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## Does Conceptual Understanding of Limit Partially Lead Students to Misconceptions?

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**Abstract.** This article talks about the result of preliminary research of my dissertation, which will investigate student's retention of conceptual understanding. In my preliminary research, I surveyed 73 students of mathematics education program by giving some questions to test their retention of conceptual understanding of limits. Based on the results of analyzing of students' answers I conclude that most of the students have problems with their retention of conceptual understanding and they also have misconception of limits. The first misconception I identified is that students always used the substitution method to determine a limit of a function at a point, but they did not check whether the function is continue or not at the point. It means that they only use the substitution theorem partially, because they do not consider that the substitution theorem  $\lim_{x \rightarrow c} f(x) = f(c)$  works only if  $f(x)$  is defined at  $x = c$ . The other misconception identified is that some students always think there must be available of variables  $x$  in a function to determine the limit of the function. I conjecture that conceptual understanding of limit partially leads students to misconceptions.

### 1. Introduction

Calculus is a compulsory course for students of mathematics education. In the curriculum of mathematics education of Universitas Sriwijaya, Calculus is divided into three parts, which are Calculus I, Calculus II, and Calculus III (Advance Calculus). Students of mathematics education are not allowed to take Calculus II if they are not passed Calculus I, and they cannot join Calculus III if they have not taken Calculus II yet. In my opinion, students of mathematics education should have good conceptual understanding of Calculus I before they are going to continue learning the next parts of Calculus.

As we know that students learning Calculus start from studying the concepts of limit and function, and according to Purcell [1], Calculus is the study of limits. **5** means limits as the main topic studied in learning Calculus. According to Bokhari and Yushau [2], if mathematics majors do not understand the concept of limit, they are not likely to understand the concepts of continuity, uniform continuity, convergence, derivative, and they are not likely be ready to take other Analysis courses. **6** reality, based on my experience as a lecturer of Calculus students of mathematics education still had **problems with their conceptual understanding and their misconception of limit.**

According to Muzanga and Chifamba [3] misconceptions in learning calculus are because of incompletely or poor understanding of fundamental topics of calculus, such as limits of functions and



the representations of limits. According to Ugulu [4] misconceptions of a topic which a student experienced are caused by inappropriately instructional techniques of teaching, and the misconceptions would retain in the student's memory approximately one to two years since the student got the misconceptions even the student learn other subjects related to the topic.

In my opinion students' abilities of memorizing of what they learned are still effective to make their learning much better. Therefore, in my dissertation I am interested in investigating more deeply whether students of mathematics education whom already passed Calculus I having good retention of conceptual understanding of limits, and I also want to identify the quality of their conceptual understanding and their misconceptions of limits.

## 2. Method

To figure out my research questions I did a preliminary research in February 2017, by giving 16 questions to survey 73 students of mathematics education of Universitas Sriwijaya without any rehearsal for the survey test. All students whom were the respondents had already passed Calculus I, and the test was taken approximately three months since they finished Calculus I. I analyzed students' answers descriptively and classified the students' answers into some categories.

## 3. Result and Discussion

In this article, I will discuss the questions of survey and the findings related to misconceptions. The students' answers on the first question "**Do you remember definition of limit?**" reveal that there are 6 students (35.84%) claiming that they do not remember the definition of limit. It shows that some students have problems with their retention of what they learned.

Actually, the students just finished and passed the class of Calculus I, which is approximately two months before they took the survey test. I assumed that they should had still remembered the definition of limit, but in fact they did not shows my expectation 10 could be that the students did not maintain what they already learned. According to Ugulu [4], without any effort the longer term retention of students' knowledge fade away over time. It means that students should maintain and be aware of their responsibility of their learning by always practicing and applying what they already learned and understood. I think that students should have good retention of what they learned to make their studying other related concepts more effective. Even though students could remember the definition of limit, it does not guarantee the students understood the concept as well. If they have problems with the retention of what they learned then they will find difficult to understand what they learned and to connect with other related concepts.

