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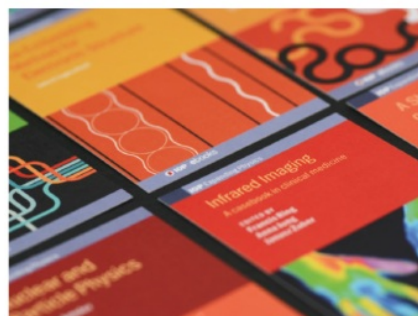
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Mathematical proof analysis using mathematical induction of grade XI students

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Abstract. The purpose of this study is to provide an overview on how students do mathematical proof using mathematical induction through the activity of compiling and validating proof. It is known that mathematical induction is a proof used to prove the truth of a mathematical statement related to natural numbers. The subjects of this study were 34 students of grade XI SMA Negeri 13 Palembang. The results showed that (1) students were able to prove on the base of the induction step by showing the truth when n is assumed by one member of a natural number, (2) students were able to make a mathematical induction hypothesis by assuming the statement $n = k$ is true, (3) students still make mistakes in performing mathematical induction steps precisely on the part when doing algebra manipulation to prove $n = k + 1$ is true from the statement $n = k$ that has been considered true, and (4) Students still do not understand the concept of mathematical induction as indicated by the large number of students unable to do the activities of compiling proof by mathematical induction and there are still some students who write conclusions on the activity of compiling proof that the statement is proven true without showing the truth of the induction step

1. Introduction

In learning mathematics one of the activities that students must master is the activity of conducting proof. Proof has an important role in mathematics learning [1-4]. Proof is an absolute and fundamental part of learning mathematics or an inseparable part of learning mathematics [5-7]. Proof is a series of logical arguments where the argument here comes from theorems, premises, definitions and postulates which later from this argument will explain the truth of a statement [8]. Several benefits can be obtained from conducting proving activities including being able to improve students' skills in solving a problem, persuasive, creativity, reasoning, and mathematical thinking [8,9]. Also, students who are accustomed to carrying out proof activities can verify a truth [10]. From the statement above it is known that proof is an ability that students should have in learning mathematics.

In carrying out proof, several methods can be used, one of them is by mathematical induction. Mathematical induction is a standard proof technique in mathematics [11]. It is known that induction is used to prove the truth of a mathematical statement related to natural numbers [11-14]. A statement can be proven true if it has shown the principle of mathematical induction. It is known that there are 3 principles of mathematical induction, namely the principle of standard induction, the principle of strong induction and the principle of induction which is confiscated [11]. The principle of mathematical induction is generally formalized as follows: suppose $P(n)$ a statement relating to natural numbers. To prove that $P(n)$ is true it must show that if (1) $P(1)$ is true, (2) $P(k)$ is true $\rightarrow P(k + 1)$ is true for every $k \in \mathbb{N}$, then $P(n)$ is proven true for every $n \in \mathbb{N}$ [12,14]. Mathematical induction is the third goal of the school curriculum where the teacher is expected to give more

attention to learning proof by mathematical induction [15]. Mathematical induction is one of the most powerful tools in proving statements related to discrete mathematics [16]. From the statement above it can be seen that the activities of proof by mathematical induction are important for secondary school students.

According to Selden and Selden [17] the evidentiary activity consists of the activity of compiling proof and validating proof. In carrying out the activities of compiling proof students must have the ability to use methods of proof, entries, definitions, axioms to show the truth of a mathematical statement. In validating proof students must have the ability to criticize proof related to the types of proof that often appear in mathematics. There are several activities that can be carried out during the validation of proof, including reading mathematical proof to determine the truth and error of the proof by paying attention to the suitability of the axiom system, the premise, existing mathematical results (theorems or entries) with deductive reasoning flow, completing the proof if mistakes are found and compare the effectiveness of the proof with each other [17,18].

