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DESIGN RESEARCH: LOOKING INTO THE HEART OF MATHEMATICS EDUCATION

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Abstract

Design research can be characterized as a research approach in which the design of educational materials is interwoven with the development of theory. Design research aims at educational innovation and it has a cyclic character: development and prediction, teaching experiments and reflection and revision form an iterative process.
In design hypotheses, as in other research approaches, hypotheses are formulated preliminary to the data collection. However, in design research these hypotheses are continually tested and revised during the teaching experiment.
Design research cycles typically consist of three phases: preparation and design phase, teaching experiment and retrospective analysis. The results of the retrospective analysis normally lead to new designs and a follow-up cycle. The hypothetical learning trajectory is an essential instrument during each phase of a design study, though plays a different role in each phase. Design research is not an easy approach but valuable as it offers a unique opportunity for learning to understand students' thinking and learning.

Keywords: *innovation, mathematics education, cyclic character, role of hypotheses, hypothetical learning trajectory*.

MAIN FEATURES OF DESIGN RESEARCH

Intertwinement of design and theory

The main aim of design research is to develop instruction theories about the learning of students and to develop educational materials that are designed to support that learning (Gravemeijer & Cobb, 2006). Design research results both in useful products (educational materials) and related scientific insights into how these products can be used in education (McKenney & Reeves, 2012; Van den Akker et al., 2006). Therefore it has the potential to bridge the gap between educational practice and theory.
The design of educational materials is a crucial part of the research and aims at the investigation of how the design works. Thus, the intertwining of design and theory development is an essential feature of this approach.
These theories are not general theories about learning but domain specific theories. In the case of design research in mathematics education the theoretical result would be a contribution to domain specific theory on learning and teaching mathematics. This means theory on certain mathematical domains such as proportions, measurement or geometry. In design studies we mostly focus on specific 'local' aspects within such a domain: for example in the domain of geometry on the measurement of length or area, or even on more specific mathematical topics such as the development of a unit of measurement. Therefore these specific theories can be

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called 'local' instruction theories.

Design research differs from other approaches. A comparison between design research and other research approaches can be found in Bakker & Van Eerde (submitted).

Aiming at innovation

Before starting a study researchers chose a certain research approach depending on the function of the study they plan to do. So the aims of a study determine the choice of a research approach. Plomp & Nieveen (2007) distinguish the following research functions:

- *to describe*. Approaches such as surveys, correlational studies, and case studies usually have descriptive aims. An example of a research question: What conceptions on area do 5th-grade students have?
- *to compare.* Experiments often have a comparative aim. An example of a research question: Does textbook series A lead to better test scores than textbook series B?
- *to evaluate.* Evaluative studies investigate the results of certain teaching practices. An example of a research question: How well do students develop an understanding of percentages in this textbook?
- *to explain or predict*. An example of a research question: Why do so few students choose to study mathematics education?
- *to advise.* An example of a research question: How can primary school students be supported to learn to understand decimals?

Design research mainly has an advisory aim, namely to give theoretical insights into how innovative ways of teaching and learning can be promoted. Design researchers intervene in current educational practices with the purpose to improve education. To understand why students' current learning is unsatisfactory researchers design and enact new materials such as learning activities. Then they investigate to understand how these new activities lead to learning.

Since in design research something new has to be created it sometimes is characterized as a form of (didactical) engineering (Freudenthal, 1978). Gravemeijer (1994) used the French term 'bricolage' for this creative process, i.e. what a handyman does.

In design research changing and understanding a situation are intertwined in line with the following adage that is also common in sociocultural traditions: 'If you want to understand something you have to change it, and if you want to change something you have to understand it' (Bakker, 2004a, p. 37).

Although design research projects have an overall aim for innovation, several phases of a study can have different aims. So apart from having an overall advisory aim a design study can also include phases with a descriptive, comparative, or evaluative function.