Students' answers on the second question "**Write down what you remember related to definition of limit.**" reveal that there are 32 students (43.84%) giving unclear answers, which means they just wrote symbols and notations used for the definition of limit but they did not make any correct relations between the symbols and notations. Figure 1 shows a student could remember some symbols and notations related to definition of limit, like  $c, x, f(x), L, \varepsilon, \delta, <, >, \rightarrow$ , and  $| |$ , and the student tried to build the connection between the symbols and notation but the student failed to construct any correct connections between the symbols and notation to be a definition or concept of limit. The worst thing is that the student wrote meaningless expressions, like  $0 > |x - c| < \delta$ ,  $\delta > fx$ , and  $|f(x) = L| < \varepsilon$ . It shows that the student did not understand the meaning of symbols and notations.

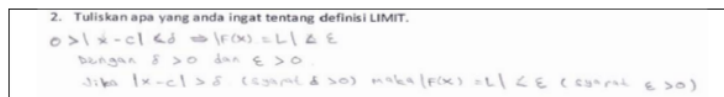


Figure 1. Two examples of students' answers on the second question.

The students' answers on the third question "**Do you understand every symbol and notation related to definition of limit?**" show that there are 38 students (52.05%) answering that they do not really understand every symbol and notation related to definition of limit. This finding supports a reality of students' problems of making connections between symbols and notations of limit. For instance, the

student wrote  $0 > |x - c| < \varepsilon$  which is meaningless in the context of definition of limit, because  $0 > |x - c|$  is an incorrect mathematical expression, as we know that any absolute value will never be lesser than zero. It shows that the students have problem with their understanding of absolute value.

The students' answers on the fourth question "**Did you find difficult to understand definition of limit? Give your reason**" reveal that there are 58 students (79.45%) responding that they found difficult in learning and to understand definition of limit. The first example of student's answer is "Yes, at the beginning of learning it was difficult for me to understand symbols used to definition of limit, and relation between  $\varepsilon$  and  $\delta$ ". The second example of student's answer is "Yes, at the beginning of learning I experienced difficulty to understand definition of limit. Especially at the part of formal definition of limit, because when at senior high school it was never taught and the symbols look strange". It shows that symbols and notations of formal definition of limit become the main problem of learning limit for students to understand. Most of the students said that they were taught how to solve questions about limit but not really taught how to understand definition of limit. This problem could be a cause of misconception of limit students experienced in learning limit.

Based on the findings of students' answers described above, I could identify that some students found difficult to understand the concept or definition of limit through symbols and notations even they could remember the symbols and notations. It is the same as Blaisdell's [5] finding that students whom take Calculus course find difficult in learning limit which only 21.6% of 111 students could identify the correct formal definition of limit from some given choices of definitions. According to Vosniadou [6], to understand other advanced knowledge of many disciplines, students could not be depended on their simple memorization of what they learned or their intuitive theories, but they need to do conceptual change which means they need to restructure their previous knowledge based on their daily experiences and lay cultures.

The students' answers on the fifth question "**Write down your algorithm to solve questions about limit. Give your reason why you prefer to the algorithm.**" inform that 50 students (68.49%) could write down their steps for solving questions about limit. Most of the students used a step of substituting a value of variable  $x$  into a function as the main step of the algorithm, and they said that by substituting the value  $x = c$  into  $f(x)$  is the easier way to solve questions about limit.

Here it is the example of student's algorithm for solving questions about limit, which are the first step: look at the function  $f(x)$ ; the second step: look at  $x$  closes to what number; the third step: substitute the value of  $x$  which closes to  $n$  (for instance) into the function  $f(x)$ ; the fourth step: we assume  $x = n$  (because closes to); and the fifth step: calculate the result of substituting the value of  $n$  into the function  $f(x)$ . The algorithm supports that in reality most of the students solve  $\lim_{x \rightarrow c} f(x)$  by directly substituting  $x = c$  into  $f(x)$  without checking whether the function  $f(x)$  is defined or not at  $x = c$ . In my opinion it is a misconception of limit that the students have, even though we could solve  $\lim_{x \rightarrow c} f(x)$  by substituting  $x = c$  into  $f(x)$  for any function  $f(x)$  continuous at  $x = c$ . The example of student's algorithm shows that the students partially understood the substitution theorem in [1], which is "If  $f$  is a polynomial function or a rational function, then  $\lim_{x \rightarrow c} f(x) = f(c)$  provided  $f(c)$  is defined. In case of a rational function, this means that the value of the denominator at  $c$  is not zero".

