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# Mathematical thinking of 13 years old students through problem-solving

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**Abstract**. This study aims to describe the mathematical thinking of 13-year-old students through problem-solving. Mathematical thinking consist of several aspects, namely: reasoning, abstraction, symbolization, and representation. The respondents were 6 students. The instruments used were tests and interviews. Tests are used two problem-solving. The data collection used qualitative approach. The conclusion is deductive dominant in reasoning. The formal form and geometrization are dominant in mathematization. All aspect of abstraction appears on students' answer. Reasoning seen when students use prior knowledge and prior concept to solve problems. Abstraction appears when students find a particular form (pattern) of the problem being solved. mathematization appear when students visualize problems and use algebraic concepts to solve the problem.

## 1. Introduction

All of the students should be made to acquire the behavior of building mathematical knowledge, being able to solve problem-solving, because problem-solving makes it possible to structure knowledge and to bring into connection with the other knowledge [1]. Mathematics is the single method of thinking [2] and the problem be determined for thinking to develop by correlating among the concepts for the solution of problem [3]. In this aspect, thinking involves critical and creative aspects of the mind and identified as ability to comprehend, separate, merge, as well as to connect and understand forms [4]. One of reason why mathematics taught in primary school to university is for teaching how to think mathematically. It is well known as mathematical thinking.

A mathematical thinking is the way in which an individual prefers to present, to understand and to think through, mathematical facts and connections by certain internal imaginations or externalized representations [5]. According to Mason mathematical thinking is a dynamic process that broadens the scope and depth of mathematical understanding [6]. Mathematical thinking generally requires deep mathematical knowledge, general thinking skills, and knowledge of heuristic strategies. Making guesses, reasoning, proof, abstraction, generalization, and specialization are important aspects of mathematical thinking [7-10]. mathematical thinking is realized not only in cases with numbers and abstract mathematical concepts but also in daily life. Mathematical thinking needs many skills such as logical and analytic thinking as well as quantitative reasoning [11].

Mathematical thinking defined as thinking style and also as a process, which includes; problem-solving, looking for patterns, making conjectures (conjecturing), testing the border, the data inference, abstraction, explanation, justification, specializing, generalizing, and convincing [12,13].

Mathematical thinking evokes contradiction, tension, excitement, and supported by the atmosphere of questioning, difficulties and reflection, because of that mathematical thinking has relations to problem-solving. According to Ersoy and Guner, problem-solving has a positive effect on mathematical thinking [4]. According to Delima, there is a significant correlation between problem-solving ability and students' mathematical thinking and to improve students' mathematical thinking, it would be better, to enhance the problem-solving ability at first [14].

One of the important components of the problem-solving process is, on the other hand, the problem-solving strategies. The use of suitable problem-solving strategy is significant in terms of being successful in problem-solving. Problem-solving strategy is a plan made as to how a question can be solved, and a perspective and pattern in the events. The most common problem-solving strategies included in the literature are guessing and testing, making a systematic list, finding a pattern, drawing a diagram, solving a equation inequation, estimating, solving a simplier problems, working backwards, making a table and logical reasoning.

A mathematician use mathematical thinking to solve a mathematical problem-solving. They used some processes and action, like: exemplifying, specializing, completing, deleting, corrected, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing and refuting [15,16].

The PISA study in 2012 put the mathematical abilities of students are at level 1, 2 and 3 are at the level of low-level thinking. Therefore, the duty of a teacher of mathematics, as educators, to develop the ability to think from a low level at a high level. The achievement of students in subjects related to thinking skills is still minimal. Students are still difficulties in problems related to verification, analysis, representation in the form of graphs, and no routine matters.

According to Toheri, students is still weak in deductive reasoning, the visual representation of the geometry and graphics, symbolic representation in trigonometry, direct evidence, generalizing from patterns of relationship variables, and think analogy, using special process [17]. Types of mathematical thinking and mathematical thought processes used to depend on the content of math problems. Problems related to the geometry of the dominant use of visual thinking. Students to think mathematically very personal (intelligence, interests experience), and Depending on the situation at hand (mathematics content, type of problem) [18]. Both activities are important for the mathematical knowledge building, which is an essential basic cognitive action for the education theory and practice. For that reason, the concept of number and the problem-solving is considered the backbone of the school mathematical knowledge [19]. Problem-solving is a basic component for learning, as well as for knowledge acquisition.

In this research, researchers are interested in analysis mathematical thinking of 13 years old students in problem-solving like a mathematization, abstraction and reasoning. Mathematization consists of geometrization, formalization, and connection. Abstraction consists of specialization, generalization, conjecture, and conjecture test. Reasoning consists of inductive, deductive, and logical reasoning.

