Code: P-47

FIRST CYCLE ON DESIGNING THE TANGRAM GAME ACTIVITIES AS AN INTRODUCTION TO THE CONCEPT OF AREA CONSERVATION
Game Activity for 3rd grade (9-10 years old)

Shofan Fiangga
Surabaya State University
shofan.rizvia23@gmail.com

Abstract

Most of the teachings in the topic of area measurement tend to give the formula too early to the students. This situation results the condition where the teaching of the teacher is not appropriate with what the students understand in their state results what van Hiele called as mismatch situation. The students could not have the meaning of measuring area except remembering formula. This situation should be administered properly by developing a well design learning activities that could help the students to understand area conservation in area measurement. To develop a good local instructional theory, RME theory suggests that we must apply the parts of each constructed among guided reinvention, didactical phenomenology, and emergent modeling well. Here, the model of tangram by using "price" is developed to reason with the concept of area especially for the big idea of conservation. Then, we expect that it will become the model for reasoning the concepts of area and help them to solve area related problems.

The study has been conducting by framing the learning design on the aforementioned as the guideline to develop the local instructional theory. The ongoing analysis on this study suggests that area conservation for 9-10 year-old pupils is a difficult task. They tend to still use mechanistic reasoning than logical reasoning. By what I mean, they have difficulty in reasoning logically using geometrical picture or drawing. Thus, they could use the price and calculate the total as the reasoning (mechanistic approach). However, they have a better performance in doing area conservation using geometrical reasoning when they deal directly with hands-on activity using geometrical figure (tangram puzzle). This finding is also parallel with the first 2 levels development of van Hiele's level of geometrical reasoning, visualization and Analysis. The ongoing analysis also suggests that the use of scaffolding is important to help the students in measuring an area using the concept of area conservation.

Keywords: *Area Conservation, Tangram, Van Hiele, Realistic Mathematics Education*

INTRODUCTION

Most of the teachings in the topic of area measurement tend to give the formula too early to the students (e.g. Kordaki & Balomenou, 2006; Kospentaris et al., 2011; Papadopoulos, 2010). This results in the students understanding of area as merely procedural, as the use of the formula. Nevertheless, these studies provide a suggestion of the importance of learning the concept of area conservation in the school curriculum. They agreed with the idea of why we need to focus on conservation as the core in understanding area. Accordingly, the understanding of conservation of area is an important prerequisite concept for the understanding of area measurement using area-units and area-formulae. In fact, most of the pupils deal with confusion about rearrangement problems (Kordaki & Balomenou, 2006). They cannot see that if we decompose a shape into another form, then the area remains invariant. The consequence of this is that the students will have difficulty while dealing with the area of more complex 2 dimensional figures. Kordaki & Balomenou (2006) concludes from several studies that, in teaching the topic of area, the integration of teaching conservation area, unit iteration, and area formula is important for children.

The fact that the geometry topic in Indonesia heavily relies on the use of formula can cause a detrimental effect. Learning geometry in Indonesia is defined as learning only the formula and the properties of geometrical concepts without any meaningful knowledge (Fauzan, 2002b). S. M. Amin (personal communication, Augusts 9, 2012) stated that the learning activities teach the students about the properties and the formula of geometrical figures without considering the students' spatial ability and their environment. This idea is in line with what Soedjadi (in Fauzan, 2002a) found which is that the students in elementary and secondary school have difficulties in recognizing and knowing geometrical objects. In fact, according to my experience, the students tend to be interested only in the formula and remembering it. This ensures a non-meaningful learning that lessens their understanding of the concepts of area.

In Indonesia, especially area measurement topic, there is no space provided in the school curriculum to teach the students about area conservation (Departemen Pendidikan Nasional [Depdiknas], 2003). Meanwhile, the teaching in the topic of area requires the integration of teaching conservation area, unit iteration, and area formula which is important for children. Unless, the condition where the teaching of the teacher is not appropriate with what the students understand in their state will occur. This condition is what van Hiele called as mismatch situation (van Hiele, 1985). This forces the students who do not fully understand the area measurement to remember the formula. Thus, in this study we will design a sequence of learning activities which could provide appropriate experience for the students to learn about area conservation.

