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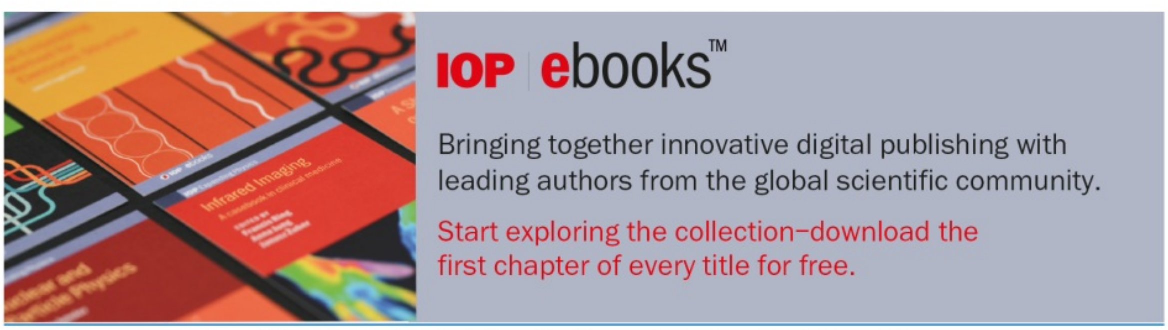

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Analysis of student's proof construction on matrix determinants

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Abstract. This research is a descriptive qualitative research which aims to describe the construction of student evidence on the determinant matrix material. Construction of evidence is compiling evidence from a statement based on the definitions and rules used. The research subjects were students of class XI IPA Palembang 11 Public High School consisting of 42 people. The learning process takes place in accordance with the steps of direct learning. The data collection technique used is a written test consisting of three problem descriptions. The data collected was analyzed by evaluating the evidence from the results of student work consisting of four assessment categories as follows: valid arguments, proofs and proofs that were completed (K_1) valid arguments there were mathematical manipulation errors in the proof process (K_2), valid arguments but there are mathematical manipulation errors in the process of proof and proof of unfinished (K_3), empirical arguments, invalid or incomplete (K_4). Student proof construction in K_1 category was 34.52%, K_2 category was 16.67%, K_3 category was 22.62%, and K_4 category was 26.19%.

1. Introduction

One aspect that must be considered in learning mathematics is proof of mathematics. Evidence is an important component in learning mathematics at all levels of the school, namely working, communicating, knowing, and understanding mathematics so that evidence is recognized as the core of mathematical thinking where students are required to be able to recognize, develop, and use several methods of proof so that the arguments that have been prepared can stated the truth [1,2]. Evidence and proof is an important part of mathematics because proof and proof become the main foundation or basis in building mathematical knowledge [3].

The ability to construct evidence is very important for those who are in the field of mathematics [4,5]. This is because construction is one of the eight roles in mathematical proof, there are (1) verification, (2) explanation, (3) systematization, (4) discovery, (5) communication, (6) exploration, (7) construction, and (8) unification [6]. Evidence construction ability will have an impact on students in secondary school for the next level, namely tertiary institutions in the ability to prepare evidence [6,7].

One of the materials taught at school is the determinant matrix. Matrix theory is a branch of Linear Algebra which is an important discussion in Mathematics [8]. Matrix material is usually applied by the community in recording population data, exchange rates, and so forth. The matrix can be applied in economics and business [9]. In the field of economics the application of the matrix is found in leontive economic models and in the business field the application of the matrix is found in game theory. The field of mathematics study is the theory of the association [10]. In addition, the matrix material is a prerequisite material in the transformation material [11].

Research related to the construction of student evidence in the field of mathematical studies that is geometry at the student and student levels including research [12-14]. Research is also carried out in the field of trigonometry [1]. The ability of students in constructing evidence of geometry material in secondary schools is low or students have not been able to construct the evidence [12]. Students have not been able to construct evidence of students' trigonometry material in secondary schools [1], in that study, it provides suggestions for other researchers to conduct further research on the ability to construct mathematical proofs in the field of other mathematical studies because the ability to construct evidence on trigonometric material has not yet developed. Therefore, researchers will conduct research on the determinant matrix material because in this material the proof is very important to do or be applied so that students understand more about the evidence for the next level. In addition, there is no research on the ability to construct student evidence on the determinant matrix material which shows that the ability to construct student evidence on the determinant matrix material is difficult.