## 2. Research methodology
The participant in this study were 6 students with category 13 years old. This research used qualitative approach. This research focuses on three aspects: mathematization, abstraction, and reasoning. Mathematization consists of geometry, formalization, and connection. Geometrization is defined as using geometrical concepts and techniques in solving problems. formalization is defined as using concepts and mathematical symbols when solving problems. Connections are defined as using previous mathematical concepts and ideas in solving problems. The instruments used tests and interviews. Two of problem-solving used in this study. The following problems used in this study.

**Table 1**. Problem used in research.

| No | Problems |
|---|---|
| 1 | House ceramic floors are arranged in the following pattern. |

| | | | | |
|---|---|---|---|---|
| Number of black squares | 1 | 2 | 3 | .. |
| Number of white squares | 4 | 6 | 8 | ... |

| No | Problems |
|---|---|
| | Determine how many white squares if there are 350 black squares! |
| 2 | In the picture below AB and CD are perpendicular diameters that meet at point O. There are any points E on the AC arc, and form two perpendicular lines namely EF and EG, where points F and G are in line AB and CD. If the length of the diameter AB 16 cm looks for the length of FG! |

## 3. Results and discussion

### 3.1. Results

Figure 1(a) tells about students' mathematization, reasoning, and abstraction. The first student uses connection and formalization forms, because changes the information from the table into sentences forms and used mathematical symbol. The first student sees a separate pattern on both the black square and the white square. The first student use allegations and generalizations before make conclusion. The first student sees the number of pattern. The original number pattern for black square and even number patterns starting from 4 for white square. The first student suspects that many white squares when the black square is 350 are even numbers. The first student sees the pattern on the white square starts from the second even number, so concludes that the number of white squares when the number of black squares is 350 pieces is even after that. The first student sees even number is twice an original number, so even number 350 is 700, so even number after 700 is 702.
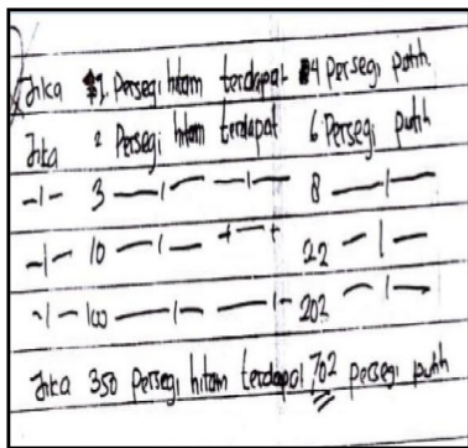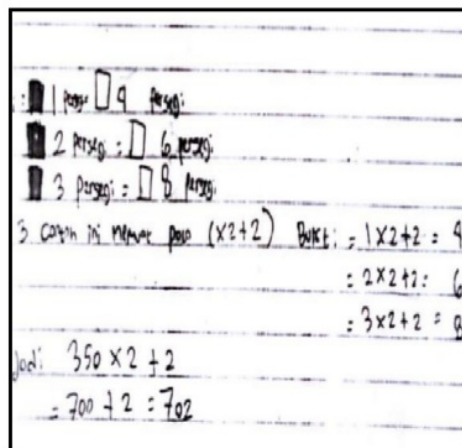


|     (a)     |     (b)     |
|---|---|

**Figure 1.** Student answer of first problem. (a) student 1st and (b) student 2nd.

The second student uses all three forms of mathematization, namely: geometry, formalization and connection. Geometrization can be seen from the square images that are white and black. Formalization

be seen from the existence of a mathematical symbol, while the connection can be seen from the use of the concept of multiplication and the summing of numbers in solving problems.

The second student use deductive and inductive reasoning. The second student uses the formula "x.2 + 2" to solve the problem. however, to ensure that the formula made is true or false, student 2 proves it for several numbers. The second student also do specializations, generalizations, and conjectures when solving problems. The second student explains two number of patterns. The left side (black square) has a pattern of natural numbers, while the right (white square) has a difference 2. The second student sees all numbers on the right (white square) were even numbers. The second student combines even numbers with the difference 2. The second student guess that the number of white squares is obtained from "twice the black square plus two". Indirect generalizations also arise when students write "x.2 + 2".
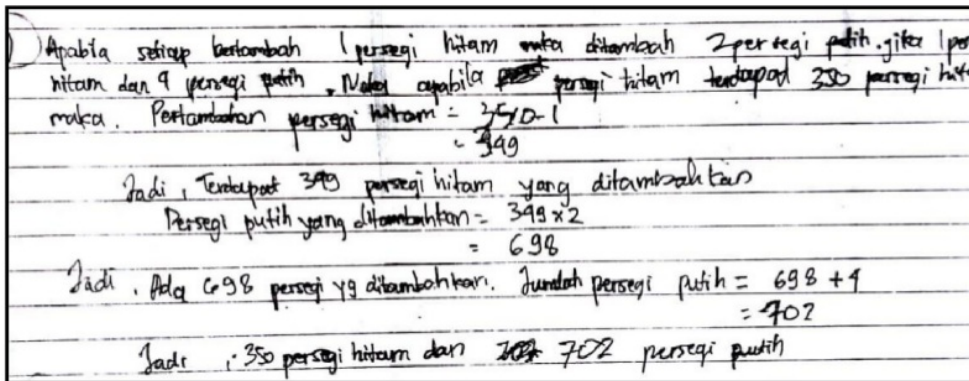