Area of an object remain invariant in size regardless whatever changes on its shape or positions (Fauzan, 2002). Egocentric thinking makes the reality seen subjectively by the children. When they could see the measurement are just the same from any point of views, it is the process of de-centering thinking because the main point of measurement is that the object's size is remain invariant no matter the positions are changed (Piaget, 1960). Conservation means that the quantitative value of an area remains unaltered while its figure can be qualitatively new (Piaget et al., 1981). Piaget also noted the difficulties of the students in understanding the area conservation. He found that (1) that subtracting and adding of the same parts operationally and (2) moving a portion of shape to make it into another shape are hardly understood by the students, that these will make the area invariant, because its continuity is lost.

Understanding the concept of area conservation is a process of giving meaning to its different representations: for example numerical, visual and symbolic. Students have the opportunity of expressing their own knowledge of the above concept by selecting from among representations those most appropriate to their cognitive development and of constructing a broader and more abstract view of this concept by selecting more than one representation system. Moreover, the meaning that students can give to the concept of the conservation of area is closely related to the tools, the tangram pieces, that they use and to the shapes, irregular polygon, on which they have to study. Therefore, we define understanding of area conservation as activities of (1) using the tools, tangram, to examine the occurrence of area conservation by rearranging the shape formed by the tangram pieces and (2) using the concept of area conservation to quantify the area of irregular polygon

To change the package of mathematics into interesting and fun activities, the use of games or puzzles seems promising as non routine problem for the students. Van Hiele (1999) claimed that playful exploration could occur when using a mosaic puzzle in dealing with certain shapes and their properties, symmetry, parallelism, and area. One of the famous mosaic puzzle games is Tangram, an ancient Chinese puzzle. Its seven geometric movable pieces can be assembled to make more elaborated shapes (Jovanovic et al, 2009). Bohning and Althouse (1997) introduced the word "tangramming" as the activity of assembling a figure using tangram pieces. The tangram experience is essential for the students' development of a positive attitude towards mathematics, especially for recognizing and appreciating geometry in their natural world, because the students deal with the geometrical vocabulary, shape identification and classification, by which the hands on activities allows the discovering of the relationship among those seven pieces. Furthermore, the property of tangram namely that among those seven pieces a relationship can easily be drawn as comparison where we can compare the size of each tangram pieces. This can be used for a promising activity in teaching about area conservation.

To develop a well design learning activities that could help the students to understand area measurement better, RME theory suggest that we must apply the parts of each constructed among guided reinvention, didactical phenomenology, and emergent modeling well (Gravemeijer, 2004). Freudenthal (1991) said that guiding the learners to reinvent the both valuable knowledge and abilities will help them to learn those concepts easier. The use of tangram puzzle will lead the students to deal with the reinvention of area conservation by rearranging and restructuring the pieces of tangram. The didactical phenomenology emerges along the activities where the students experience rich stimulating geometric activities by playing with the tangram. We expect that the students will gain the insight of area conservation in area measurement through the contextual learning activities. Here, the model of tangram by using price is developed to reason with the concept of area especially for the big idea of conservation. Then, we expected that it will become the model for reasoning the concepts of area and help them to solve area-related problems.