In the previous research, it was known that mathematical proofing activities usually only looked at how the students did the activity of compiling proof. As conducted Netti researchers [19] who saw the stages of mathematical thinking of students from the activities of proof construction on the material function. There is no previous research that discusses how students conduct proof through the activities of compile and validation proof on mathematical induction material which is an important material for secondary school students. For the reasons above, the researcher is interested in taking the purpose of the study " *Mathematical Proof Analysis Using Mathematical Induction of Grade XI Students* ". The purpose of this study is to describe how students do mathematical proof using mathematical induction through the activities of compiling and validating proof.

2. Method

This research is a descriptive study that aims to provide an overview of the results of mathematical proofing analysis using mathematical induction. This research has been carried out in SMA Negeri 13 Palembang in 2019/2020 academic year recorded from July 2019 to September 2019. The research subjects used were students of class XI MIPA 1 of SMA Negeri 13 Palembang. The procedure of this study consisted of three stages, namely the preparation phase, the implementation stage and the data analysis stage. In the preparation stage, the thing to do is to observe the school of SMA Negeri 13 Palembang to determine the class that will be used as the subject of research and prepare a research instrument consisting of RPP, LKPD and test questions consisting of 2 questions with different types. Problem number 1 asks about how students validation proof related to series and problem number 2 asks how students compile proof related to division. At the stage of implementing things done is teaching mathematics induction material in class according to the RPP that has been designed, giving LKPD during the learning process and at the end of student learning will be given test questions. While at the data analysis stage, the test results sheets that have been done by students are then grouped based on the similarity of errors which are then analyzed to find out how students validate the proof and arrange proof by looking at the principles of mathematical induction. The principle of induction is divided into several steps including (1) the induction base step, (2) the induction hypothesis, (3) the induction step and (4) the conclusion [12]. From the validation questions, the students can do the induction base step if the students can show the truth in statement 1, for the induction hypothesis of truth in statement 2, for the induction statement on 3, 4, and 5 as well as the conclusion in statement 6.

3. Results and Discussion

The test results sheets that students have done first are grouped based on similarities and wrong answers which are then analyzed to find out how students can validate the proof and arrange the proof by looking at the principles of mathematical induction. From the validation of the proof activities students can perform induction basis steps if students can show the truth in statement 1, for the induction hypothesis of truth in statement 2, for the induction step in statements 3, 4, and 5 as well as the conclusions in statement 6. For the activity of compiling the results of the answers students who

have been classified based on similarity and error answers obtained several categories. The categories are described in Table 1 below.

Table 1. Categories of activities for compiling student proof.

Category	Answer type
A1	Compile proof by fulfilling the principle of induction correctly and completely.
A2	Compile proof by fulfilling the principle of induction but make mistakes.
A3	Compiling proof only fulfills the principle of mathematical induction on the basis of the induction step and the induction hypothesis.
A4	Compiling proof does not fulfill the principle of induction but keeps writing conclusions.
A5	Unable to conduct compile proof.

The results of the data analysis validated the proof and compiled the proof of class XI students of MIPA 1 with mathematical induction as follows.

3.1. Validation of Mathematical Induction Proof

The results of data analysis from the student test answer sheets validating the proof of question number 1 related to the series material is presented in Table 2 below.

Table 2. Distribution of student proof validation.

Principle of Induction	Validation of Proof	Correct	Incorrect
Induction Basis Step	Statement 1	28	6
Induction Hypothesis	Statement 2	22	12
	Statement 3	8	26
	Statement 4	20	14
Induction step	Statement 5	5	29
	Statement 5	30	4
Conclusion	Statement 5	30	4
Total students		34 People	

From the results of data analysis validating the proof in Table 2 it is known that the average student already knows that statement 1 is an induction basis step which is shown from the test answers of students who write information that statement 1 is an induction basis step. Then students also can do the induction basis step by showing the truth in statement 1 that contains $P_2: 2a + b = 2a + b$, when suppose with $n = 2$. Students write the reason behind the statement by adding up the first term and the second term to show the truth of the left side so that $a + a + b = 2a + b$. In the right side, students substitute the value of $n = 2$ to the statement $\frac{n}{2} [2a + (n - 1)b]$ so that it is evident the number of series of the left side is the same as the result of statement $\frac{n}{2} [2a + (n - 1)b]$ when $n = 2$. In the statement 2, the average student has been able to validate by giving reasons from $P_k: a + a + b + a + 2b + a + 3b + a + 4b + \dots + a + (k - 1)b = \frac{k}{2} [2a + (k - 1)b]$ is an induction hypothesis.

