DEVELOPING GRADE 5 STUDENTS' UNDERSTANDING OF MULTIPLICATION OF TWO FRACTIONS THROUGH TAKING A PART OF A PART OF A WHOLE ACTIVITY

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Abstract

Many studies found that students have difficulties in learning about multiplication of two fractions. Students only tend to apply the procedure to solve the problems without an understanding of the process. Therefore, this study aimed to contribute to the development of a local instruction theory in supporting students' understanding of multiplication of two fractions. Some ideas such as partitioning, taking a part of a part of a whole unit within contexts and using array model are concerned in this study. Using the design research method, a sequence of five lessons was designed grounded by the Pendidikan Matematika Realistik Indonesia approach. The hypothetical learning trajectory (HLT) became the base for conducting a teaching experiment and designing its learning instruments. The participants in this study were five students in grade 5 of SDI Al Hikmah Surabaya. The main data were collected through video registrations and students' written works. Then, a retrospective analysis was conducted by comparing the HLT with the actual learning process. This study shows that using students' informal knowledge about partitioning and the use of sharing chocolate block context in which an array model was introduced promote students' understanding about taking a part of a part of a whole unit. It is also shown that the students started to use the array models to reason about the concept of taking a part of a part of a whole. The understanding of taking a part of a part of a whole unit could be used as a starting point in learning about multiplication of two fractions.

Keywords: multiplication of two fractions, partitioning, part-part- whole, Pendidikan Matematika Realistik Indonesia (PMRI), design research

INTRODUCTION

Learning about fractions is important for students because the understandings of fractions become a basic foundation to learn about proportions, decimal numbers and percentages which are very useful in the daily life (van Galen, et al, 2008). However, fractions itself is a complicated topic (Streefland, 2008). Fractions have multifaceted interpretations and this condition makes students struggle in developing their understanding about fractions (Kieren 1993; Pantziara and Philippou, 2012). Therefore, we need to design an instructional theory to apply in mathematics classrooms in order to help students (and teachers) develop the understanding of fractions. To make it specific, in this present study we only focus on supporting students' understanding of multiplication of two fractions. The aim of this study is to contribute to the development

of students' understanding of multiplication of two fractions. We formulate a research question as follows: *"How can model support students' understanding of taking a part of a part of whole?"* We use par-part-whole relation as a starting point for the students to understand about multiplication of two fractions.

Informal knowledge of students about multiplying fractions

Students learn to know fractions at home before they learn it at school, which means that they already have initial knowledge about fractions itself. Studies focusing on developing students' understanding have documented that the informal knowledge of students should be an important consideration (Carpenter, Fennema, Peterson, Chiang, and Loef, 1989; Mack 1990). Brown (1993) has documented that students tend to use their informal knowledge of fractions to form a meaningful understanding of the algorithms. It also implies that in developing students' understanding of multiplication of two fractions, teachers have to connect to the informal knowledge of the students as the starting point in the learning activity.

The informal knowledge of students that we can use in developing the understanding of multiplication of two fractions is partitioning. Supporting students to build their informal knowledge on partitioning may lead to the development of the understanding about multiplication of two fractions (Mack, 2000). Specifically, in this study we provide students to experience partitioning activity in the taking a part of a part of a whole context. Later on, they can interpret the taking a part of a part as taking a fraction of a fraction or multiplying a fraction with another fraction.

Realistic Mathematics Education

We grounded our study by using Realistic Mathematics Education approach (RME) which in Indonesia is adapted as *Pendidikan Matematika Realistik Indonesia (PMRI)*. The main point of RME that we used in this study are mathematizing and also the use of model.

Freudenthal (1968, in Keijzer, 2003) argued about learning mathematics as "mathematising". He stated that mathematising is "watching the world from a mathematical perspective to thus make it more mathematical". Moreover, Keijzer (2003) proposed that "When discussing this mathematising processes, we actually discuss the process of modeling, symbolizing, generalizing, formalizing, and abstracting". He said that these kinds of activities reflect the journey of the students in reaching the formal and abstract structures of the mathematical concepts. They experience every part of the activities by themselves which leads to meaningful learning. In this present study we only focus on modeling and symbolizing process since we only address the informal part of the students' journey on understanding multiplication of two fractions.