The model is used to help in solving the problem. Gravemeijer (2004) define an **emergent model** as a model that cannot distinguished clearly with the concept which is being modeled. Furthermore, Gravemeijer define this process as the development of the *model-of* informal mathematical knowledge into *model-for* formal mathematical reasoning. Here, the model of tangram by using using price is developed to reason with the concept of area especially for the big idea of conservation. Then, we expected that it will become the model for reasoning the concepts of area and help them to solve area-related problems. Gravemejer define the development of the model will pass through 4 levels: **(1) Situational level**, the informal model will arise from the contextual problem given in the beginning of model development. The interpretation of the model really bounded into the context given as the background of the problem. Here, the use of the "Pricing the tangram pieces" context is considered as situational level modeling. **(2)**

Referential model, the model becomes available to be used for other context. In referential level, the interpretation of the model is no longer bounded with the former context. Here, the flexibility usage of tangram pricing model in determines which shape has the same price or not. The *model-of* the activity is formed. **(3) General level**, the mathematical concepts start to appear as a part of the model which is no longer need a context to be interpret. The model involves the mathematical relationship to derive its meaning. Here, the conjectured relationship between the price of a shape and its area start to arise as the generalization of tangram pricing model. The *model-for* mathematical reasoning emerges. **(4) Formal level**, when the reasoning becomes independent from the model means that the formal level is obtained. A new structure of mathematical reality is formed and ready to be used for reasoning. Here, the discussion of area conservation becomes salient as the main goal of the learning sequence.

The teaching and learning of geometry in Indonesia is conducted in a formal way. The tangram with its properties in which we can freely manipulate it in various meaningful ways might break the formal way of learning geometry in Indonesia and make a fun and meaningful activity for children. The tangram pieces as geometrical plane figures can support the students' understanding of plane figure properties as well as their understanding of plane figures' area. The main focus will be how to measure the area of polygon using the idea of area conservation. The use of hands on activities in grasping the concept of area is seems promising that the students can do something real with the notion area conservation. Then, we propose the following research question for this study, how can tangram game activities help the students in understanding the concept of area conservation in area measurement?

In answering that question, we use design research approach as the framework in developing local instructional theory. Because this study concerns about geometrical thinking, we also consider how Van Hiele level of geometry reasoning development could suggest a good learning trajectory. By which and the Realistic Mathematics Education Domain specific theory as the design guidance, the hypothetical learning trajectory was designed.

Implementing the RME theory and van Hiele level of geometric thinking to bridge the students into conceptual understanding rather than memorizing formula results this study which aimed to describe how we can help the students understanding of the concept of area conservation. The main concern of the researcher is to improve education in Indonesia by designing innovation and change on development of PMRI especially on the topic of area measurement. The RME approach that provides many opportunities to develop a meaningful mathematics activity is starting to be adopted in Indonesia. We, then in ongoing process, try to develop local instruction theory which is aimed to develop Pendidikan Matematika Realistik Indonesia (PMRI). To reach this objective, we, then, decide to conduct design research for answering the question. Here, we not only can elicit the way of the methods work well, but also we can understand why they work well. The resulting outcome of this study is a sequence of learning activities which is crucial as the answer of the research question (Bakker & van Eerde, submitted). Accordingly, we are interested in designing a learning activities sequence using tangram in the scope of RME theory regarding van Hiele level of geometrical thinking to help the students to understand the concept of area conservation.

 ISBN : 978-602-17465-1-6

MAIN SECTION

In this section, we will describe briefly about how we design the learning sequence in HLT and the result of retrospective analysis on the first cycle. This cycle was conducted with a small class of 6 students with the researcher as the teacher.