Role of hypotheses

Some research approaches such as in experiments with randomized control trials aim at testing hypotheses. These hypotheses are well-defined before data collection starts and not changed anymore during the rest of the study. In design research the role of hypotheses is quite different. Before they start an experiment design researchers do a thought experiment in which they try to imagine and predict how students will

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respond to a particular problem or activity, based on their knowledge of the topic of the study (Freudenthal, 1991). These thought experiment result in the formulation of hypotheses.
These hypothesis about students learning are tested continually during the experiment. If the observed learning is different from the expected learning this implies that the learning activities have to be changed during or after a lesson and that the hypotheses must be adapted to the new situation.
The role of hypotheses is discussed in more detail in the section on the first phase of design research when the hypothetical learning trajectory is discussed.

Cyclic character

Another crucial feature of design research is its cyclic character. We distinguish macro cycles and micro cycles. A macro cycles consists of three phases: design, teaching experiment and retrospective analysis. And the retrospective analysis feeds forward to a new macro cycle starting with a design phase (Gravemeijer & Cobb, 2006).
The cyclic character can also relate to micro cycles. Such a cycle only refers to a set of problems and activities during one lesson. Figure 1 represents several micro cycles.

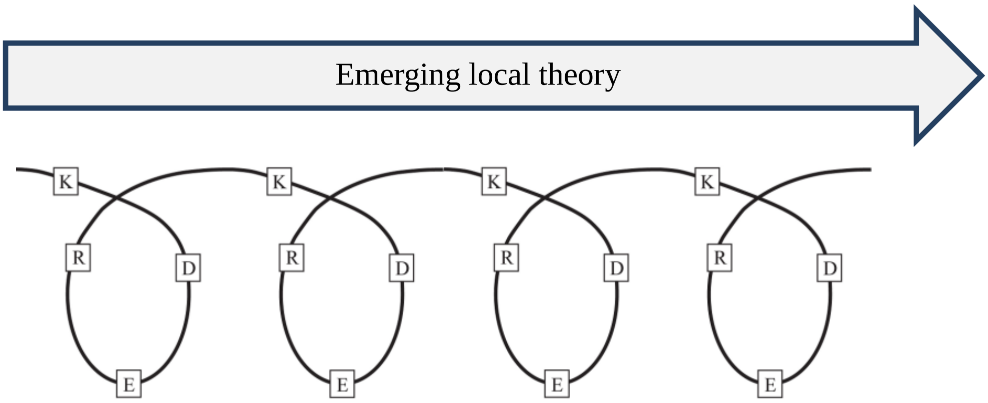

Fig. 1 Cyclic process of knowledge, design, experiment, reflection and (new) knowledge.

Based on the current knowledge (K) the researcher conducts thought experiments and designs problems and activities (D), conducts an experiment (E) with these problems and activities and reflects (R) on the experiment. This results in new knowledge (K) .
In a macro cycle the new knowledge results in a redesign of the HLT at the end of a lesson series. This can be the start of a new study, a new macro cycle.
In a micro cycle the new knowledge resulting after activities in one lesson might cause the need to make small changes in the design of a follow-up lesson. In this case the redesign takes place in between lessons. The need for redesign could be unforeseen events in the classroom for example when activities are too difficult of too easy for the students.
Both during macro- and micro cycles the development of a local instruction theory takes place.

The role of the teacher

The role of the teacher in design research is often confined to conducting the

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designed activities. However, not only the students but also the teacher learns from participating in a design study. The innovative character of the design always requires that the teacher discusses new problems with the students often employing new ways of teaching. One could also chose to involve the teacher as co-designer of the activities, in this way expanding the role of the teacher and at the same time enlarge his of her ownership of the design.

Moreover, one could use design research as a way to promote the teacher's learning and at the same time study the teachers' learning process. This special type of design research that examines both the students' and the teachers' learning processes is called dual design research (Gravemeijer en Van Eerde, 2009; Smit en Van Eerde, 2011).

