

EDUCATIONAL DESIGN RESEARCH: SUPPORTING FIFTH-GRADE STUDENTS TO LEARN ABOUT PROPORTION

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Abstract

Nowadays, the way of teachers in teaching proportion usually stands at giving the students a ready-made formula, such as cross multiplication, to solve proportional problems involving fractions. As a result, the students tend to memorize the formula and just apply it in solving the problems. This will be meaningless for them because they do not even know and do not understand the meaning of proportionality. Additionally, this will make students difficult to develop their proportional reasoning. Therefore, there is a need for us to support the students to learn about proportion and to develop their proportional reasoning to solve problems through innovations in teaching proportion. To deal with this situation, design research was chosen as the research approach of this study. It highlights a new way for fifth grade students in primary education to learn about proportion which includes fractions. Additionally, it underlies an innovative way for mathematics teachers to teach proportion since this study also employs the heuristic of Realistic Mathematics Education (RME) as the main theory in which we designed five instructional activities within a learning trajectory. Consequently, the aim of this study was to investigate how we can support students to learn about proportion and to develop their proportional reasoning. In conducting this study, 6 fifth-grade students from SD Al – Hikmah were involved. Thus, the students' activities during the teaching and learning process were recorded so that they can be analyzed. The retrospective analysis of the teaching and learning process showed that models, such as: students' visualization, can support students to learn about proportion and to develop their proportional reasoning.

Keywords: Students' Visualization, Proportion, Proportional Reasoning, Realistic Mathematics Education (RME), Design Research

INTRODUCTION

Background

In general, many mathematicians define a proportional situation as the equality between two ratios (e.g. Tourniaire & Pulos, 1985; Langrall & Swafford, 2000; Lo & Watanabe, 1997; Piaget & Inhelder, 1975). According to van Galen et al. (2008) pointed out that a proportion can be defined as a comprehensive concept since it includes fractions, percentages and decimals. Meanwhile, Ma (1999) emphasized that fractions are the most complicated numbers to deal with arithmetic. Subsequently, solving proportional problem requires students more knowledge about the relation between one unit and other units, such as the number of t-shirts and the price, the number of kilogram of rice and the number of days and others. This can be a possible reason that students feel

difficult to deal with proportional problem. Thus, it is necessary for the children to learn about this topic.

The results of recent studies also identified that it is very difficult for fifth-grade students to solve proportional problems (e.g. van Dooren et al., 2010; Lo & Watanabe, 1997; Widjaja et al., 2010; Silvestre & da Ponte, 2012). According to these studies, there are three factors that can lead most students to errors, such as: the existence of an integer ratio, the complexity of the numbers, such as: fractions and decimals, and the order of the number. Moreover, most of the young children are required to use the fixed mathematical procedures and algorithms in order to solve problems related to proportion (e.g. Soenaryo, 2007; Sumanto et al., 2008; cited in Sumarto, 2013). For instance, Sumarto (2013) pointed out that most of the mathematics teachers teach formal procedure, such as: cross multiplication to solve proportional problems. As a result, it is difficult for students to develop their proportional reasoning. Consequently, there is a need to support children develop their mathematical thinking about proportion. For these reasons, we attempt to conduct a study which is aimed to investigate the development of students' proportional reasoning which includes fractions by creating a meaningful teaching and learning situation, such as: making the topic real in students' mind through the use of concrete models.

Dealing with this, many researchers suggest the mathematics teachers to use concrete models to develop students' mathematical thinking about proportion that includes fraction, such as: the use of students' representation (Widjaja et al., 2010). However, it is rarely found that mathematics teachers use model in order to support students to learn about proportions. Therefore, it is necessary to conduct more researches in order to find out how model, such as: students' visualization, can develop the students' proportional reasoning which includes fractions.

Research Aim and Research Question

Basically, this study aims to investigate the development of students' proportional reasoning which includes fractions by using models. However, this study, in this case, focuses on conducting an intervention to the students to construct their proportional thinking. To be more specific, this study discusses about how the students get the idea of proportion and able to explain by using their own model/visualization. Moreover, it also highlights about how the students shift their model into proportional forms.

