# SUPPORTING STUDENTS' COMPUTATIONAL ESTIMATION SKILL BY INFORMAL DAN FORMAL STAGE OF ROUNDING NUMBERS

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### Abstract

*Estimation is an important skill that should be owned by the students due to the relevancy to the everyday life computations. However, in reality students are better in doing exact computation rather than computational estimation. Students prefer to use the exact computation rather than estimation even when they were asked to estimate. This study aimed to acquire learning trajectory of computational estimation. The Indonesian Realistic Mathematics Education (PMRI) approach was applied to the subject of this study (i.e 4th grade students. Design research was chosen to reach the research goal by designed a Hypothetical Learning Trajectory (HLT) which is consists of three components: the goals of activity, the description of activity, and the conjectures of students' thinking. The HLT is applied to 38 students at MIN 2 Palembang in two cycles, namely pilot experiment and teaching experiment. The results showed that the application of learning trajectory which is begin with informal and formal stage of rounding numbers can support students to use rounding strategy and to apply (use/find) compatible number strategy in solving computational estimation problems.*

**Keywords**: *Computational estimation, PMRI, compatible number strategy, rounding strategy, design research.*

## INTRODUCTION

Estimation is an important skill that should be owned by the students due to related to mental computation which is applied in everyday life. According to Reys (2009) more than 80% of all mathematical computations in daily life involve mental computation, which is an important aspect on numeracy (Callingham, 2002) and estimation.

The importance of the estimation skill for students are: the student can control the correctness of the answer without doing a re-calculation and also control the occurrence of misconceptions based on the logic (Rizal, 2011), students can answer in a short time to determine whether an electronic calculators works or not (Tsao, 2011), and students can find approximate answers before doing the calculation (Lan et al, 2010). However, in reality students prefer to use exact computation rather than estimation even when they were asked to estimate (Boz & Bulut, 2012).

Therefore, this study aimed to acquire learning trajectory of computational estimation by using Pendidikan Matematika Realistik Indonesia (PMRI). One of characteristic of PMRI is using contexts related directly with the students (Sembiring, 2010). Therefore, the context of this research is a situation problems which is related to informal and formal stage of rounding numbers as the first stage of learning estimation based on Van den Heuvel-Panhuizen (2005).

**THEORETICAL FRAMEWORK**
1.  **Computational Estimation**
Computational estimation is defined as finding an approximate answer to arithmetic problems without actually (or before) computing the exact answer (Lemaire&Lecacheur, 2002). According to Van de Walle (2007) computational estimation is determining a number that is an approximation of a computation that we cannot or do not wish to determine exactly. In line with that, Rizal (2011) describes the meaning of computational estimation as a close approximation calculation results by using informal methods,  methods that are not related with the algorithm, but with a intuitive understanding and not tied to a single method. Not tied to one method means that the number can be used to estimate a different method, but the numbers taking the approach that is easy to do with mental arithmetic. The example is rounding numbers.

2.  **The Learning-Teaching Trajectory by Van den Heuvel-Panhuizen.**
Van den Heuvel-Panhuizen (2005) stated that there are several phases of learning to use estimation, appear as follows:
    -   Rounding off numbers
    -   Estimation in addition and subtraction
    -   Estimation in multiplication and division
    -   Estimation in case of incomplete data.

The phase of learning to round off appears as follows:
    -   The informal stage: the students can round off numbers in an informl way. In this stage, students indicate the numbers of approximations of quantities and measure numbers.
    -   Rule-directed or formal stage: the students arrive at the standard rule for rounding off numbers and learn apply this rule.

Moreover, Van den Heuvel-Panhuizen (2001) said that there are three characteristics of problems in estimation, namely:
    -   Are there enough?
    -   Could this be correct?
    -   Approximately how much is it?

3.  **Pendidikan Matematika Realistik Indonesia (PMRI) Approach.**
PMRI approach is an Indonesian version of Realistic Mathematic Education (RME) approach developed by the Freudental Institute in the Netherlands. This approach has five tenets (Zulkardi, 2002), namely:
a.  The use of contexts in phenomenological exploration
    The mathematical activity is started from local contexts situation that are experientially real for students.  In this study, context of school field and shopping situation are  used to support student find their own strategies for computational estimation.
b.  Using models and symbols for progressive mathematization.
    A variety of context problems and symbols can support the development of progressive mathematization gradually from intuitive, informal, context-bound notions towards more formal mathematical concepts (Jupri, 2008). In this study, the context problem and symbols can support the development of progressive mathematization to understand computational estimation.
c.  The use of students' own constructions and productions

Students are given the freedom to come up with their own construction and strategies in solving estimation problems (Jupri, 2008). Thus, there will be various of strategies.

d. Interactivity.