The students' answers on sixth question "**Write down the value of  $\lim_{x \rightarrow 1} 3x$ .**" reveal that 68 students (93.15%) could give the correct value of  $\lim_{x \rightarrow 1} 3x$ , which is 3. However, there are some students failed to give a correct answer to the sixth question. Most students just simply substituted the value  $x = 1$  into  $f(x) = 3x$  and they wrote the correct result, which is 3. Of course, the answer is accepted and also correct because  $f(x) = 3x$  is a polynomial function which is defined at  $x = 1$ , so the student could apply the substitution theorem in this case.

The students' answers on the seventh question "**Write down the value of  $\lim_{x \rightarrow 1} 3$ .**" reveal that there are 28 students (38.36%) giving incorrect answers. Some incorrect answers on the seventh question are the first one said "it is not limit"; the second one claimed "the limit does not exist"; the third one answered " $\lim_{x \rightarrow 1} 3 = 0$ "; the fourth one argued " $\lim_{x \rightarrow 1} 3$  does not exist because there is no a function so it cannot be determined the value of its limit"; and the last one stated " $\lim_{x \rightarrow 1} 3$  cannot be done because it

has no variable  $x$  next to 3". It shows that some students still had problem of conceptual understanding of a constant function,  $f(x) = a$  where  $a \in R$ . All the incorrect answers imply that the students have misconception of limit because they only could calculate limit of a function if the functions have variables of  $x$ .

The students' answers on the eighth question "**Is it correct that  $\lim_{x \rightarrow 1} 3x = \lim_{x \rightarrow 1} 3$ ? Give your reason.**" show that 31 students (42.47%) did not give the correct answer. Some examples of students giving incorrect answers on the eighth question are the first student wrote "*It is not equal, because  $\lim_{x \rightarrow 1} 3x = 3(1) = 3$  its limit exists, meanwhile  $\lim_{x \rightarrow 1} 3$  its limit does not exist*"; the second student answered "*It is different, because  $\lim_{x \rightarrow 1} 3x$  has the value of  $x$  substituted into the function to get the value of limit. The function  $f(x)$  in  $\lim_{x \rightarrow 1} 3x$  is  $3x$ , meanwhile  $\lim_{x \rightarrow 1} 3$  has no any function*"; the third student wrote "*It's not correct, because  $\lim_{x \rightarrow 1} 3x$  has a variable  $x$  which by the value of  $x$  we can calculate its limit, meanwhile at  $\lim_{x \rightarrow 1} 3$  there is no variable  $x$  therefore we cannot calculate it*"; and the last one stated "*It is not equal, because 3 is not a function  $f(x)$  in  $\lim_{x \rightarrow 1} 3$ . Meanwhile in  $\lim_{x \rightarrow 1} 3x$  there is a function which is  $3x$ .*"

The students' answers on the ninth question "**Write down the value of  $\lim_{x \rightarrow 1} \frac{1}{(x-1)}$** " inform that there are 60 students (82.19%) giving incorrect answers. Some incorrect answers on the ninth question are the first student wrote " *$\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \frac{\lim_{x \rightarrow 1} 1}{\lim_{x \rightarrow 1} (x-1)} = \frac{\lim_{x \rightarrow 1} 1}{\lim_{x \rightarrow 1} x - \lim_{x \rightarrow 1} 1} = \frac{1}{0}$* "; the second student answered " *$\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \frac{1}{1-1} = \frac{1}{0} = \text{undefined}$* "; the third student answered " *$\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \frac{1}{1-1} = \frac{1}{0} = \infty$* "; and the last one wrote " *$\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \frac{1}{(x-1)} \cdot \frac{(x+1)}{(x+1)} = \frac{(x+1)}{(x-1)(x+1)} = \frac{1+1}{(1-1)(1+1)} = \frac{1}{0 \cdot 2} = 0$* ".

I identify that all the mistakes occurred because the students always use the substitution method of variable  $x$  into a function  $f(x)$  without checking whether the function is defined or not at the value of  $x$ . This finding supports a conjecture that students whom understood the substitution theorem partially would have misconceptions of limit.