In order to achieve the aim of this study, we formulate the research question of this study. Consequently, the general research question of this study is "*How do model, such as: students' visualization, support students' proportional reasoning?*"

THEORETICAL FRAMEWORK

Proportional Reasoning

In this case, proportional reasoning can be stated as the way of the students to solve proportional problems. Piaget (cited in Boyer, 2008) proposed that proportional reasoning involves an understanding of the "relation between relation" and it is a hallmark of formal operation. Relating to this situation, the development of proportional reasoning, then, can be seen as an important goal of primary school mathematics (Shield & Dole, 2002). This is because many topics in mathematics require students' proportional reasoning and thinking. For instance, it is used when students solve

geometry, percentages, fractions, ratio, decimal, scale, algebra and probability problems (Dole et al., 2008).

Realistic Mathematics Education (RME)

There are many questions among mathematics educators about what *Realistic Mathematics Education* (RME) is about. *Realistic Mathematics Education* (RME) is a domain-specific teaching and learning instruction theory for mathematics education (e.g. Treffers, 1987; Streefland 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen 1996; Van den Heuvel-Panhuizen, 2003). The term “realistic” does not always refer to the students’ real word. The word ‘*realistic*’, however, denotes more to problems situation that students can imagine and are meaningful for them (Van den Brink, 1973; Wijdeveld, 1980; cited in Van den Heuvel-Panhuizen, 2003). In other words, mathematics, in classroom practices, can be related to problem situations as long as these situations are real in students’ mind.

In relation with this study, we create several daily-life problems so that the students can easily recognize and imagine them. In this case, rice problem, therefore, was chosen as the consideration that the students can find this problem in their everyday life. In this case, this situation was expected to lead the students to a more formal mathematics situation.

Participants

This study was conducted at SD Al – Hikmah, Surabaya. In this case, this study took place in the fifth grade within the second semester. In conducting the teaching and learning process, we involved six fifth-grade students from class 5B as the participants. In this semester, students in the fifth grade will learn about proportion and kinds of proportions, such as: direct proportion and indirect proportion.

Research Method

This study proposes to make an innovation in teaching proportion in primary education. Thus, *Realistic Mathematics Education* (RME) is chosen as the main theory and “design research” as the research method in this study. As we stated before, we chose RME as the main theory of this study that the students recognized the situations in their daily life that can guide them to a more formal mathematics situation.

Furthermore, Bakker and Van Eerde (2013) proposed that “design-based research is claimed to have potential to bridge between educational practice and theory”. This is because the aim of design research is to develop local instruction theories about both the process of learning and the means designed in order to support that learning (Bakker & Van Eerde, 2013; Gravemeijer & Cobb, 2006). Moreover, it has also its cyclic character for preparation, teaching experiment and retrospective analysis (Van Eerde 2013; Gravemeijer and Cobb, 2006).

In the preparation phase, we collect several data, such as: class observation, interview with the teacher and pretest. The Hypothetical Learning Trajectory (HLT) is implemented within two cycles, the preliminary teaching experiment (cycle 1) and the teaching experiment (cycle 2). In this case, the activities during the teaching and learning process are recorded. The recording also includes the activities of the students.

In order to answer the main question of this study, we, therefore, designed five instructional activities which will be conducted within five meetings. In order to answer the main question, this paper will describe the students' activities of the first cycle for lesson 2. In analyzing the data, we observed the teaching experiment by looking at the whole video recording. We selected some important data which can be used in order to answer the main question of this study. Subsequently, we analyze the data by adjusting and comparing the actual teaching and learning and the conjectures in the HLT. In this case, we provide the expectations of the students in our Hypothetical Learning Trajectory and describe the students' responses by referring to conjectures in the HLT.