2. Method

The study was conducted during 3 meetings with 2 learning meetings using direct learning and 1 written test meeting. This type of research is a descriptive study that aims to describe the construction of student evidence on the determinant matrix material. The study was conducted in class XI IPA Palembang 11 Public High School consisting of 42 students in the odd semester of the 2019/2020 school year. The data collection technique used is a test. The test questions consist of 4 questions in the form of a description that aims to determine the construction of student evidence on the determinant material matrices by evaluating the evidence of student work that consists of four assessment categories adopted from [15] as follows: valid arguments, evidence and completion (K_1), valid arguments but not proof (K_2), valid arguments but not proof and not finished (K_3), empirical / invalid and incomplete arguments (K_4).

3. Result and Discussion

The implementation of learning in research carried out 3 times meeting. The first meeting is conducted learning with learning indicators that explain the definition of matrix determinants, matrix 2×2 matrix determinants, and matrix 3×3 matrix determinants, and classify the matrix determinant properties. The second meeting carried out learning with learning indicators namely classifying the determinants of the matrix. The third meeting was held a test to students consisting of 4 problem descriptions, the test was conducted to determine the construction of student evidence. The following will summarize the results of the construction of student evidence on the determinant matrix material.

3.1. The Summary Results of The Construction of Students Proof The Determinant Matrix.

Table 1 shows that the summary results of the construction of student evidence on the determinant matrix material were analyzed using evidence evaluations based on the four categories described earlier.

Table 1. Distribution of student's proof construction.

Category of Proof Evaluation	Question Number				%
	1	2	3	4	
K_1	31	6	17	4	34.52
K_2	4	6	12	6	16.67
K_3	2	19	10	7	22.62
K_4	5	11	3	25	26.19

Based on Table 1, obtained students with categories of valid arguments, proof and completion (K_1), valid arguments but not proof (K_2), valid arguments but not proof and not finished (K_3), empirical /

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invalid and incomplete arguments (K_4). Students who are categorized as having proof construction with valid arguments, proof and finish (K_1) basically have proven the statement correctly (using definitions, properties and rules) and have finished doing the proof. Students who are categorized as having proof construction with valid arguments, but not proof (K_2) basically have proven the statement correctly (using definitions, properties and rules) and have finished proofing, but there is a mathematical manipulation error in the proof process. Students who are categorized as having construction of proof with valid arguments but not proof and not yet completed (K_3) basically have proven the statement correctly in the initial step, but there is a mathematical manipulation error in the proof and proof process that has not yet been completed. Students who are categorized as having proof construction with empirical / invalid and incomplete arguments (K_4) basically cannot prove the statement correctly (using definitions, traits and rules) and do not understand the order of the questions given so that the proof carried out is invalid or not finished. The following findings will be presented from the results of student work as follows:

3.2 Student Work

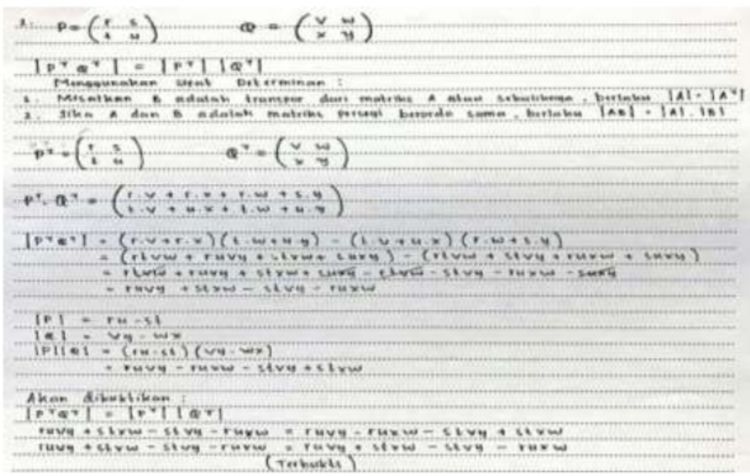

Figure 1. MRA work results in the first problem.