In the statement 3, only a small proportion of students can validate correctly. This is known from the results of the student evidence validation activity where only 8 students answered correctly and

correctly in validating the statement 3. Students who validate correctly can give reasons $P_{k+1}: a + a + b + a + 2b + a + 3b + a + 4b + \dots + a + (k-1)b + a + bk = \frac{k+1}{2} [2a + bk]$ is an induction step. Students also explain that the statement 3 is a step used to show the truth of $n = k + 1$ by using the statement $n = k$ which has been assumed to be true. Students also have shown the statement $a + a + b + a + 2b + a + 3b + a + 4b + \dots + a + (k-1)b$ because it adds 1 term from the statement $n = k$ which we have assumed to be true in the 3rd statement by substituting $k + 1$ into the statement $a + (k-1)b$ to obtain $a + bk$. For statements $\frac{k+1}{2} (2a + bk)$ obtained by substituting $k + 1$ into the statement $\frac{k}{2} [2a + (k-1)b]$. While students who are not quite right in validating statement 3 are only giving reasons that statement 3 is an induction step used to show the truth of $n = k + 1$ using the statement $n = k$ which we already consider to be true in the statement.

konsep yg digunakan ialah pemfaktoran aljabar

$$\begin{aligned} \frac{k}{2} [2a + (k-1)b] + a + bk &= \frac{k}{2} [2a + bk - b] + a + bk \\ &= \frac{k}{2} [2a + bk - b] + a + bk \\ &= \frac{2ak + bk^2 - bk}{2} + \frac{2a + 2bk}{2} \\ &= \frac{2ak + bk^2 - bk + 2a + 2bk}{2} \\ &= \frac{bk^2 + 2ak + bk + 2a}{2} \\ &= \frac{(k+1)(2a + bk)}{2} \\ &= \frac{k+1}{2} (2a + bk) \end{aligned}$$

Pemfaktoran aljabar terbukti benar
karena kedua ruas bernilai sama, maka penulisan $n=k+1$ terbukti benar

Figure 1. The test answers validate student 1 proof.

.....(5) Konsep yang digunakan ialah pemfaktoran aljabar

$$\begin{aligned} &= \frac{k}{2} [2a + (k-1)b] + a + bk \\ &= \frac{k}{2} [2a + (k-1)b] + a + bk \\ &= \frac{k}{2} [2a + bk - b] + a + bk \\ &= \frac{k}{2} [2a + bk] + a + bk \\ &= \frac{k}{2} [2a + bk - b] \\ &= \frac{k+1}{2} [2a + b] \end{aligned}$$

Figure 2. The test answers validate student 2.

For the statement 4 students make a lot of mistakes from giving reasons because $a + a + b + a + 2b + a + 3b + a + 4b + \dots + a + (k-1)b + a + bk = \frac{k}{2} [2a + (k-1)b]$, then the statement on the left side can be changed to $\frac{k+1}{2} [2a + bk]$ even though students should write on the left side it can

be changed to $\frac{k}{2} [2a + (k - 1)b] + a + bk$. Furthermore, in the statement 5 it is known that only 5 students can validate the statement $\frac{k}{2} [2a + (k - 1)b] + a + bk$ is equal to $\frac{k+1}{2} [2a + bk]$ by showing the truth of the statement (figure 1). Almost all students cannot show the truth of the statement. It is known that students often make mistakes in doing algebraic manipulation shown in Figure 2. From Figure 2 it is known that student 2 cannot apply the distributive nature of algebra. This is shown by student 2 who writes the result of $\frac{k}{2} [2a + bk - b] + a + bk = \frac{k}{2} [3a + bk^2 - b]$ where the result should be $\frac{2ak+bk^2-bk+2a+bk}{2}$. As a result of mistakes made by students in doing algebraic manipulation, students cannot show that $\frac{k}{2} [2a + (k - 1)b] + a + bk$ is the same value as $\frac{k+1}{2} [2a + bk]$. Because students cannot show the truth of the statement, students cannot validate the proof of the statement 5 which is an induction step.