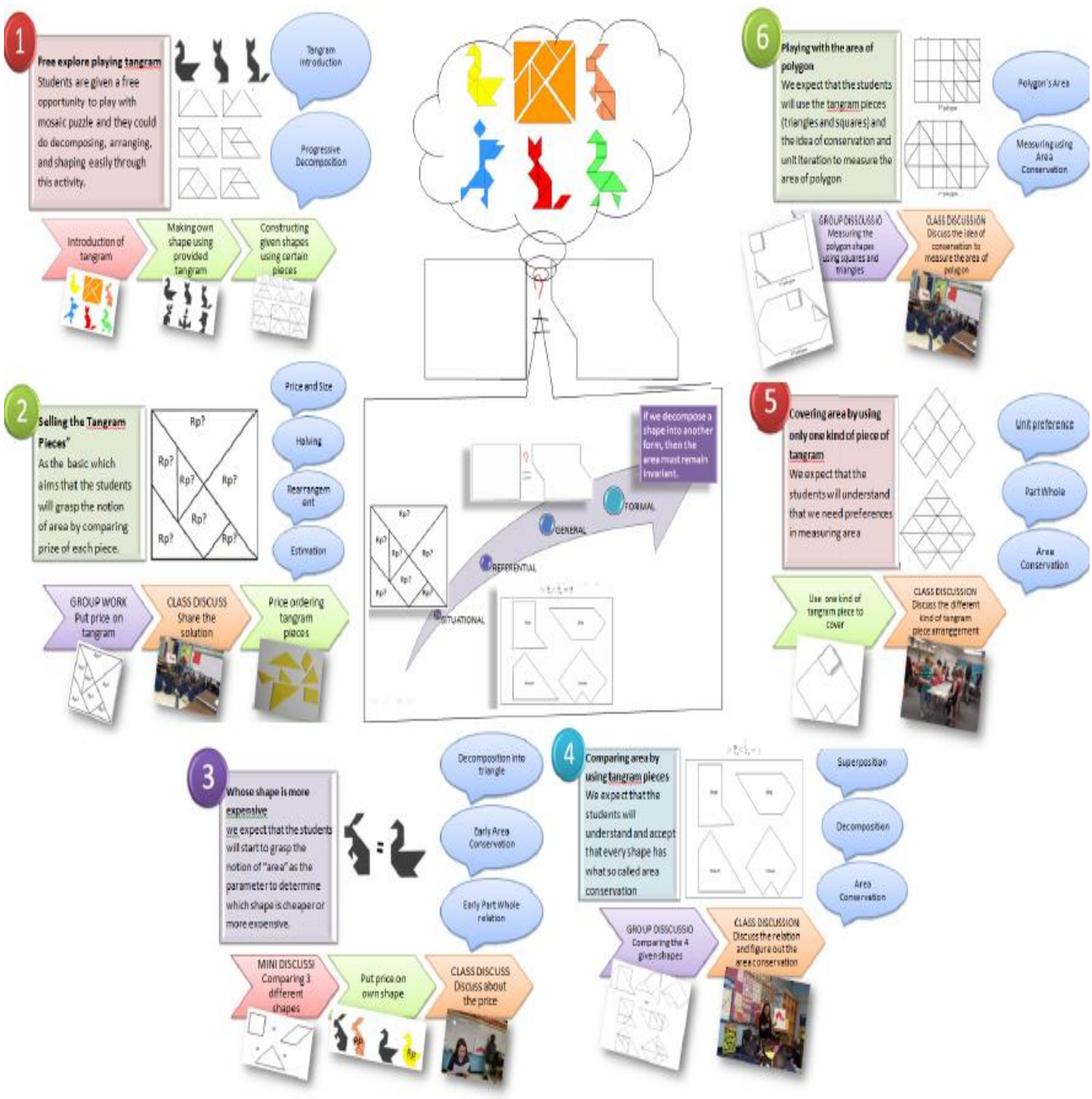
The Learning Design

The first thing that we have to prepare to design such learning activities that can really help the students is literature review. There are a lot of researches on how the students learn about area measurement (e.g. Kordaki and Balomenou, 2006; Kospentaris, Spyrou & Lappas, 2011; Piaget & Inhelder, 2001). Beside, the research on how geometry topic should be taught (e.g. van Hiele, 1985; Freudenthal, 1991). From those literatures, we hope that we can find the information about what is the students' difficulties dealing with the area measurement topic and what might be the solution for them to develop a good learning sequence.