Figure 1 shows that the work done by the MRA in the first problem as a whole has been able to declare the statement correctly, in that problem the MRA has been able to use definitions, properties and rules in carrying out proof, this can be seen from each step of the answer, namely as follows : in step 1, MRA can determine the determinant properties of the matrix used in the proof which consists of 2 properties, namely (1) for example B is the transpose of matrix A or vice versa, applies $|A| = |A^T|$, and (2) if A and B are square matrices of the same order, then $|AB| = |A||B|$, this shows that MRA can use the definition, the determinant properties of the matrix. In step 2, the MRA can change $P = \begin{pmatrix} r & s \\ t & u \end{pmatrix}$ and $Q = \begin{pmatrix} v & w \\ x & y \end{pmatrix}$ to $P^T = \begin{pmatrix} r & t \\ s & u \end{pmatrix}$ and $Q^T = \begin{pmatrix} v & x \\ w & y \end{pmatrix}$, then do the matrix multiplication that has been transposed to obtain $P^TQ^T = \begin{pmatrix} rv + sx & rw + sy \\ tv + ux & tw + uy \end{pmatrix}$, then do the matrix determinant that has been operated by the matrix multiplication so we get $|P^TQ^T| = ruvy + stxw - tsvy - urxw$, this shows that MRA can use transpose definitions, matrix multiplication rules and matrix determinant rules. In step 3, the MRA can do the determinant rules of the transposed matrix to obtain $|P^T| = ru - st$ and $|Q^T| = vy - xw$, then do the matrix multiplication to obtain $|P^T||Q^T| = ruvy + tswx - stvy - urwx$, this shows that the MRA can prove it.

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Based on the explanation above, the results of MRA's work on the first problem can be classified in the K_1 category (valid arguments, proof and completion). This is similar to the study of Stylianides [14], which explains that if students can use definitions and rules in proving, then the proof carried out can be categorized as valid arguments, proofs and proofs that are completed.

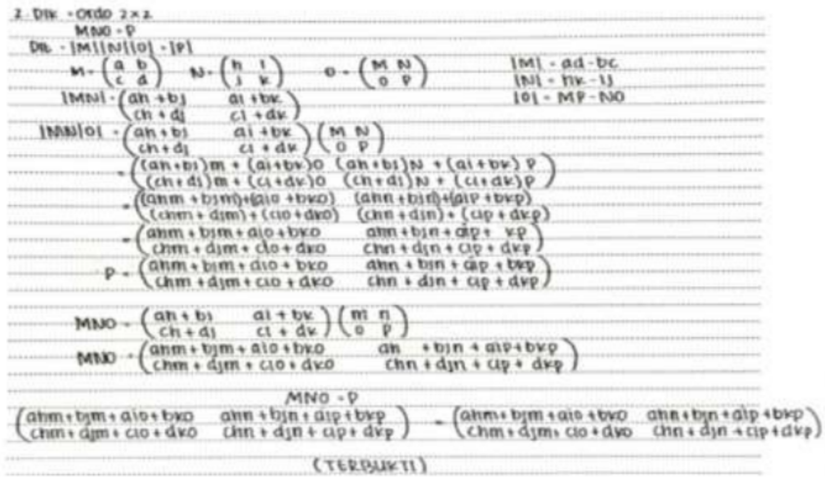

Figure 2. The work of OF in the second problem.