Furthermore, the use of model took an important role in this study. Gravemeijer (1994) stated that basically models are used as a concrete starting point for developing a formal notion of a concept in mathematics. In this study, in order to support students in developing their understanding of taking a part of a part of a whole activity we provide them with a chocolate block context in which the array model was introduced.

METHODOLOGY

We used design based research (DBR) approach in this study. We use this approach because we have considered the characteristics of DBR. Cobb, Confrey, diSessa, Lehrer, and Schauble (2003, in Bakker and van Eerde, in press) identified five characteristics of

DBR, of which in this study we pointed one of them that is regarding the aim of the design based research. They stated that DBR not only develops theories about learning, but also the instruments which are designed to support the learning.

Following the DBR phases which are argued by Gravemeijer and Cobb (2006), there are three main steps in this study; preparing for the experiment, experimenting in the classroom, and conducting a retrospective analysis. The study was conducted in the 5th grade of SDI Al Hikmah Surabaya involving 30 students and also the mathematics teacher of that class. There would be 2 cycles where the designed hypothetical learning trajectory (HLT) of five lessons was implemented. However in the present paper we only focus on the analysis and the result of an activity of the second lesson in the first cycle with 5 students involved. The five students were Abdul, Arjuna, Adrian, Izudin and Kelvin (not the real names). The researcher took role as the teacher in this lesson. The data were collected through video registrations and students' written works. Furthermore we conducted a retrospective analysis by comparing the HLT with the actual learning process of the students.

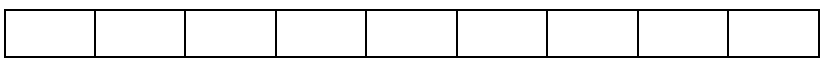
ANALYSIS AND RESULT

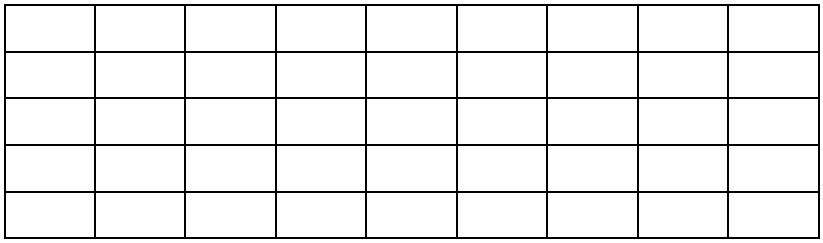
In this part, we first will give an overview about what students already learn in the first lesson as a starting point in the second lesson. Further, we will provide the task that should be solved by the students along with our aims and our expectation of students' answers and strategy to solve the problem. We will provide a transcript of an interesting fragment we took from the video registration of the activity of sharing chocolate block in lesson 2. Then, we compare the students' actual learning process with the HLT of this activity. At the end, we will explain about the conclusion and the discussion of the lesson.

In lesson 1 the students were already introduced into the idea of partitioning and interpreting the result of the partitioning into a fraction notation. The context in this lesson was the committee of a hiking event wants to locate 6 flags in each kilometer of the hiking trail with 6 km length. There will also 4 game post in equal distance along the trail. There is no flags and game post at the starting line. The students' task was to locate the flags and the game posts in the given trail figure then they should determine the fraction notation of each position of the flags and the game posts. The model that is used in the process of the partitioning in this first lesson was a bar model as the representation of a hiking trail. The students can do a partitioning activity properly in the bar model. They recognized that the result of the partitioning should be in equal size. The students recognized about the use of fraction when the teacher asked about *"In what part of the trail …?"*

Furthermore, in the activity of sharing chocolate block in lesson 2, the context was about sharing a chocolate block between three children, Hafidz, Aufa and Siraj, who wants join the hiking event. We provided the problem as follow.

1. *Suppose that the following grid is the chocolate block that is given by Hafidz's father. Indicate by shading the part Aufa, Siraj and Hafidz will get!*





2. *What part of the chocolate block is for Hafidz? Write your answer in a fraction notation!*
3. *After sharing the chocolate with Aufa and Siraj, Hafidz remember his sister Nazifah who also like chocolate very much. So he thinks to split his part and give it to Nazifah.*
   Instruction: can you show the Nazifah's parts in the given drawing!
4. *What part of the chocolate block did Nazifah get? Write your answer in a fraction notation!*

The aim of this activity is to assure that our students can do the partitioning process in the array model properly. Further, they can interpret the result of the partitioning into a fraction notation. We also expected that the students could use the model to reason about taking a part of a part of a whole unit.