The Hypothetical Learning Trajectory (HLT) Simon and Tzur (2004) describes that HLT consist of 3 parts which are the goals of learning, the description of the learning activities and the hypothesized students thinking. In the end of the description, there is a plan for implementing these 6 activities in around 4-5 meetings with 2x35 miutes for each meeting. It is really dependent with the actual condition later. The guidance for the teacher that I have designed is aimed for 4 meetings learning sequences. However, there is still possibility to compose and decompose the activities but still in the right order.

The understanding of area conservation that has been defined in the beginning of this section is implemented operationally as activities of (1) using the tools, tangram, to examine the occurrence of area conservation by rearranging the shape formed by the tangram pieces and (2) using the concept of area conservation to quantify the area of irregular polygon. (1) The first 4 activities were designed for guiding the students through experiential hands-on activities using tangram to grasp the idea of area conservation. In the first, second, and third activities, we use tangram with price as the model to help the students in reasoning (developing model of). Whereas, the fourth activity was intended to start the model for in case that the students no longer need to use price to reason about area. (2) The last 2 activities were designed for guiding them in measuring a given polygons using the fact of area conservation. Here, the tangram model should have been developed into formal model that they could use it freely as an independent concept in their mind.

Scheme 1. *Playing Tangram as the Introduction of Area Conservation*



The Analysis of the First Cycle

The retrospective analysis for the first cycle that had been conducted by deliberately analyzing the data from video, note, and students' worksheets results the fact that they have difficulty in reasoning using geometrical picture or drawing (symbolic reasoning). The students could not easily compare given shapes. In fact, the use of tangram puzzle and pricing context during the learning sequence was quite helpful. They could use the price and calculate the total as the reasoning (enactive reasoning). However, the learning sequence still needs to be revised especially in measuring activity. The diagram on the following page shows how my learning trajectory from the learning sequences was carried out in the first cycle.

The first activity was designed to give an opportunity for the students to get accustomed with the tangram puzzle. Various activities of composing, arranging, and shaping were experienced by the students easily. However, we expected that they would come out with more solutions as we conjectured. In fact, most of them had a tendency of finding similar solutions with what their friend did.

Lack of creativity in reasoning also occurred in the second activity. This activity was designed to guide the students to build the model of tangram with price from the context given to reason with. The context was about finding how to sell tangram in pieces if the price of a set of tangram was Rp. 40.000. The conjectured solutions from students were not taken place as expected. Using simple geometrical reasoning like comparing the pieces which is the area is twice of another could not easily be done by the students. They preferred to use numerical reasoning which made them to guess and check the price of each piece until the total amount of them was Rp. 40.000. Guidance from the teacher was quite intensive here. Yet, some of them still could not understand correctly about the idea of halving and doubling the shapes which was intended to guide them in entering the idea of area conservation.

In the third activity, the students dealt with various shapes constructed by each of them and discussed which shape is cheaper or more expensive. We expected, by experiencing this comparing activity, that they could notice that the concept of area determine how expensive or how cheap the price is. Even though not all conjectured solutions were taken place, the students successfully understood that there is a relation between price and size. Moreover, they also considered that bigger the shape. The idea of shapes whose pieces are the same but it is not necessarily mean that the shapes have the same size was not discussed thoroughly.

The fourth activity was intended to guide the students in using area conservation to compare the given area without heavily rely on the use of pricing model. This phase was designed to guide from 'model of' to 'model for'. The fact that the students could not solve the problem unless they use the price as reasoning was undeniable. This suggests that more mini lesson should be added properly as the scaffolding activities. This addition successfully helped the students to do area conservation on the given objects in the next activity. They were be able to compare the given shapes without using price as reasoning.

In the fifth activity, the students were invited to use the tangram pieces as measurement unit. We expected them to be able in understanding the fact that, in conserving area, the use of different construction or measurement will result the same area. Choosing the best piece that could be used to measure a given shape was done correctly by the students. They also did not have difficulty in comparing which measurement was correct because either using square piece or small triangle piece was the same measurement.