MAIN SECTION

In this part, we will provide the description how the teaching and learning process took place at the beginning of the second lesson. Before the second lesson was conducted, the students had discussed how to solve a proportional problem in the first lesson. During the first lesson, the students just discussed their strategies in order to deal with proportional problem.

Lesson 2

The aim of this lesson is to shift students' visualization into a ratio table and to introduce the ratio table to the students.

In conducting the second meeting, the teacher asked the students to solve the following problem.

Mrs. Sahri buys 20 one-kilogram packs of rice. In one day, she cooks $\frac{3}{4}$ kg of rice. Can you determine 20 one-kilogram packs of rice are enough for how many days?

In our HLT, we expect the students get the idea of proportion with their own understanding and the teacher can directly guide the students to the idea of ratio table. However, it, in this study, will be described how the teacher guided the students to the early stage of ratio table by asking them questions.

Initial activity

In our HLT, we assumed that the students were still working on their work in the previous meeting. Thus, the beginning of this lesson stood at finishing the students' work and discussing the work within the whole-class discussion. However, this situation did not happen in the actual teaching and learning process. This is because the students work faster than we expect. Moreover, some students can give a meaningful way such as they used their own model, such as: their own visualization, to solve problem. Therefore, the teacher only told the students that he would continue the lesson at the beginning of this meeting. In order to do that, he asked them to recall what they had learned in the previous meeting by asking them several questions. The question was aimed to remind the students about the problem that they solved in the previous lesson and to discuss the way how they solved it.

Dealing with this, some students still remembered that they were working with a problem which asked them to determine the number of days that 20 packs of rice would be enough if $\frac{3}{4}$ kg was cooked everyday. Subsequently, the teacher continued asking them the second question. The second question was intended to remind them about how they solve the problem yesterday. In this case, all the students also still recognized how

to deal with the problem. They said that drawing 20 boxes and each box was divided into four parts. We took $\frac{3}{4}$ part from each box and so on. The next question was asking the students about the conclusion that they had agreed in the previous meeting. Raihan came to the front of the class and stated that “so the conclusion is that Mrs. Sahri can consume 20 packs of rice for 26 days with $\frac{2}{4}$ kg remains”. Then, the teacher asked one of them to write it on the white board.

Main Activity

After making a short conversation, the teacher tried to guide the students to an early stage of ratio table by making short discussion. At the beginning of the discussion, the teacher asked them to explain how to draw $\frac{3}{4}$ of one kilogram. Afterward, the teacher asked them 3 kg = ... days and 6 kg = ... days. According to our initial HLT, we conjecture that the students still do not know how to deal with those problems as what happened in lesson 1 or they will apply what they understand to deal with those problems. For instance, the students use multiplication, subtraction and division. However, we also assume that the students used their model that they have learned in the first lesson.

Here, the description of what happened in classroom. In this case, we asked several students to solve the three problems by using their understanding in front of the class. We found that all the students that came to the front were able to answer the problems correctly by using their own model. For instance, Raihan can explain it by using picture to represent $\frac{3}{4}$. He drew a box divided into four parts and shaded three parts of it. Subsequently, the teacher continued the questions if he had 3 kg of rice and 6 kg of rice. He asked the students that these amounts of rice are enough for how many days. To deal with this, the following short fragment showed how the teacher guided the students to the idea of proportion.

1. Teacher : Now, if I have 3 packs of rice, then for how many days 3 packs of rice will last?
2. Hvl : Four days.
3. Abr : How come?
4. Teacher : Can you explain it?
5. Hvl : (*Start drawing three boxes, divide each box into four parts, and shade each three parts of it with different color*)
: This is $\frac{3}{4}$ and I draw $\frac{3}{4}$ where each box is divided into four parts and three parts of : it are shaded. So, this is the first day, this is the second day, this is the third day : and this is the fourth day. (*Pointing from the first three parts until the last three : parts*)
6. Teacher : So, can you write the conclusion?
7. Hvl : So, three packs of rice can be consumed for four days.
8. Teacher : Ok, now... If there are six packs of rice, then for how many days they will last?
: Dhiya... (*Asking Dhiya to explain it*)
9. Dhy : (*Drawing six boxes, dividing each part into four parts and shading each three : parts*)

: Each kilogram of rice is divided into four. Then, one day there must be $\frac{3}{4}$.
 So, $\frac{3}{4}$: is taken from one pack of rice. The rest can be cooked for the second day and I : just take two parts from the second box. Then, the rest can be cooked for the : third day. And so on until the last three parts can be cooked in the eighth day.