In finding strategies for computational estimation, students always work in group and will share their idea to others. By this interaction, students can develop their thoughts and can learn to respect each other.

e. Intertwinement.

This topic, computational estimation, is apparently integrated in whole number topics, specially the thousand.

**METHOD**

Design research was chosen to achieve the research goal. Design research consists of three phases; those are preliminary design, teaching experiment, and retrospective analysis (Gravemeijer & Cobb, 2006). In preliminary design, the researcher designs a hypothetical learning trajectory (HLT) which is made up of three components: (a) learning goals, (b) planned instructional activities, and (c) a conjectured learning process in which one anticipates how student's thinking and understanding (Gravemeijer, 2004). The HLT is applied to 38 students at MIN 2 Palembang in two cycles, namely pilot experiment and teaching experiment. The aim of the pilot experiment is to see how the HLT works in order to be elaborated and refined for conducting the teaching experiment. In the retrospective analysis, the HLT and the result of teaching experiment were compared.

**Table 1. Overview of The HLT in Learning informal and formal stage of rounding numbers.**

| Goals | Activities | Conjectures |
|---|---|---|
| 1. Students are able to round numbers on informal stage. | Approximate how many people can be accommodated in the field school | • Students are able to describe how many people in the field school by using words such as: *approximately, almost,* and *about.*<br>• Students are able to describe how many people in the field school with right answers such as: *approximately 1000 people/almost 1000 people/about 10000 people.* |
|  | Choose the appropriate answers to indicate the numbers of people in the field school. | • Students are able to choose the approximation numbers, namely: about 1000 people, slightly more than 1000 people, and approximately 1000 people.<br>• None of students choose 2.5000 people as an appropriate answer. |
| 2. Students are able to round numbers on rule | Conclude the formal rules of rounding numbers. | • Students are able to round a number to the right nearest number.<br>*13 round down to 10*<br>*79 round up to 80*<br>*289 round up to 300* |

| Goals | Activities | Conjectures |
|---|---|---|
| directed or formal stage. | | *720 round down to 700*<br>*1.899 round up to 2.000*<br>*2.230 round down to 2.000*<br>*4.550 round up to 5.000*<br>• Students are able to round the numbers by consider the one. If the one is more than 5 then they round to the numbers up and vice versa.<br>• Students are able to round the numbers by consider the ten. If the ten is more than 50 then they round to the numbers up and vice versa.<br>• Students are able to round the numbers by consider the hunderd. If the hunderd is more than 500 then they round to the numbers up and vice versa. |
| 3. Students are able to doing estimation. | Solve the estimation problems | • Students are able to solve the problems by using their own strategies. |

**RESULT AND DISCUSSION**

**1. Activity 1 (Approximating how many people can be accommodated in the school field)**

The goal of this activity is students are able to reach informal stage of rounding numbers. Through this activity students can globally determine answers without using the standard rounding off rule, but only indicate the approximation of quantities. For the purpose, students were given a question which is approximately how many people can be accommodated in the school field. In the end this activity students are expected to find the answers that indicate approximation numbers.

The following dialog show the discussion between teacher and students.

Teacher: "Approximately how many people can be accommodated in the school field?"
Ilham : "100 people"
Andri: "250 people"
Teacher: "Ilham said that there is 100 people in the school field, now, I want all of you consider about the number of people when we attend a ceremony on Monday, all of stundents of grade 1 until grade 4 and all of the teachers here. Is there a large number of people in the school field?"
Students: "Yes, there is"
Teacher: "So, how many people?"
Rani: "900"
Ilham: "1000"

Rani's and Ilham's answer indicate that students did not answer the question with an approximation numbers, they were still answer by using excat numbers. Therefore, the teacher continued the discussion by giving more question.

Teacher: "Is it right that the answer is 900 or 1000 people?"
Andi    : "No"
Teacher: "why?"
Andi    : "Because we did not count exactly"
Teacher: "That is right, we can not count the number of people, so what we supposed to do?"
Andi    : " I think we can only estimate the numbers of people"
Teacher: "Ok, please estimate how many people can be accommodated in our scholl field"
Andi    : "It is… maybe.. about 900 people or about 1000 people"

      In this phase students are able to answer the approximation problems with approximation numbers. Students realize that they only need to estimate instead answer by using excat numbers.
After that, students were given a worksheet and they have to choose some of the appropriate answers to indicate the number of people in the field school. The answer shows in the following figure.



**Figure 1. Students choose the appropriate answers.**

It can be seen that students are able to solve problem by choosing the appropriate answers.