The last activity was designed to guide the students to use their knowledge in area conservation to measure a given area. The shapes of polygon, actually, could be decomposed easily by the students after experiencing the series of conserving activities. Yet, they had difficulty in measuring the area using the tangram piece. Conserving one shape into another shape such that they could measure easily was not occurred. They deliberately covered the area of the given shapes by using the tangram pieces. Small mini lesson should be administered for the students to bridge the knowledge slowly. Asking them to measure directly without providing enough scaffolding was the main reason why they unable to perform area conservation.

The use of tangram as media to do area conservation was quite helpful for the students. The price model also provided an opportunity for them to understand that the area remain the same if we compose and decompose the shape into another shape by relating the price with size of the shape. The second, the third, and the fourth activities show that the students could do freely reasoning using price.

In the other side, the students had a tendency to use price as reasoning than using geometrical reasoning. In the second and the fourth activity, we can see that price model had become more salient than reasoning in geometrical area conservation. The activity to support the students' development of knowledge seems not enough. The scaffolding activities should be provided more extensively.

CONCLUSION

In summary, the design could help the students in developing an idea of area conservation. Nevertheless, teacher guidance is really needed to ensure that the students really understand the concept. However, The design could not help much the students to use the concept of area conservation in area measurement. In my design, at least, they were busy in doing covering activities and not conserving the area.

The ongoing analysis on this study suggests that area conservation for 9-10 year-old pupils is a difficult task. The use of price context is really helpful to help the student in dealing with the notion of area conservation. However, unfortunately, they tend to still use mechanical approach to solve the problem than geometrical approach. By what I mean, they have difficulty in reasoning using geometrical picture or drawing. Nevertheless, they could use the price and calculate the total as the reasoning. This implies that the students were accustomed with mechanistic teaching approach. The conclusive standard procedure is heavily used by the students which result the fact that they could not solve the problem independently (Nelissen & Tomic, 1993). The context was also not considered as helpful idea in proceeding to solve the problem.

This finding is also parallel with the first 2 levels development of van Hiele's level of geometrical reasoning. The dealt with visualization level quite extensively. They did trial and error regardless the fact of geometrical fact of halving and combining shapes that can be used to help them in reasoning about the problem. However, they have a better performance in doing area conservation using geometrical reasoning when they deal directly with hands-on activity using geometrical figure (tangram puzzle).

The tangram puzzle serves as hands on activity on conserving area to help the students the real feeling of area conservation. The use of tangram puzzle appends an experiential knowledge for them to enter the analysis level, second level of van Hiele's level.

The ongoing analysis also suggests that the use of scaffolding is important to help the students in measuring an area using the concept of area conservation. Asking a problem followed by supporting small activities as the scaffolding would help the students to understand the concept easily regarding the fact the students are accustomed with mechanistic teaching approach. This finding supports what Gagne believes (in Nelissen & Tomic, 1993) that, in mechanistic class; the students should experience a series of smaller progressive tasks before dealing with the complex one.

REFFERENCES

Bakker, A., & van Eerde, D. (submitted,2012). *An introduction to design-based research for Master and PhD students*. FISME UU

Bohning, Gerry and Althouse, Jody Kosack. (1997). Using Tangrams to Teach Geometry to Young Children. *Early childhood education journal*, 24. 239-242

Depdiknas. (2003). *Standar Kompetensi Mata Pelajaran Matematika Sekolah Dasar dan Madrasah Ibtidaiyah*. Jakarta: Balitbang

Fauzan, Ahmad. (2002a). APPLYING *REALISTIC MATHEMATICS EDUCATION (RME) IN TEACHING GEOMETRY IN INDONESIAN PRIMARY SCHOOLS*. Thesis University of Twente. ISBN 90 365 18 43 1

Fauzan, Ahmad. (2002b). TEACHING MATHEMATICS IN INDONESIAN PRIMARY SCHOOLS USING REALISTIC MATHEMATICS EDUCATION (RME)-APPROACH. *2nd INTERNATIONAL CONFERENCE ON THE TEACHING OF MATHEMATICS* . Hersonissos, Crete , Greece, July 1-6, 2002