10. Teacher : What is the conclusion?
 11. Dhy : Six packs of rice are enough for eight days
 : (Writing 6 packs are enough for 8 days)

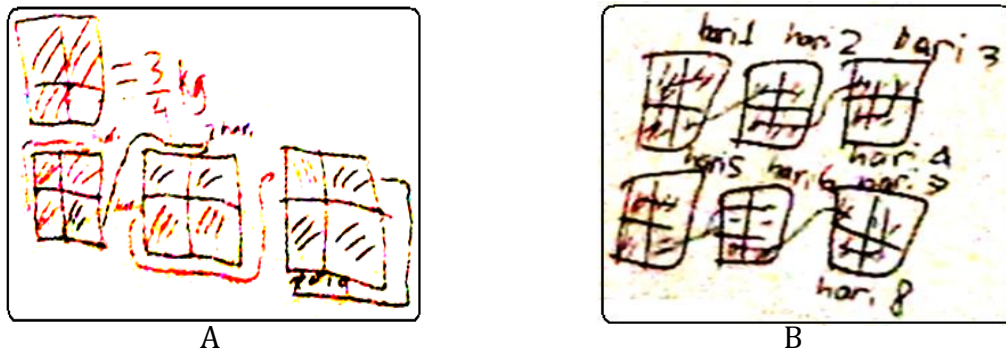


Figure 1. Students' visualization

The short video transcript shows that students used model, such as: their own visualization, to solve the problems. To determine the answer of 3 kg = ... days, the second student used the same way as the first student did. Based on the figure 1A, it can be explicitly seen that the student draw three boxes. He divided each box into four parts and shaded three parts of each box. Subsequently, he counted every three parts to get the answer that 3 kg = 4 days (see also lines 1 – 7). At the same time, the second figure also illustrates the student's strategy to solve the third problem which is 6 kg = ... days. To solve this problem, the third student did the same strategy as the two students before. Based on the figure 1B, we can explicitly see that the student recognized that 6 kg = 8 days (see also lines 9 – 11).

After the students got the idea of proportion, the teacher continued the lesson by guiding the students to the early stage of ratio table. In this case, the early stage of ratio table means the moment when the students build the sense of proportion by using their model. Thus, the following short fragment shows how the teacher did it.

12. Teacher : Now, I want to ask another question. Do you see a pattern?
 : 3 kg of rice are enough for 4 days. Dhiya gets that 6 kg are enough for 8 days.
 : Do you see the way?
 13. Hvl : Just add them. (Going to the white board to explain it)
 : Three packs are equal to four days and six packs are equal to eight days.
 : (Writing 3 packs = 4 days, 6 packs = 8 days)
 : And 12 packs are equal to... (Thinking)
 14. Teacher : Are there any one able to help him?
 : 3 packs are enough for 4 days and 6 packs are enough for 8 days.
 : 3 packs become 6 packs. How come?
 15. Abr : Multiply them.

- 16. Rhn : We can also add them.
- 17. Teacher : Can you write it on the white board? (*Asking Akbar to explain his answer*).
- 18. Abr : This one and this one are multiplied with 2. (*Pointing 3 and 6*)
- 19. Teacher : Do you agree with his answer?
- 20. Rhn : I also want to try it. Why don't we just add them?
- 21. Teacher : Can you write it on the white board?
- 22. Rhn : From 3 to 6 we can add 3 and from 4 to 8 we add 4. (*Pointing 3 to 6 and 4 to 8*)