Figure 2 shows that the work done by OF in the second problem seems valid and finished, but the proof carried out by OF is not yet finished and is not proof. In this problem, OF cannot use definitions, characteristics and rules in proving it, this can be seen from each step of the answer, which is as follows: in the answer to the problem, OF does not write the determinant properties of the matrix used, namely if A and B are square matrices with the same order, then $|AB| = |A||B|$ applies. in step 1, OF writes the matrix elements using capital letters $O = \begin{pmatrix} M & N \\ O & P \end{pmatrix}$, OF should use lowercase in determining the elements of the matrix $O = \begin{pmatrix} m & n \\ o & p \end{pmatrix}$, this shows that OF cannot use the matrix definition. In step 2, OF makes a mistake in determining the symbol, it should not use a matrix determinant symbol like $|MN|$ and $|MN|O|$ however, using MN and MNO which states matrix multiplication, this shows that OF cannot use the matrix definition and the determinant definition of the matrix. In the third stage, OF does matrix multiplication but the matrix is used using a matrix whose elements are capital letters, but at the final completion of matrix multiplication, OF has correctly written the matrix elements in matrix multiplication, this shows that OF has not can use a matrix definition. In step 4, OF writes a proven word that explains that the proof is correct and complete but the proof carried out by OF is not finished, OF proves by equalizing the two segments but in that step the results obtained are not valid because $MNO = P$ is not proof, but the MNO matrix multiplication which states the value of P, this shows that OF can not use the definition of matrix determinants, matrix determinant properties and matrix multiplication rules. Based on the explanation above, the work of OF in the second problem can be classified in the K_3 category (valid argument but not proof and not finished). This is similar to the study of Stylianides [15], which explains that if students have not been able to use definitions and rules in proving and proofing that has not been completed, then the proofs carried out can be categorized as valid arguments but not proofs and proofs that are not finished.

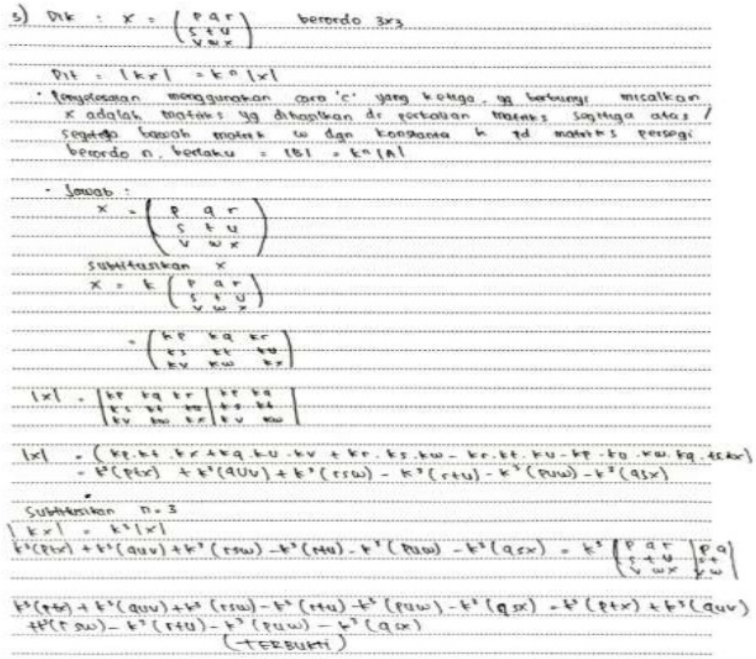

Figure 3. Results of UP work on the third question.

Figure 3 shows that the work done by UP on the third problem seems logical and valid, but the proof is that there is an error in mathematical manipulation. In this problem UP can already use the determinant properties of the matrix, but cannot use the matrix definition and the matrix determinant definition. This can be seen from each step of the answer, which is as follows: in step 1, UP is correct in doing matrix multiplication, but in the order stating that substitute X, UP should write kX indicating that the constant k is multiplied by matrix X, this shows that UP cannot use the matrix definition and definitions of determinants matrix. In step 2, UP is correct in doing the multiplication of matrix determinants, but UP is wrong in writing a command that states the multiplication of matrix determinants, UP writes $|X|$, which should UP write $|kX|$ which indicates that the determinant of the constant k and matrix multiplication X, this shows that UP cannot use the definitions of matrix determinants, and mathematical manipulations. In step 3, UP can do what is known in the problem, namely substitution $n = 3$ because the matrix is used 3×3, then UP is correct to do matrix multiplication so the results obtained are correct and valid. Based on the explanation above, the results of UP is work on the second question can be classified in the K_2 category (valid argument but not proof and complete). This is similar to the study of Stylianides [15], which explains that if students have not been able to use definitions and rules in proving but the proof has been completed, then the proof can be categorized as valid arguments but not proof and proof is complete.