In the statement 6 it is known that students have been able to give reasons from the conclusions given why the statement $P(n)$ can be proven to be true because it has shown the induction basis step and the induction step is proven to be true. But in the concept of mathematical induction the activity of validating evidence in the statement 6 made by students is wrong. Because most students make mistakes and mistakes in validating statement 5 which is an induction step in mathematics. So it can be said that students cannot validate correctly in statement 5.

From the above explanation it can be concluded that students have not been able to validate the evidence in statements 3, 4 and 5 because they still make mistakes in showing the truth of the statement. In addition, it is known that students' understanding of algebraic concepts is still weak which is indicated by students who cannot validate the truth of the 5th statement. This is in line with research conducted by Kong [20] which explains that the lack of knowledge students have on certain topics in metaphors affects the ability of students to complete proof with mathematical induction.

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3.2. Compile Proof by Mathematical Induction

The results of data analysis from the answer sheets of student tests compiling proof with mathematical induction on item number 2 related to the subject matter of division are presented in Table 3 below.

Table 3. Distribution compile student proof

Category	Frequency (People)
A1	5
A2	2
A3	11
A4	6
A5	10
Total students	34

From Table 3 which is the distribution of activities to compile student proof it is known that only 5 people fall into the A1 category. Prove the truth of the statement $a^{2n} - b^{2n}$ divide by $(a + b)$ by fulfilling the principles of mathematical induction correctly and completely, which shows the truth of the induction basis step, makes the induction hypothesis, shows the truth of the induction step and makes conclusion. In the A2 category, students have proven the truth of the statements $a^{2n} - b^{2n}$ divided $(a + b)$ by fulfilling the principle of induction but making mistakes and mistakes. Students who make mistakes can be seen in Figure 3 and students who make mistakes can be seen in Figure 4 below.

$$\begin{aligned}
 a^{2k+2} - b^{2k+2} &= a^{2k} \cdot a^2 - b^{2k} \cdot b^2 \\
 &= (a^{2k} - b^{2k})(a^2 + b^2) \\
 &= [(a+b)^{2k} + b^{2k} - a^{2k} - b^{2k}](a^2 + b^2)
 \end{aligned}$$

Figure 3. Student error 3 in compiling proof at the induction step.

Bukti
Langkah awal
misalkan: -1
diperoleh P: a - b : a - b habis dibagi a + b

Figure 4. Student error 4 in compiling proof at the basis step.

It is known from Figure 3, student 3 made a mistake in the induction step to show the truth of $n = k + 1$. Student 3 made a mistake in changing the form $a^{2k} \cdot a^2 - b^{2k} \cdot b^2$ to $(a^{2k} - b^{2k})(a^2 + b^2)$. It is known that the result of the multiplication factor $(a^{2k} - b^{2k})(a^2 + b^2) = a^2 a^{2k} + a^{2k} b^2 - a^2 b^{2k} - b^2 b^{2k}$ not $a^{2k} \cdot a^2 - b^{2k} \cdot b^2$. From Figure 4 it is known that student 4 made a mistake at the basis induction step. To show the truth at the basis step of student induction 4, suppose $n = -1$. While the problem is known that n is a member of a natural number where the member of a natural number starts from number 1. Negative numbers are not included in the original number.

In category A3 students can only carry out activities to compile proof up to the step of the induction basis and the induction hypothesis. Students have not been able to show the truth at the induction step, leading statements $a^{2(k+1)} - b^{2(k+1)}$ can be completely divided $(a + b)$. In addition, there are some students who can only do induction steps limited to writing $a^{2(k+1)} - b^{2(k+1)}$. The student cannot continue, herding statements $a^{2(k+1)} -$ so that they have been divided $(a + b)$ and there are also some answers of category A3 students in showing the truth in the induction step of making a mistake in applying the distributive nature of algebra. The tendency of students to make mistakes in applying distributive algebra is supported by Kurniati research [21]. In Kurniati research explained that many students have difficulty in working on algebraic form problems is to apply the distributive nature of algebra.