**Figure 2**. Third student answer of first problem.

The third student doesn't use images to solve problems. The third student use logical reasoning. The third student tells more about how to find a solution. The third student tends to guess the number of relationships that are in trouble to find a solution. The other student uses same strategies with the first and the second student.
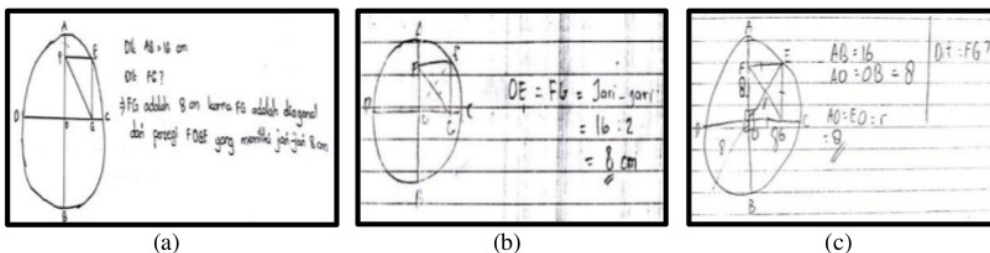


| (a) | (b) | (c) |

**Figure 3**. Student answer of second problem. (a) student $1^{st}$, (b) student $2^{nd}$, and (c) student $3^{th}$.

Figure 3 tells about the second problem. The first student also uses formalization, connection, and geometrization. The first student changes the information from the problem into an image form, then writes down a mathematical symbol, and sees the connection between the radiuses of the circle with a diagonal of square. The first student tends to use logical reasoning because tells more about how to find a solution. The first student sees and finds two plane, namely square and circle. The first student explains the length of FG is equal to OE and OE is the radius of the circle. The first student sees and writes that "FG = 8 cm because the FG is diagonally from the FOGE and equal to the radius of the circle which is 8 cm".

The second student changes the information from the problem into an image, uses mathematical symbol, writes down the information, and sees the connection that FG is the radius of the circle and the value of FG is half of the diameter.

The second student tends to use deductive because the second student uses a mathematical formula "radius = diameter: 2". The second student sees and finds that FG is the radius of the circle. The second student explains the length of FG is equal to OE and OE is the radius of the circle. The second student writes that "radius = diameter: 2".

The third student uses all mathematization. The third student draws the circular image and OGEF square. The third student uses mathematical symbols and an explicit connection is seen from "AO = EO". The third student uses deductive reasoning, because uses mathematical concepts and procedures.

*3.2. Discussion*

Types of mathematical thinking and mathematical thought processes used to depend on the content of math problems. Problems related to the geometry of the dominant use of visual thinking. Students to think mathematically very personal (intelligence, interests, experience), and Depending on the situation at hand (mathematics content, type of problem) [18].

The students do something now is influenced by their past habits. The students use deductive reasoning, accustomed to memorizing formal calculation methods. They always think about which operations will be used to solve problems. Conversely students who use inductive reasoning, use trial and error strategies in special cases before providing a final solution. The students use logical reasoning sometimes do not know the method used. The students that use logical reasoning actually know surely the principle of the solutions. Sometimes, some of people are questionable the accuracy of inductive reasoning [20]. The completion given by the student is not only adhered to relevant and logical to answer the question, but there is a connection between data and questions, and reflected in the model made, and finally solving the problem using concepts represented in mathematical form [21].

Mathematization depends on students' understanding of the problem. Students who have not been able to think abstractly must tend to use geometry in describing what is contained in the problem. but on the contrary, students who have thought abstractly tend to directly use mathematical symbols as a form of the results of pouring ideas. Mathematical connections are usually used by students when students read and translate the information contained in the picture.