DIFFERENT NAMES

In Western Europe during the 1980s and the 1990s discussions took place on the relation between research and design in mathematics education. This evolved into a new approach integrating design and research. During its relatively short history the approach has become known under the following names:

- Developmental research (Freudenthal, 1988)
- Design experiments (Brown, 1992; Cobb et al., 2003)
- Design-based research (Educational Researcher, 2003)
- Educational design research (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006).
- Design research

The term *developmental research* is a translation of the Dutch *ontwikkelingsonderzoek*, which Freudenthal introduced at Utrecht University in the 1970s (Freudenthal, 1988). He formulated the approach as follows: "Developmental research means: experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience". (Freudenthal 1991, p. 161).

The core idea was that development of learning environments and the development of theory are intertwined. This idea was elaborated in the following decennia when different groups used different names. Although these approaches may differ in focus, such as a learning perspective (Gravemeijer & Cobb, 2006) or a curriculum perspective (McKenney & Reeves, 2012), but they all refer to educational research and bear the initial idea of integrating design and research.

In this paper we chose a learning perspective on mathematics education and for practical reasons we use the general name design research.

PHASES IN DESIGN RESEARCH

Design research cycles typically consist of the following phases: preparation and design phase, teaching experiment and retrospective analysis. The results of such a retrospective analysis normally lead to new designs and a follow-up cycle.

A Hypothetical Learning Trajectory (HLT) is an essential instrument during these phases, having a different function in each of them.We will elaborate this in our description of each phase. We now describe each phase illustrated with some examples.

The Hypothetical Learning Trajectory

An hypothetical learning trajectory is made up of:

- Starting points
- Learning goals that define the direction
- Mathematical problems and activities
- Hypotheses on students' thinking and understanding

The HLT has different functions in the three stages of a design study. It is designed in the preparation phase and during the teaching experiment it informs researcher in carrying out the experiment: what to focus on in teaching, interviewing, and observing. During the retrospective analysis, the HLT functions as a guideline in determining what the researcher should focus on in the analysis.

The HLT is like a global plan, the direction stays the same but during the experiment the plan needs to be adapted to unforeseen circumstances. Thus, the HLT is the backbone of a design study.

Phase 1: Preparation and design

In this phase the following step are taken: a literature review, the formulation of research aim and the general research question, and the development of a Hypothetical Learning Trajectory

Literature review and research question

Once a topic is chosen, the first step is to carry out a literature review in order to find what is the relevant knowledge about the topic. Important questions to answer are: what did relevant studies on the topic reveal, how is the topic traditionally taught, what are common problems students have, what kind of innovations have been made to improve the learning?

As an illustration we give *an example on area measurement.* The following dialogue reveals a students' misunderstanding of the concept of area:

Teacher: What is the area of this island?
Student: It has no area.
Teacher: Why do you think that?
Student: It has no length and no width.

In area measurement common problems of students are that they:

- can globally compare the area of objects; but what do they know about a unit of measurement for area?
- know the formula 'area = length x width'; but do they understand the meaning of area?
- do not recognize that they have to multiply the units of measurement in a problem on area,
- often do not understand how *area* can be computed by measuring *length.*

The literature review results in defining the knowledge gap and in formulating a research aim and general research question. Learning and teaching *processes* are central to design research. Therefore research questions often start with: *How* …..? An example of a research question on area would be: 'How can we support students in learning to use a unit of measurement to measure area?'

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The development of a Hypothetical Learning Trajectory (HLT)

The *starting points* are to be determined to connect the new learning activities to students' current knowledge and understanding. For this reason students should be assessed before the classroom experiment starts for example by conducting a written assessment. This could function as a pre-test that is repeated as post-test after the experiment creating the opportunity to compare both tests and getting at least some data of the changes in understanding of all students.

