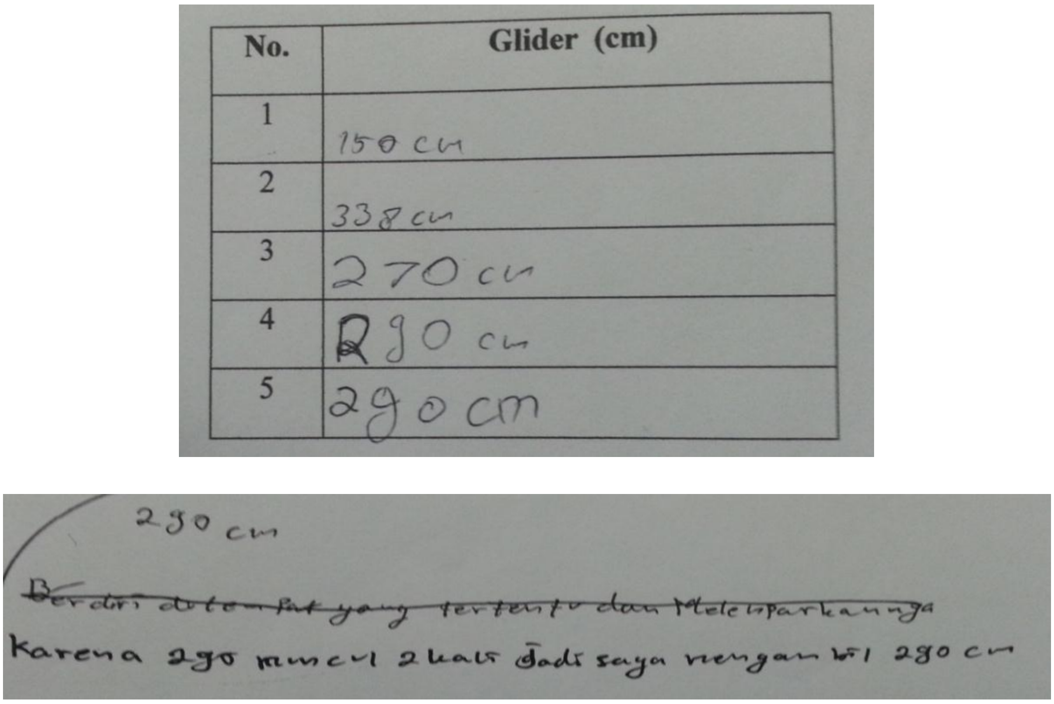
1 Student A : Add then divided it by 5 (Scratching on paper)
2 Student B : mm… (looking at her friend's writing)
3 Student A : Subtract it … Zero … eight minus two ..hmm six .. six minus five is one … one minus two hmm it cannot … (Crossing the scratch paper)

The two students tried to applyrandom formulas to decide the prediction. Interestingly, they used the idea of mean – add all of the data and divided it by the number of data (line 1). However, during execution, she changed her mind to subtract the data (line 3). Since she did not succeed, they tried another strategy. This short discussion showed that the formula of mean did exist in students' discussion and strategy. However, since there is no teacher around and also the students faced difficulty with formula, they did not continue their thinking. The following transcript showed the two students next strategy.

4 Student A : I think, it will be between this (pointing 230 cm) and this (pointing 320 cm)
5 This (pointing 450) is not possible because it was an accident. (looking at her friend with smile)
………
6 Student A : So,330 and 320. Do I write it down? (looking at her friend)
7 Student B : (Nodding) write down "or"

As we can see from the transcript, the students predict the next glider by considering all of the data. This is what we expect from the HLT that the students will consider all of the data in deciding the prediction.

Furthermore, there were interesting strategies to predict the glider using the idea of a mode. The students have the same result and they chose this data as their prediction (Figure 3).



No.	Glider (cm)
1	150 cm
2	338 cm
3	270 cm
4	290 cm
5	290 cm

Figure 3. Students' strategy using mode

Besides, there was also a group that used the idea of mode but they used the boundary (Figure 4). Since in their data they saw that 400 cm appeared three times, the prediction was 400 cm.

As the conclusion, the glider activity was rich context that can allow students to have their own strategy. The students realized that they had to consider all of the data in order to predict the next throw. From the students' strategy, we can start to introduce the idea of measures of central tendency such as the mode, the median, or even the mean. In addition, since this is a hands-on activity, the students were interested to play with a glider.