Sometimes, they make a representation such as graphs, maps, averages, and equations were not only useful as inscriptions (representations in some permanent medium, usually paper), but also as conscription devices in the construction of, and through which, students engaged each other to collaboratively construct meaning [22].

Abstractions appear when they are able to plan a solution. Specialization appears when students are able to find a relationship between information on the problem. Furthermore, by classifying, analyzing or perhaps by changing the conditions of the problem a little, students will make a conjecture or suspect that all information in the problem forms an abstract idea or situation [23]. Generalization is obtained if student is sure of the suspicion he has obtained. In addition, an important core of problem-solving is allowing students to gain an understanding of the new mathematical concepts contained in the problem. argue that new mathematics and understanding will be acquired by students as they actively search for relationships, analyze patterns, find out if the method is appropriate or inappropriate, test the results, and assess or criticize their own thoughts [24].

According to Ayllon et al, that when an individual encounters a problem, they must think, analyze critical formulations, check formulation and handling data, and such a solution strategy that allows obtaining solutions to the problem [19]. Someone also tends to have sensitivity, flexibility in seeing unclear relationships between facts; relationships that were not intertwined between different experiences. Besides that, it also has originality and capacity to represent new models.

## 4. Conclusion

The conclusion is deductive dominant in reasoning. The formal form and geometrization are dominant in mathematization. All aspect of abstraction appears on students' answer. Reasoning seen when students use prior knowledge and prior concept to solve problems. Abstraction appears when students find a particular form (pattern) of the problem being solved. mathematizations appear when students visualize problems and use algebraic concepts to solve the problem.

## References

[1]     NCTM 2000 *Principles and Standards for School Mathematics* (Reston VA: The National Council of Teachers of Mathematics Inc)
[2]     Yildirim C 2004 *Mathematical Thinking* (Istanbul: Remzi Kitabevi)
[3]     Ersoy E 2012 *PhD Dissertation* (Dokuz Eylül University: Social Science Institution, Izmir)
[4]     Ersoy E and Guner P 2015 *The Online Journal of New Horizons in Education January* **5**
[5]     Cai J 2000 *Mathematical Thinking and Learning* **2** 309
[6]     Mason J, Burton L and Stacey K 2010 *Thinking Mathematically, 2nd Edition* (Prentice Hall)
[7]     Argyle S F 2012 *Mathematical thinking: From cacophony to consensus* (Kent State University)
[8]     Schoenfeld A H 1992 *Learning to think mathematically: Problem-solving, metacognition, and sense making in mathematics* (Handbook of research on mathematics teaching and learning, 334)
[9]     Al-Khateeb K 2004 *Doctoral dissertation* (Amman, Jordan: Amman Arab University for Graduate Studies)
[10]    Breen S and O'Shea A 2010 *Bulletin of the Irish Mathematical Society* **66**
[11]    Tasdemir A and Salman S 2016 *Ahi Evran University Journal of Kirsehir Education Faculty* **17** 785
[12]    Karadag Z 2009 *Analyzing Students' Mathematical Thinking in Technology-Supported Environments* (Toronto ON: University Toronto)
[13]    Mason J, Burton L, and Stacey K 2010 *Thinking Mathematically Second Edition* (London: Pearson Education Limited)
[14]    Delima N 2017 *Infinity* **6**(21)
[15]    Stacey K 2006 *What Is Mathematical Thinking and Why Is It Important*?
[16]    Breen S and O'Shea A 2010 *Irish Mathematical Society Bulletin* **66**(39)
[17]    Toheri 2017 *Mathematical Thinking Undefended on The Level of the Semester for Professional Mathematics Teacher Candidates*
[18]    Argyle S F 2012 *Mathematical thinking: From cacophony to consensus* (Kent State University)
[19]    Ayllón M, FGómez I A and Claver J B 2016 *Propósitosy Representaciones* **4**(169)
[20]    Meng H 2002 *The Relationship between Reduction and Deductive and Inductive Reasoning* [Online]                    Retrieved                    from: https://pdfs.semanticscholar.org/fa34/2b77370c6743519c2432f13654dd952525ac.pdf
[21]    Kirkley J 2003 *Principles for Teaching Problem-solving: Technical* (Indiana University: Plato Learning)
[22]    Roth WM and Bowen GM 1994 *Journal of Research in Science Teaching* **31**(293)
[23]    Van De Walle J A 2007 *Matematika Sekolah dasar dan Menengah Edisi Keenam* Terjemahan Oleh Suyono 2008 (Jakarta: Erlangga)
[24]    Sullivan 2011 *Teaching Mathematics: Using research-informed strategies* (Australian Council for Educational Research)

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