Mathematical learning goals are to be defined to give direction to the design of new activities and redesign of existing ones.
For the design of *mathematical problems* and activities a variety of sources can be used. The specific tenets (Treffers, 1987) and design heuristics (Gravemeijer, 1994) for realistic mathematics education (RME) offer a general base for the design of mathematical problems. Other sources can be innovative mathematical problems and lesson series that have been developed in former studies, textbooks, and websites. The process of developing problems takes several rounds of design and redesign. An idea leads to the creation of a problem that is critically analysed and redesigned several times. Criteria for accepting a problem would be whether the problem is challenging students' thinking and reasoning, whether the context is meaningful and if models could support students' learning.
Not only the problems should be described clearly but also how one problem and the intended learning is connected to the next problem and intended learning process.

The formulation of *hypotheses about students' thinking and learning and the role of the teacher* during the learning activities is one of the most difficult element of designing a HLT.
These hypotheses or assumptions of what might happen during the classroom activities are formulated before the start of the classroom experiment as a result of thought experiments. These hypotheses are twofold. They relate to what students might think and understand or might not understand. Moreover they include suggestions on how the teacher could promote and guide the students learning, including suggestions for questions to ask and for topics to discuss.
To give a clear view on the intended learning trajectory the relations between the problems and assumed learning processes should be made explicit.

For the development of an HLT different rounds of design and redesign are needed, discussions in the research team and with colleagues can support to create a sound HLT before the experiment starts.

As an illustration of a problem we give *an example on area measurement*. The teacher give the following problem for students to discuss in small groups.
The figures below are two bars of chocolate.

a. If the price of these chocolate bars are same, which chocolate do you want to buy? Why do you choose that chocolate?

b. What is your strategy in choosing the chocolate you want to buy? Explain your answer. Use the paper given to help you.

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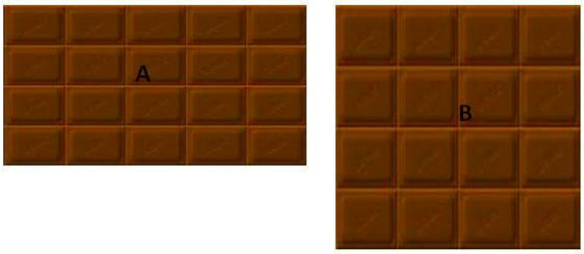

Hypotheses about students' thinking and learning, and the role of the teacher.

- Some students will cut and paste the chocolate bars and find the biggest bar.
- Other students will at first also chose the square bar. But if they count the squares of the chocolate bar as a unit of measurement they might get confused, because the smaller bar has more units.
- The teacher should promote a discussion and have students explain their arguments for their choice. Then the teacher will discuss the unit of measurement.

Once the design is ready a guide for the teacher should be developed with practical information on how to conduct the lessons.

Phase 2: Teaching experiment

Before the experiment starts the researchers decides what kind of data will be collected and how these will be collected in order to in the end answer the research questions.

In design studies many different data can be collected:

- Lesson observations students and teacher:
  - Video registrations of whole class/group discussions
  - Participating observations (notes)
- Interviews
  - Students: individuals/group work (critical learning moments)
  - Teachers (pre-post experiment, between lessons).
- Written work of students and teacher
  - Students' notations, calculations, explanations
  - Teachers diaries during the experiment

Combining different methods for data collection enables data triangulation.

For master students who conduct a design study and have very limited time for data analysis it is wise not to collect too many data. Moreover, they could collect data of the whole group but concentrate on the data collection of a focus group of 3-5 students. This enables them to at least analyse the data of this focus group and only use data of the whole group as source for additional interpretation of what is found in the focus group and as long as the time allows further analyses.

Before the teaching experiment the researcher discusses the lessons with the teacher as described in the teacher guide. During the experiment the researcher might make necessary adaptations in the HLT based on the lesson observations.

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Phase 3: Retrospective analysis

Once the data are collected they have to be prepared for the retrospective analysis, for example transcribing video and audio registrations.