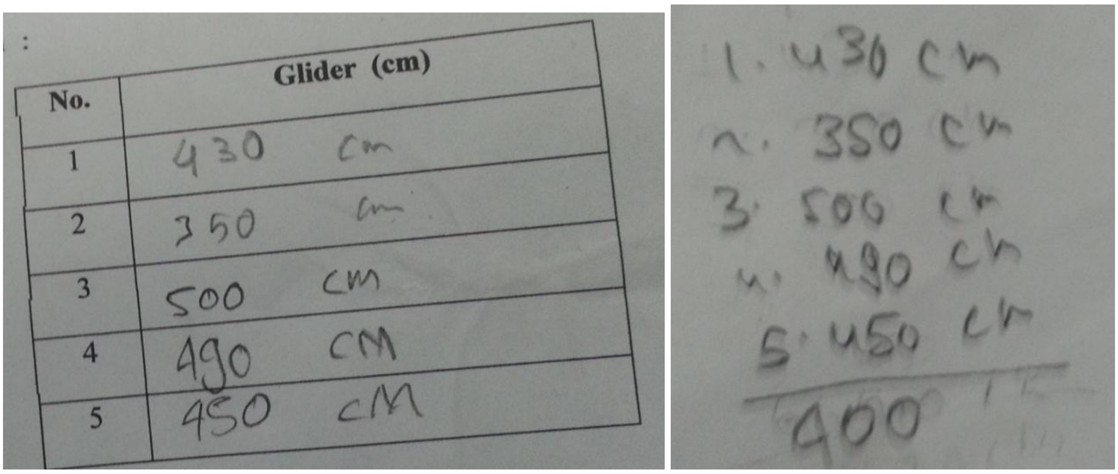


Figure 4. Students' strategy using mode and boundary

Activity 3 : Glider Experiment on Bar Chart

Instead of playing with students' data, this activity played with the data from the teacher's glider. In the beginning, the teacher gave two data of glider measurements. And the students were asked to predict the next throwing and also interpret the way they predict into the bar. In the next question, the students were given the third data, and again asked to find the fourth throwing. Similarly, they have to interpret their prediction into the bar. Lastly, they were given the fourth throw data and ask for the next throw and also the bar. The aim of this activity is that the students can be able to derive the formula of mean by using compensation strategy on the bar.

As the result, since this is the first time students encountered with the chart, they firstly have different drawing (Figure 5). However, the way they predict the next throwing used the idea of median. They took the middle of those two data. This is what we expect students predict the next throw. In order to have similar drawing, the teacher drew the bar on the whiteboard and showed how they interpret the next throwing by using the bar. At this time, the teacher interpreted the prediction with the compensation strategy on the bar.

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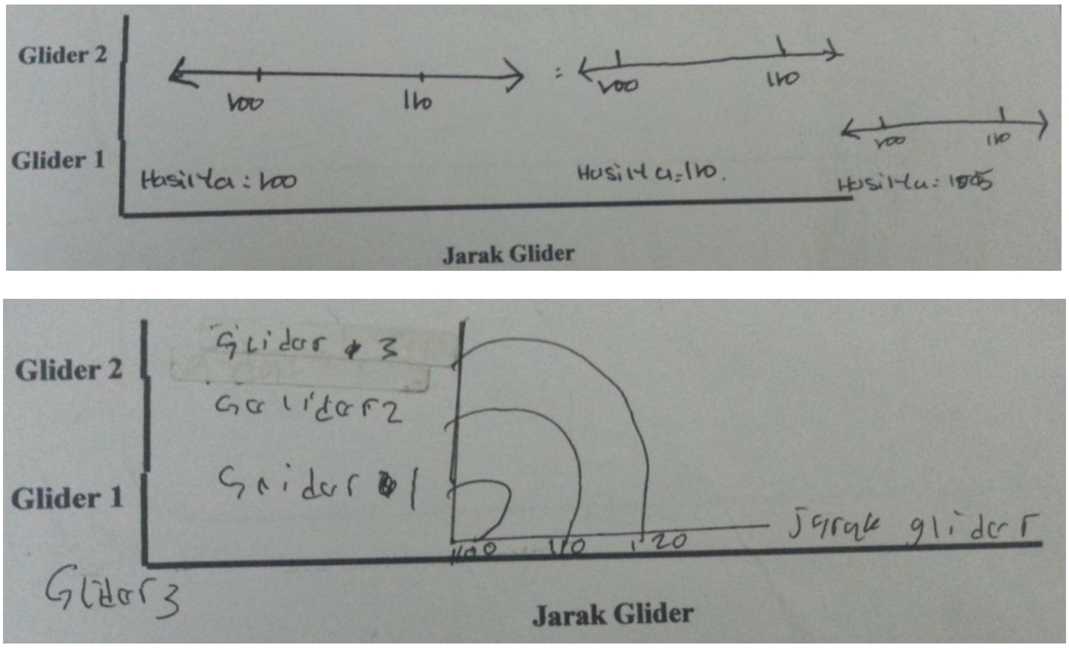

Figure 5. Students' first drawing of the graph.

The following transcript shows how the students discuss on compensation strategy.

1	Student C :	Adding five .. (start writing)
2	Student D :	Adding ten, idiot (taking a pen from student A)
3	Student C :	Adding ten? … hmm… adding ten, here (taking a pen from student B)
4	Student D :	Adding ten's here (start writing) .. Finish … So, a hundred is the result (based on the picture)
5	Student C :	(Looking at the whiteboard) This is the first, right? (pointing at the picture)
6	Student D :	One hundred and ten minus ten… ten.. after that put the ten here (Pointing at 90 and explaining the drawing)
7	Student C :	(Writing the formula 90 + 100 and stop)
8	Student D :	Add it by 100 ..
9	Student C :	What is the divisor here?
10	Student D :	Add it by 100 …
11	Student C :	(Adding 100)
12	Student D :	What is the sum?
13	Student C :	(Scratching on his hand) Wait a minute I calculate it first.