Freudenthal, H. (1991*). Revisiting mathematics education*: China lectures. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Gravemeijer, Koeno. 2004. *Creating Opportunities for Students to reinvent Mathematics*. Regular Lecture in ICME X 2004

Gravemeijer, K., & Cobb, P. (2006) Design research from the learning design perspective. In Van den Akker, J., Gravemerijer, K., McKenney, S., & Nieveen, N (Eds.), *Educational Design Research*. London: Routledge.

Gravemeijer, Koeno and Terwel, J.. 2000. Hans Freudenthal: a mathematician on didactics and curriculum theory. *Journal of Curriculum Study*. 32 (6): 777-796

Jovanovic, Jelena, Dragan Gašević,& Vladan Devedžić. (2009). TANGRAM for Personalized Learning Using the Semantic Web Technologies*. Journal of emerging technologies in web intelligence*, 1. 6

Kemdiknas. (2006). *Lampiran-Permendiknas-Nomor-22-Tahun-2006-Tentang-Standar-Isi-Lampiran-SD-MI*. Jakarta: Kemdiknas

http://www.scribd.com/doc/48620155/Lampiran-Permendiknas-Nomor-22-Tahun-2006-Tentang-Standar-Isi-Lampiran-SD-MI

Kordaki, Maria and Balomenou, Athanasia. (2006). Challenging Students to View the Concept of Area in Triangles in a Broad Context: Exploiting the Features of Cabri-Ii. *International Journal of Computers for Mathematical Learning*. 11:99–135

Kospentaris, George, Panagiotis Spyrou & Dionyssios Lappas. (2011). *Exploring Students' Strategies in Area Conservation Geometrical Tasks*. Springer Science Business Media B.V.

Nelissen, Jo, and Tomic, Welko. (1993). Learning and Thought Processes in Realistic Mathematics Instruction. *Curriculum and Teaching*. 8 (1): 19-37

Papadopoulos, Ioannis. (2008). Complex and Non-Regular Shapes: Their Evolution in Greek Textbooks (1749–1971). *Science & Education*. 17: 115-129

Papadopoulos, Ioannis. (2010). "Reinventing" Techniques for the Estimation of the Area of Irregular Plane Figures: From the Eighteenth Century to the Modern Classroom. *International Journal of Science and Mathematics Education*. 8: 869-890

Piaget, J., Inhelder, B., Szeminska, A. (1960). *The child's conception of Geometry.* London: Routledge and Kegan Paul

Simon, M. A. and Tzur, Ron. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking and Learning*. 6 (2): 91-104

Swadener, Marc and R. Soedjadi. (1988). Values, mathematics education, and the task of developing pupils' personalities: an indonesian perspective. *Educational Studied in Mathematics*. 19 (2): 193-208

Van Hiele, Pierre. (1985). *STRUCTUR AND INSIGHT*. A Theory of Mathematics Education. Orlando: Academic Press

Van Hiele, Pierre. (1999). Developing Geometric Thinking through Activities that Begin with Play. *NCTM*. 16. 310

Widjaja, Yenni B and Heck, Andre. (2003). How a Realistic Mathematics Education Approach and Microcomputer-Based Laboratory Worked in Lessons on Graphing at an Indonesian Junior High School .*Journal of Science and Mathematics Education in Southeast Asia*. 26 (2), 1-51

Zulkardi. (2002). *Developing a 'rich' learning environment on Realistic Mathematics Education (RME) for student teachers in Indonesia*. Accessed on November 6th 2012.

http://projects.gw.utwente.nl/cascade/imei/publication/PaperZulkardi.pdf

This paper can be cited as Zulkardi(2013). Proceeding The First South East Asia Design/Development Research (SEA-DR) International Conference, Sriwijaya University, Palembang, April 22nd-23rd.