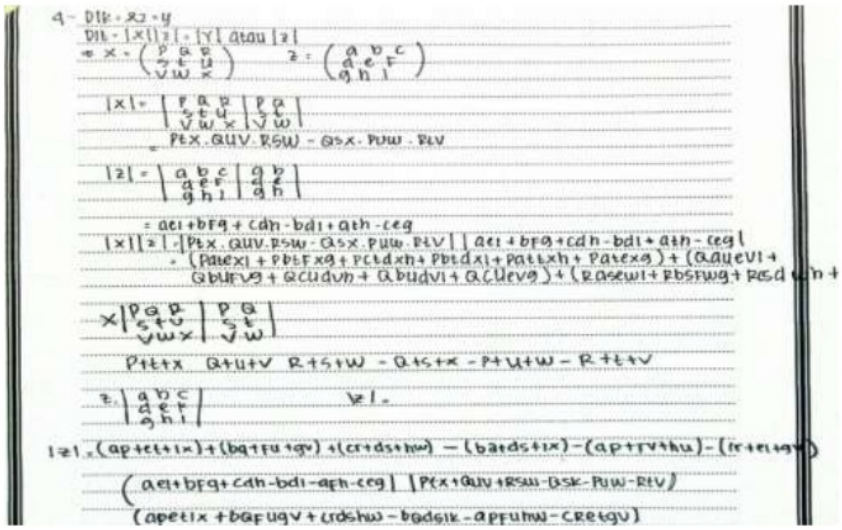

Figure 4. The work of OF in the fourth problem.

Figure 4 shows that the work done by OF has not yet been completed because it has not stated proof. In this problem OF cannot use the definition of matrix determinants, the determinant properties of matrices, and the rules of determinant matrices of 3×3 order. This can be seen from each step of the answer, which is as follows: in step 1, OF writes the matrix elements using capital letters in the first line that is $X = \begin{pmatrix} P & Q & R \\ s & t & u \\ v & w & x \end{pmatrix}$, OF should use lowercase letters in determining the matrix elements that is $X = \begin{pmatrix} p & q & r \\ s & t & u \\ v & w & x \end{pmatrix}$, then OF writes $\left|X\right| = ptx.quv.rsw - qsx.puw.rtv$ and $\left|Z\right| = aei + bfg + cdh - bdi + ath - ceg$, OF should write $\left|X\right| = ptx + quv + rsw - rtv - puw - qst$ and $\left|Z\right| = aei + bfg + cdh - ceg - afh - bdi$, this shows that OF has not been able to use the matrix definition and the 3×3 sequence matrix determinant rule. In step 2, OF performs an algebraic operation obtained from the results of the previous matrix determinant, but the results of the multiplication are incorrect and have not been completed, this shows that OF can not use algebraic operations. In step 3, OF writes X which states a matrix should OF write $\left|X\right|$ which states the determinant of the matrix, the result obtained is $\left|X\right| = p + t + x \;\; q + u + v \;\; r + s + w - q + s + x - p + u + w - r + t + v$, this shows that OF cannot use the matrix determinant rules. In the last stage, OF writes $\left|Z\right|$ but the results obtained are also wrong because the previous stage contained an error in mathematical manipulation, which resulted in invalid and incomplete proof. Based on the explanation above, the work of OF in the fourth problem can be classified in the K_4 category (empirical / invalid and incomplete arguments). This is similar to the study of Stylianides [15], which explains that if students have not been able to use definitions and rules in carrying out proofs and proofs that have not been completed, then the proofs carried out can be categorized as empirical / invalid and incomplete arguments.

4. Conclusion

Construction of student proof is divided into 4 categories as follows: valid, proof and complete (K_1), valid arguments but not proof (K_2), valid and unfinished arguments (K_3), empirical / invalid and incomplete arguments (K_4). Student proof construction in K_1 category was 34.52%, K_2 category was 16.67%, K_3 category was 22.62%, and K_4 category was 26.19%. The construction of student proof on the determinant matrix material is still in the categories K_3 and K_4, in line with previous research which states that the construction of proof in secondary schools is still in the low category.

5. Acknowledgments

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