We made conjectures of students' answer in our HLT for these problems. In our prediction the students would divided the array into three equal parts vertically, then they divided one of the three parts into two. Moreover, to determine the fraction notation, we predicted that the students would count one by one the small parts inside the array.

All of the students could answer problem 1 and 2 correctly. As our conjecture for the problem 1, they could show the parts for Hafidz, Aufa and Siraj by dividing the given array into three equal parts vertically. Moreover, there were two different strategies that were used by the students in solving problem 2. Most of them count the total number of small pieces in the chocolate blocks and then count the small pieces of the Hafidz parts. They did not count one by one as we predicted in our conjecture, but they multiplied the dimension of the block, the number of rows multiplied by the number of columns. Then they wrote $\frac{18}{54}$ and $\frac{1}{3}$as the final result (see figure 1).

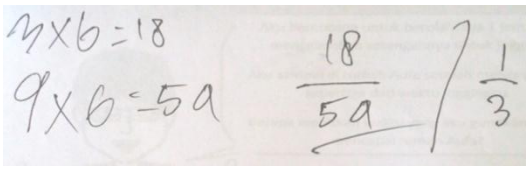


Figure 1 Multiplying rows and columns strategy in determining the fraction notation of problem 1 and 2 in worksheet 2

Based on figure 1, we can see that the students got the fraction notation for answering the problem is the number of small pieces of Hafidz over the total number of small pieces in the chocolate block. They came up with $\frac{18}{54}$. Furthermore, in our interpretation, the students simplified the fraction by dividing both the numerator and the denominator of that fraction by 18 and got $\frac{1}{3}$ as a final result.

Only one students who directly interpreted the drawing of Hafidz' parts as $\frac{1}{3}$. When the researcher asked why he directly answer like that without counting the number of small pieces like the other did, the student answered he knew that the chocolate block was divided into three, so each part is the same as $\frac{1}{3}$ (see transcript 1). In the discussion, the researcher asked the students to compare the answer of problem 2, whether they are agree or not or did they understand about the way each of them got the fraction notation.

Transcript 1

Adrian : It is divided by 3, Hafidz, Siraj … There are three children.
The researcher : There are three children, then?
Adrian : Hafidz gets one over three of it.
The researcher : One over three of it, Aufa gets?
Adrian : One over three of it.
The researcher : Also gets one over three. Siraj also like that?
Adrian : Yes.
The researcher : Why each of them get one over three? Because what does it said in the problem?
The students : Divided into equal parts.

Based on the description and the transcript 1 above we conclude that the students already know how to make three equal parts and interpret it into a fraction notation. They know about taking a part of a whole unit. They also can relate the part and the whole unit in a form of a fraction. They also get used of simplifying the fraction in the final result.

For problem 3, all of the students could show the part of Nazifah correctly. They split up the parts of Hafidz into two equal parts and shaded it. It is implied that the students could do the partitioning activity correctly.

Furthermore, for problem 5 most students except Abdul answered by $\frac{9}{18}$and then simplified it became $\frac{1}{2}$. Abdul answered by $\frac{1}{6}$. In the discussion, the students explained the way they used. They counted the small pieces of the intended part, over the small pieces of the unit. The difference between the two answers for problem 5 was due to the unit that they refer to. The students with answer $\frac{1}{2}$said that they refer to the parts of Hafidz. The meaning of $\frac{1}{2}$they wrote on the answer box is the parts for Nazifah is $\frac{1}{2}\ of\ Hafidz's\ parts$.

Meanwhile, Abdul said that to determine the fraction notation of Nazifah's parts he not only refer to the Hafidz's part but to the whole chocolate block. As can be seen in figure 2, Abdul did not count the small pieces of the chocolate block. He started with the part for Hafidz is $\frac{1}{3}$of the chocolate block then the part for Nazifah is $\frac{1}{2}$ of that $\frac{1}{3}$ part. However, the way he got $\frac{1}{6}$ as the answer is not correct. We can see that Abdul used the strategy of solving the subtraction of fractions. He subtracted $\frac{1}{2}$ with $\frac{1}{3}$ and by using the procedure he got $\frac{1}{6}$ as the answer. To correct that strategy, the researcher asked all of the students to determine the fraction notation of Nazifah respect to the whole chocolate block.