The HLT functions as a guide in the retrospective analysis. During the analysis the hypothesized learning, the assumptions about students' learning are compared with the actual learning as observed during the lessons.

Such an analysis in which researchers go back and forth between the evolving HLT and empirical observations forms the basis for developing an instruction theory. After the retrospective analysis, the HLT can be reformulated, often more drastically than during the teaching experiment, and the new HLT can guide a subsequent design phase and start for a follow-up cycle.

Generally speaking researchers can chose between two different approaches for making a retrospective analysis in design research.

The first approach is task oriented, the analysis is done on the level of the activities. First the video's of a lessons are watched with the research questions and HLT as guidelines. Notes are made of interesting fragments and observations.

For the analysis a data analysis matrix as described in Dierdorp et al. (2011) could be useful.

Table 1 Data analysis matrix for comparing HLT and actual learning trajectory (ALT)

Hypothetical Learning Trajectory			Actual Learning Trajectory	
No. of problem	Formulation of problem	Conjectures of how students would respond	Transcript excerpt	Clarification

The left part of the table summarizes the problems and the assumed learning process. In the right part is for excerpts from relevant transcripts and clarifying notes from the researcher. One could also include a quantitative impression of how well the assumed and the observed learning matched. This could for example be done by -, 0, +.

This task-oriented analysis does not include the role of the teacher. However, this role of the teacher should be included to explain inconsistencies, differences between the hypothesized and observed learning or when the teaching was fundamentally different from what the researcher had in mind.

A second method of analysis is based on the 'constant comparative method' (Glaser & Strauss, 1967; Strauss & Corbin, 1998) and the method of longitudinal analyses from Cobb and Whitenack (1996).

First the video's of all lessons are watched with the research questions and HLT as guidelines. Notes are made of interesting fragments and observations. After that the transcripts of all lessons are read and assumptions about students learning are generated and tested at other fragments of the lessons. This means both looking for observations that can confirm the hypotheses and instances that do not confirm them. This process of creating and testing assumptions is an iterative process. Crucial observations can be discussed with colleagues to test whether they agreed upon our interpretation or perhaps could think of alternative interpretations (peer

examination).
An elaboration of a retrospective analysis is not possible within the scope of this paper. Therefore we refer to Bakker en Van Eerde (submitted) that includes an example of a retrospective analysis from a design study on early statistics. Furthermore there is a website with the master theses of the IMPoMe students that include also retrospective analyses (see www.fisme.science.uu.nl/en/impome).

VALIDITY AND RELIABILITY

Of course in design research the common scientific criteria of validity and reliability have to be met. In design research as in all qualitative research approaches the meanings of validity and reliability are slightly different than in quantitative research. In brief we could say the following. Internal validity refers to the quality of the data collection and the soundness of the reasoning that has led to the conclusions (also labelled as 'credibility'). External validity is mostly interpreted as the generalisability of the results. Internal reliability refers to the reliability within a research project. External reliability usually denotes replicability in this case interpreted as virtual replicability. This implies that a study must be documented in such a way that it is clear how the research has been carried out and how conclusions have been drawn from the data.
For a more elaborated description of the internal and external validity and reliability we refer to Bakker en Van Eerde (submitted).

WHY WOULD PROSPECTIVE TEACHERS DO DESIGN RESEARCH?

As design research is not an easy research approach one may wonder why it would be sensible for master students as prospective teachers to conduct such a study. I argue that design research offers teachers a unique opportunity on learning to understand students' thinking and learning because they learn to:

- Design problems that promote students' thinking and learning
- Make assumptions about students' learning
- Make conjectures about their own role as teacher to promote and guide students' learning
- Ask open questions and follow-up questions
- Observe closely what students do and say
- Analyse and interpret what students do and say
- Re-design problems and assumptions about students' learning based on the data

Design research in mathematics education aims at understanding the process of learning and teaching mathematics. Thus, it touches the heart of mathematics education.

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