Students in the A4 category in carrying out the activity of compiling evidence tend to keep writing the conclusion that the statements $a^{2n} - b^{2n}$ are completely divided $(a + b)$ proven to be true, without showing the truth in the induction step as shown in the following Figure 5.

Langkah awal
misalkan a
diperoleh P: a^{2n} - b^{2n} : a + b habis dibagi a + b
Pernyataan (n genap, untuk itu P_n benar) atau untuk n=1

Langkah induksi
Misal n=k
P_k : a^{2k} - b^{2k} : a + b
diperoleh P_{k+1} : a^{2(k+1)} - b^{2(k+1)} : a + b
misal n=k+1
D_{k+1} : a^{2(k+1)} - b^{2(k+1)} = 2^{2k} \cdot a - b^{2k} \cdot b^1

Conclusion

Cannot show the truth of the induction step and wrong in rank multiplication

Figure 5. Students' 5 test answers categorized as A4.

From Figure 5 above it is known that student 5 does not fulfill the principle of induction by not showing the truth in the induction step. This is because student 5 did not complete the activity of compiling proof at the induction step and also the student made a mistake in doing multiplication of rank. Student 5 writes the result of the multiplication of $a^{2(k+1)} - b^{2(k+1)} = 2^{2k} \cdot a - b^{2k} \cdot b^1$. The result of the multiplication of the rank is $a^{2k} \cdot a^2 - b^{2k} \cdot b^2$. In addition, on the answer sheet of student test 5 it is known that the student wrote a conclusion because it is in accordance with the principle of mathematical induction when it shows the truth of the initial step (induction basis step) and the induction step then the statement of $a^{2(1)} - b^{2(1)}$ divisible by $(a + b)$. The tendency of students in writing conclusions without showing the truth of the basis and induction step is known to not

understand the concept of mathematical induction where the truth of the induction basis step implies the truth of the induction step then the truth of the induction step implies the statement can be proven true. So that if the basis and induction steps are proven to be true then it can be concluded that the statement is true but students tend to follow the rules of the mathematical induction principle which must write the basis of induction steps, induction hypotheses, induction steps and conclusions. This is in line with previous studies conducted by Harel [22], and Styliandes et al [23]. In that study explained that students tend to follow the rules of the principle of induction without understanding the concept of induction.

Students in the A5 category are known to have no understanding at all of how to carry out activities to compile proof. This is known through the results of student answer sheets in which category A5 students cannot answer at all question number 2. The inability of students to carry out the activities of compiling proof is because students are still unfamiliar with proof learning. Usually in the learning process students rely more on memorizing existing evidence without understanding the evidence and how to arrange the proof. This explanation is in line with what was said by Moore [24].

From the analysis of the student answer sheets described above it can be seen that the average student cannot do the activity of compiling proof using mathematical induction. Students still experience technical difficulties shown by students who still make mistakes in doing algebraic manipulation in the induction step. This is in line with research conducted by Kong [20]. In addition students still do not understand the concept of mathematical induction in compiling proof.

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4. Conclusion

Based on the results of this study it can be concluded that through the activities of validation proof and compile proof, students can already take an induction base step. Students also can make hypothesis of the induction step by assuming that the statement $n = k$ is true. But apparently, most students still do not understand the concept of proof of mathematical induction. It is known from many students still write the conclusions that statements are proven true without showing the truth of the induction step. Also, students still experience technical difficulties from the induction step shown from answer sheet of students who are still making mistakes in algebra manipulation when showing the truth of the statement $n = k + 1$ by using the statement $n = k$ that has been assumed is true.

Based on the conclusion above, the researcher suggests to the teacher to develop LKPD by the principle of induction to make it easier for students to understand the concept of mathematical induction. Also, it is expected that teachers will strengthen the concept of algebra in students so students do not make mistakes in manipulating algebra in the induction step and emphasizing the concept of mathematical induction that the steps in the induction principle are mutually implicating.

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