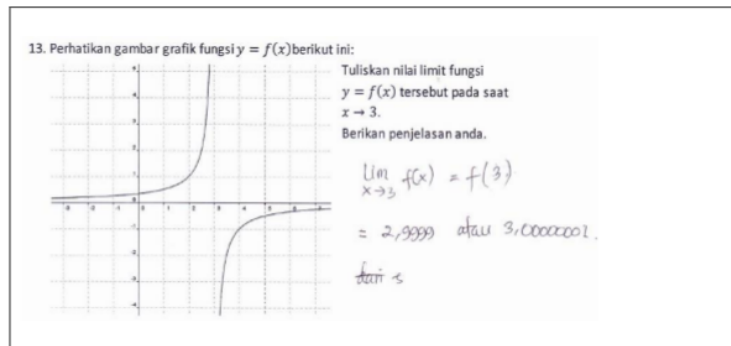
The students' answers on the tenth question "**Write down the value of  $\lim_{x \rightarrow 1} \frac{1}{|x-1|}$** " reveal that there are 53 students (72.60%) giving incorrect answers. The first example of student's answer is "*by directly substituting  $x = 1$  into the function  $f(x) = \frac{1}{|x-1|}$  and the final calculation is  $\frac{1}{0}$  then the student said that the limit is undefined*". I could say that the first student shows a misconception, because the student determined limit of a function by only substituting a certain value of  $x$  into the function without checking whether the function is defined at the value of  $x$ .

Meanwhile, the second student argued that " *$\lim_{x \rightarrow 1} \frac{1}{|x-1|}$  does not exist, even though its left-side limit and its right-side limit converge to the same number, but if the value of  $\delta$  become smaller then the result will become bigger without end*". The second student shows a contradiction of the answer, because at one side the student said that  $\lim_{x \rightarrow 1} \frac{1}{|x-1|}$  does not exist, but at the other side the student said that  $\lim_{x \rightarrow 1^-} \frac{1}{|x-1|}$  and  $\lim_{x \rightarrow 1^+} \frac{1}{|x-1|}$  converge to a certain number which means the limit exists. It implies that the second student does not really understand the concept of limit.

The students' answers on the eleventh questions "**Is it correct that  $\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \lim_{x \rightarrow 1} \frac{1}{|x-1|}$ ? Give your reason**" reveal that there are 36 students (49.32%) giving incorrect answers. Here are the two examples of students' answers. The first student wrote "*Yes it is, because each value of limit is undefined. If we check  $\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \lim_{x \rightarrow 1} \frac{1}{|x-1|}$  we get  $\frac{1}{0} = \frac{1}{0}$* ". The second student answered "*Yes it is, because when  $x \rightarrow 1$  then the value of  $(x-1) = 1-1 = 0$ , and when  $x \rightarrow 1$  then the value of  $|x-1| = |1-1| = 0$ , so by the same numerator which is 1, and then divided by the same denominator which is 0, the results will be undefined, therefore  $\lim_{x \rightarrow 1} \frac{1}{(x-1)} = \lim_{x \rightarrow 1} \frac{1}{|x-1|}$* ". Again, it shows that the students do not really understand concept of limit.

The students' answers on the twelfth question "**Write down the limit of  $f(x) = \frac{x^3+6x^2-6x-1}{2x^2-x-1}$  when  $x \rightarrow 1$** " reveal that there are 39 students (53.42%) giving incorrect answers. Some students did again the same mistakes on their answers, because they just substituted the value of variable  $x = 1$  into the function  $f(x) = \frac{x^3+6x^2-6x-1}{2x^2-x-1}$  without checking whether the function is defined or not at the variable  $x = 1$ .

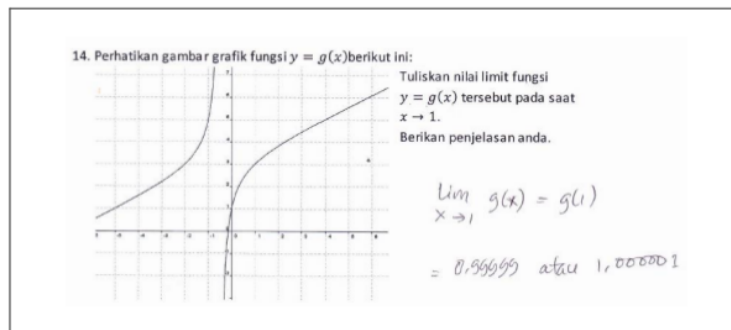
The students' answers on the thirteenth question "**Look at the graphic of function  $y = f(x)$  below: Write down the value of limit of function  $y = f(x)$  when  $x \rightarrow 3$** " reveal that there were 15 students (20.55%) giving incorrect answers.