## 2. Activity 2 (Rounding numbers to nearst ten, hundred, and thousand).

In this phase, teacher gave a worksheet to students. The worksheet consists of several numbers and students were asked to round the numbers to the nearest ten, hundred, or thousand. There was no difficulties when students solved the problem because they used the concept of distance. They chose the ten, hundred, or thousand which is nearest to the numbers by drawing an arrow. Students's work shows in the following figure.
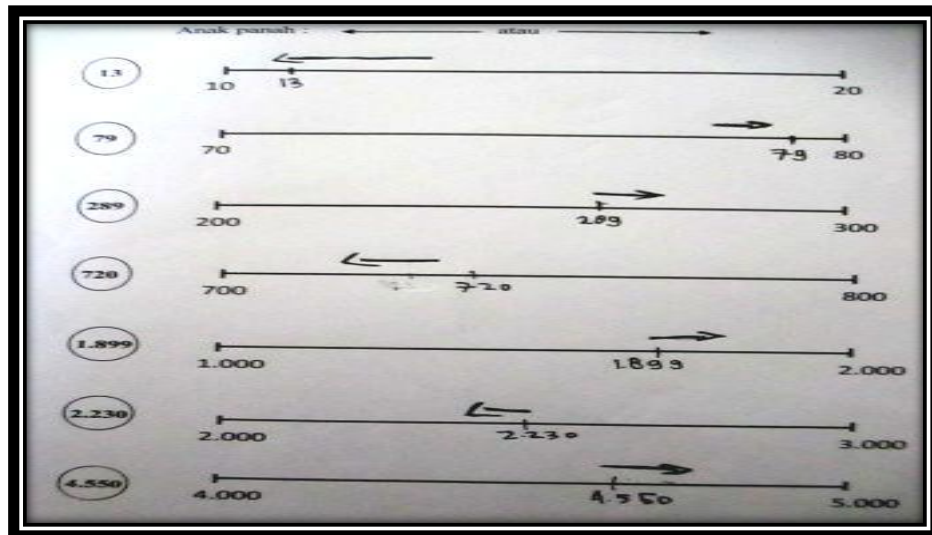
**Figure 2. Students round the numbers.**

Based on students answer, it can be conclude that students can round the numbers by using formal rules even they did not know about the rules previously. From this phase students can get knowledge about the rules of rounding numbers and also conclude the rules by their own self.

### 3. Activity 3 (Solving the estimation problems)

For the last activity, students are given some questions of estimation problems. Students solved it by their own strategies. One of the question is *"If the total price is Rp.52.550, 00 and Ambar wants to reduce the total price with restoring some fruits which the prices are Rp.8.600,00, Rp.5.550, 00, and Rp.14.600,00 respectively. Approximately how much many is then paid by Ambar?*

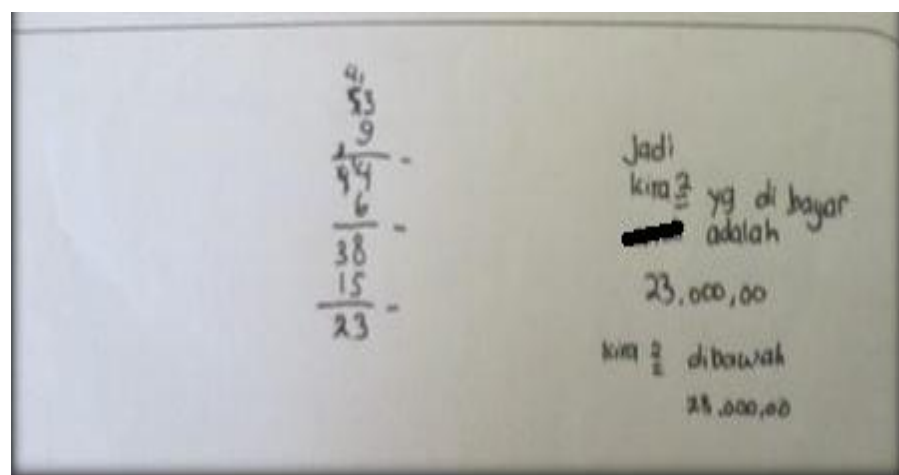The strategie that used by students shows in the following figure.



**Figure 3. Students solved the estimation problem.**

Students round all the numbers to the nearest thousand and then got the answer. For this situation, students also realize that they did not get the precise price but just the approximation price. Moreover, it clear to see that in this phase students can using their knowledge about rounding numbers and working by their own strategy.

**CONCLUSION**

In the first activity, students get knowledge about the situation that allowed them to work with estimation instead find the exact numbers. The second activity, students can get sense about rounding the numbers and in the end of activity they can realize the formal rules of rounding numbers. And in the last activity, students can use their previous knowledge to find the strategy of estimation. Therefore, it can be concluded that activities of informal and formal stage of rounding numbers can bring students to solve the estimation problems.

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