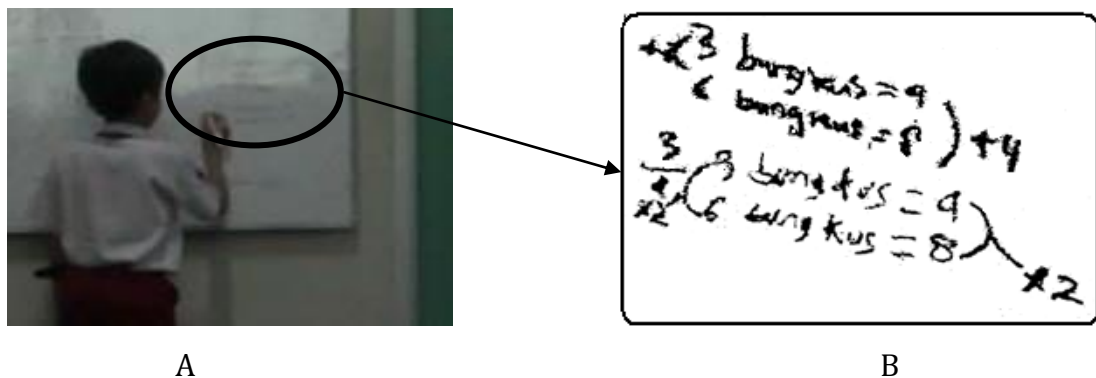


Figure2. Students get the idea of proportion

The transcript and the pictures show how the students build the sense of proportion by using their own model. In this case, we can explicitly see that Havel concluded $3 \text{ packs} = 4$ and $6 \text{ packs} = 8$ based on the model he saw. For instance, he saw “three boxes and six $\frac{3}{4}$ s” and “six boxes and eight $\frac{3}{4}$ s” (see line 13 and figure 2B).

After the students wrote their thinking, the teacher managed those answers in a table. In this case, he drew a table which consisted of two rows where the first row was filled with kilograms and the second one was filled with days. Since the first conclusion was “3 kilograms of rice are enough for 4 days” and he put 3 in kilogram and 4 in days”. Then, he continued to the second conclusion that was “6 kilograms of rice are enough for 8 days” and put 6 in kilogram and 8 in days. After that, he asked the students to continue it in group. Subsequently, the teacher, at the end of the lesson, introduced ratio table to the students.

RESULTS AND DISCUSSION

Based on the transcript and figure 1, we can notice that model plays an important role in order to help students solve proportional problem. It helps the students visualize their thinking towards the way how to solve proportional problem. In this case, we can explicitly see that the students automatically used model, such as: their own visualization, to solve the three problems. Therefore, we can say that the students’ visualization helps them when they were asked to reason how they get the answer of the proportional problems.

Based on the transcript and figure 2, we, furthermore, can notice that model also plays an important role in order to help the students build the sense of proportion. It helps the *students* visualize the proportional situations from the given problem. In this case, we can see that the students write the form of proportion based on the model that they had made before.

CONCLUSION

In this section, we would like to answer the main problem in this study which is: *How do model, such as: students' visualization, support students' proportional reasoning?*

Based on the findings and the video fragments, we conclude two that model plays an important role in two ways. Firstly, model helps students solve proportional problem. It helps the students visualize their thinking towards the way how to make their own representation of a fraction and to solve proportional problem. For instance, the students draw a box and divide it into four parts. After dividing it, they shade three parts over the four parts of the box in order to represent $\frac{3}{4}$. Subsequently, they use the model, such as their visualization, in order to show the way how they get the proportion, such as: $\frac{3}{4}$ kg = 1 day, 3 kg = 4 days and 6 kg = 8 days.

Secondly, model also plays an important role in order to help the students build the sense of proportion. It helps the students visualize the proportional situations. For instance, the student concluded *3 packs = 4* and *6 packs = 8* based on the model he saw. This is because he saw "*three boxes and six $\frac{3}{4}$ s*" and "*six boxes and eight $\frac{3}{4}$ s*" (see line 13 and figure 2B).

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