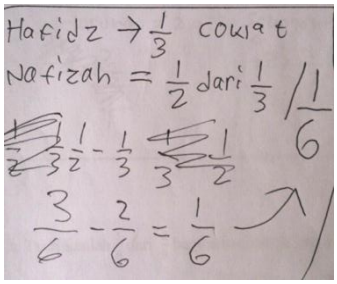


Figure 2 Abdul's work on problem 5 of worksheet 2

Kelvin tried to explain his answer in front of the class. He got $\frac{9}{45}$ as the answer. But Abdul had a question regarding the answer that was shown by Kelvin. The discussion is transcribed in the transcript 2 along with figure 3.

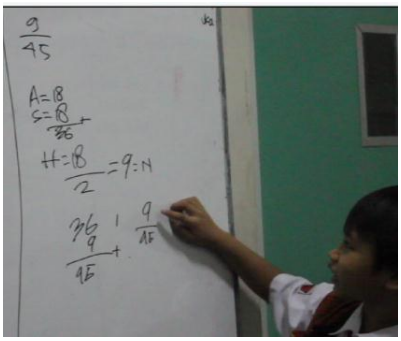


Figure 3 Kelvin explained his answer in front of the class

Transcript 2

Kelvin	*: This is the Hafidz's parts (pointed at the number 9 in figure 5.12), this is Aufa's and Siraj's (pointed at number 36). We added up became 45. It means the parts of Hafidz, Aufa and Siraj are 45 in total. And this 9 is Nazifah's (pointed at the number 9 in the numerator of the result).*
The researcher	*: Then?*
Abdul	*: Why it is 45? Since initially the Hafidz's parts is not include the Nazifah's parts.*
The researcher	*: Do you understand the question of Abdul, Kelvin?*
Kelvin	*: No, I don't.*
Abdul	*: I mean, you added up the part of Aufa, Siraj and the part of Hafidz exluded the Nazifah's part. Why don't you add it up with the initial part of Hafidz?*
The researcher	*: The part that is not shared yet [with Nazifah].*
Abdul	*: The part that is not shared yet [with Nazifah].*
Kelvin	*: Because [the information of] the problem said that the parts [of Hafidz] already cut before.*
The researcher	*: What do you think Arjuna?*
Arjuna	*: I think, it is from the whole part. Why don't you take the whole parts of Hafidz.*
Abdul	*: Because it is asked about the parts of Nazifah respect to the whole [chocolate block]*

Based on the transcript 2, we can interpret that Kelvin still got confuse on determining the whole unit in the taking a part of a part of a whole unit. Instead of just taking the part of Hafidz after he split it up with Nazifah, Abdul and Arjuna suggested that they need to consider the whole part of Hafidz before the splitting when counting the total number of small pieces of the chocolate block. It is imply that Abdul and Arjuna know how to relate the intended part (the part of Nazifah) with the whole unit. Together they corrected the answer and got $\frac{9}{54}$ and simplified it became $\frac{1}{6}$.

Furthermore, the researcher asked the students whether the strategy that they already discussed had the same meaning of the subtraction that was done by Abdul to find the fraction notation $\frac{1}{6}$ as the answer of problem 5. The students could recognize that it is not the same. They cannot use it in solving the taking a part of a part of a whole problems.

We also pointed out from the students' strategy in determining the fraction notation of the Nazifah parts, none of the students think about how many times the parts of Nazifah fit into the initial chocolate block.

CONCLUSION AND DISCUSSION

As a conclusion, in the activity of sharing chocolate block in this lesson, the students started to use the context and the array model to deal with the taking a part of a part of a whole problems. It shows that the context and the array model helps students convince each other about the idea of part-whole relationship, although they still count the number of the cells (the small pieces) in the block (the array) to determine the fraction notation. It raises the need to improve the design especially to engage students to use the model as a tool in determining the fraction notation easily without counting every cell in the array.

Moreover, the idea of taking a part of a part of a whole unit is important for the next step of understanding of multiplication of two fractions. In the next lesson, the students should experience more about the part-part-whole relation and tried to discuss about the relation between the fractions in the problem.

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