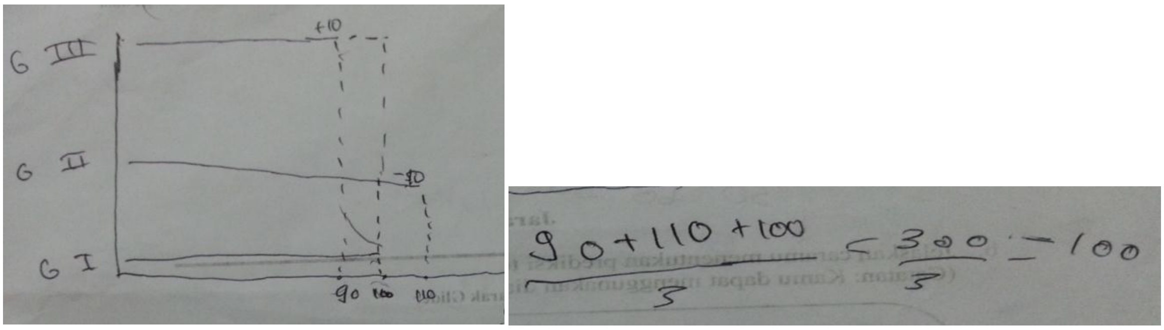

Figure 6. Students' strategy in predicting the fourth throw

The transcript showed that student C explained how to draw the prediction of the next throwing using the bar. The discussion showed that he realized the compensation strategy on the bar chart by adding or subtracting some part of a bar to another part. And then they can figure out the formula by adding all of the data and divide it by three.

The students can interpret the compensation strategy into the formula. Thus, compensation strategy supports students to visualize the formula of mean.

CONCLUSION

Before we elaborate the conclusion, it should be noticed that this study is the part of design research. The conclusion that we draw is based on the result on the second cycle. The students cannot solve the problem considering all data through repeated measurement problem. They used to choose one single measurement which was more convincing. However, the glider experiment as a hands-on activity can engage students to consider all the data to predict the next throwing of the glider. It was a rich context that can allow students to use varyingstrategies, including the idea of measures of center such as mode, median, or even mean. In addition, the students can learn how to gather their own data and they can use their own data. Regarding the concept of mean, the bar chart on glider experiment supports students to visualize the formula of the mean by using compensation strategy. Lastly, the students can also demonstrate how to find the average of a set of data.

REFERENCES

Bakker, A. (2004). *Design Research in Statistics Education: On Symbolizing and Computer Tools*. Utrecht: Wilco, Amersfoort.

Bakker, A., &Gravemeijer, K. P. (2006). An historical phenomenology of mean and median. *Educational Studies in Mathematics, 62*, 149 - 168.

Bremigan, E. G. (2003). Developing a meaningful understanding of the mean. *Mathematics Teaching in the Middle School*, 22 - 26.

Cai, J. (1998). Exploring students' conceptual understanding of the averaging algorithm. *School Science and Mathematics*. 93–98.

Cortina, J. L. (2002). Developing instructional conjectures about how to support students' understanding of the arithmetic mean as a ratio. *ICOTS6*.

Gal, I., Rothschild, K., & Wagner, D. A. (1989). Which group is better? The development of statistical reasoning in elementary school children, *The Annual Meeting of the Society for Research in Child Development*. Kansas City, MO.

Gal, I., Rothschild, K., & Wagner, D. A. (1990, April). Statistical concepts and statistical reasoning in school children: Convergence or divergence. In *annual meeting of the American Educational Research Association, Boston, MA*.

Hardiman, P. T., Well, A. D., &Pollatsek, A. (1984). Usefulness of a balance model in understanding the mean. *Journal of Educational Psychology*, 792-801.

Heiman, G. W. (2011). *Basic Statistics for the Behavioral Sciences*. Canada: Wadsworth Cengage Learning

Heuvel-Panhuizen, M. V. D., &Drijvers, P. (in press). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Dordrecht, Heidelberg, New York, London : Springer.

Kemmendiknas. (2013). *KompetensiDasarSekolahDasar (SD)/Madrasah Ibtidaiyah (MI)*.

Konnold, C., &Pollatsek, A. (2004). Conceptualizing an average as a stable feature of a noisy process. *The Challenge of Developing Statistical Literacy, Reasoning and Thinking,* 169-199.

Lestariningsih, Putri, R. I. I., &Darmawijoyo. (2012). The legend of Kemaro Island for supporting students in learning average. *IndoMS. J.M.E*, 203-212.

Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, 20-39.

Spatz, C. (2008). *Basic Statistics Tales of Distributions*. USA: Thomson Wadsworth.

Strauss, S., &Bichler, E. (1988). The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education*, 64-80.

Zazkis, D. (2013). On students' conceptions of arithmetic average: the case of inference from a fixed total. *International Journal of Mathematical Education in Science and Technology*, 204-213.