**Figure 2.** An example of incorrect answers on the thirteenth question

Figure 2 shows that the student just simply substituted  $x = 3$  into  $y = f(x)$  then wrote  $\lim_{x \rightarrow 3} f(x) = f(3) = 2.9999$  or  $3.00000001$ . It shows that the misconception of limit occurs because students have an understanding that to solve  $\lim_{x \rightarrow a} f(x)$  by simply substituting the value of variable  $x = a$  into the function  $f(x)$ .

The students' answers on the fourteenth question "**Look at the graphic of function  $y = g(x)$  below: Write down the value of limit of function  $y = g(x)$  when  $x \rightarrow 1$** " reveal that there were 9 students (12.33%) giving incorrect answers.



**Figure 3.** An example of incorrect answers on the thirteenth question

Figure 3 shows that the student just simply substituted  $x = 1$  into  $y = g(x)$  then answered  $\lim_{x \rightarrow 1} g(x) = g(1) = 0.9999$  or  $1.00000001$ . It shows that the student does not understand to read a picture of graphic of a function.

The students' answers on the fifteenth question "**Which one (based on your opinion) is much easier, solving questions about limit in algebra expressions or in picture of graphic of functions?**"

*Please give your reason.* ” reveal that there are 60 students (82.19%) claiming that questions in algebra expressions are much easier than questions in picture of graphic of functions. It could be a reason why students are not familiar with pictures of graphic of functions, because students are busy with manipulating algebra expressions of functions without knowing the picture of graphic of the functions. A student wrote *“In my opinion, it is much easier answering questions about limit in forms of mathematical expressions than in pictures of graphic of functions. Because questions in form of mathematical expressions we could see the numbers precisely.”*

The students’ answers on the sixteenth question *“Was the class of Calculus I you had taken using computer aid in learning limit? Please give your reason ”* reveal that there are 33 students (45.21%) claiming that they did not use computer aid in learning limit, 19 students (26.03%) saying that they used their own mobile phones or computers at home for learning limit, and the rest students did not answer the question. The first example of student’s answer is *“Not. In the Calculus I class we only studied it by text books.”*, and the second example of student’s answer is *“Not. But I have my own application on my mobile phone which can draw graphics of any function so that it makes easy to see the value of limit.”* The second student’s answer informs that by using technology in learning could help students to understand what they learn. According to Budi [7] learning activities in which computer use is embedded could help students reaching better achievement of they learned.

There are 12 (16.43%) students giving the correct answer on the twelfth question, but failed to answer the fourteenth question correctly. Actually, the fourteenth question is the graphical representation of the twelfth question, which means the questions are the same. The problem 4 was that the students did not have good ability of graphical representation. My findings show that students in general had more difficulty with graphical representations than mathematical expressions or algebra manipulation. It is different from Blaisdell’s [5] findings that students in general had less difficulty with graphical representations than mathematical notation or definition questions. Blaisdell [5] says that this difference could be due to student prior experience with graphical questions. In my case students were more familiar and experience with questions in mathematical expressions. Meanwhile in my case students had little opportunities to deal with graphical questions.

#### 4. Conclusion

Most of the students said that they were taught how to solve questions about limit but not really taught how to understand definition of limit. That is why most of students always consider the method of substitution as the easy way to solve questions of limit. Unfortunately, they do not comprehensively understand and apply the substitution theorem, which is *“If  $f$  is a polynomial function or a rational function, then  $\lim_{x \rightarrow c} f(x) = f(c)$  provided  $f(c)$  is defined. In case of a rational function, this means that the value of the denominator at  $c$  is not zero”*. Problems occur when they face questions of limit of rational functions’, like  $\lim_{x \rightarrow 1} \frac{1}{(x-1)}$ . Students whom have misconception of limit will directly

substitute the value of  $x = 1$  into the function  $f(x) = \frac{1}{(x-1)}$ , which results an incorrect answer  $\frac{0}{0}$ .

Finally, I conclude that students whom have partially conceptual understanding of what they learned will come to other misconceptions. Therefore mathematics teachers should be aware of students’ misconception and also should design learning activities appropriately to reduce or avoid other misconceptions. According to Liang [8] teaching the concept of limit by using conceptual conflict strategy and Desmos graphing calculator could be the answer to help students reducing their misconception of limit.

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