Code: P-10

DEVELOPING STUDENTS' SPATIAL VISUALIZATION ON VOLUME MEASUREMENT

Dwi Sulistya Kusumaningrum[1], Meiliasari[2]
Universitas Negeri Jakarta[1], Universitas Negeri Jakarta[2]
dwisulistya1989@gmail.com[1], melisari79@gmail.com[2]

Abstract

The learning of volume measurement is closely related to the learning of geometry. Being able to visualize the shape of the object measured is necessary for students to do the volume measurement. This paper discusses students' spatial visualization development within the learning sequences on volume measurement of cubes. The research used design research methodology which consists of three phases; thought experiment, instructional experiment, and retrospective analysis. The learning activities were designed in the thought experiment phase and were underpinned by realistic mathematics education theory. The instructional experiment was conducted in a grade 8 class of a Junior High School in Karawang West Java. The retrospective analysis showed that counting unstructured group of unit cubes stimulated students to organize the cubes in a structured configuration to enable them count easily. Drawing and building 3D objects by using unit cubes has enabled students to develop their spatial visualization, that is knowing how the objects look like when seen from different angles. Finally, students were able to group the unit cubes in layers or columns and use multiplicative strategy and construct an understanding of the concept of volume of a cube.

Keyword : *spatial visualization, volume, measurement, cube, PMRI*

INTRODUCTION

One of the problems in mathematics classroom in Indonesia is students are directly given the abstract concept of mathematics without enough exploration using concrete materials. Boswinkel and Moerlands (2003) as cited in Amin et al. (2010) named this phenomena as the metaphor of the iceberg. The abstract concept is the tip of the iceberg which is only a small part of the whole iceberg. The bigger part underneath the water is often unseen, that is the mathematical process of investigating real problem, modeling and making generalization for formal mathematics. The effort of improving mathematics classroom practice in Indonesia has been one of the focuses of a Pendidikan Matematika Realistik Indonesia (PMRI), the Indonesian adaptation of the Dutch Realistic Mathematics Education (RME).

In learning about volume, our initial finding indicated the iceberg metaphor happened in the classroom. Students who are only given the abstract concept of volume, have difficulties in understanding the concept of volume of concrete object. The formula of volume of a rectangular prism becomes meaningless for students when they are given a drawing of a 3D object made of unit cubes and asked to find how many unit cubes are there. When doing this task, students used counting one by one and failed to see the hidden unit cubes. This problem was identified by Ben-Haim et al. (1985) that volume measurement tasks with unit cubes are related to spatial visualization.

In the effort of improving mathematics classroom practice on volume measurement, we designed some instructional activities for grade 8 students in Junior High School underpinned by RME theory. We pose a question: how student develop their spatial visualization in volume measurement of rectangular solid? This paper discusses the sequence of the learning and how each activity support students' spatial visualization in the development of constructing the concept of volume.

THEORITICAL BACKGROUND

Spatial Visualization in Volume Measurement

Volume is often understood as the capacity of a 3D object, more formally, in mathematics volume is the number of units can be filled in a 3D object (Bennett, Nelson, 2001). In terms of units, Van de Wall (2007) explained that there are two types of units that can be used in volume measurement, solid units and small container units. A wooden cube can be used as a solid unit that the volume of the object observed is the number of wooden cube filling out the space inside the object. The smaller container is usually used for uncountable units such as liquid or small particles.

Understanding the concept of volume of a 3D object requires one's ability to visualize the object, i.e. seeing the walls that bound the space inside the object. Visualization is often a mental activity that requires one to imagine the movement of an object. Clements & Battista (1992) said that visualization is one component of spatial ability. Moreover, they explain that spatial ability has two components: (1) spatial orientation is understanding and operating on the relationship between the position of object in space with respect to one's own position, and (2) spatial visualization is comprehension and performance of imagined movement of object in two and three dimensional space.

Spatial visualization plays an important role in understanding the concept of volume. Spatial visualization support students' construction of the concept of volume. Activity such as building a 3D objects and counting the number of unit cubes in the object support students to use grouping such as counting the unit cubes by layers, rows or columns, skip counting and multiplication (Revina et al., 2011)

Pendidikan Matematika Realistik Indonesia (PMRI)

Pendidikan Matematika Realistic Indonesia (PMRI) is the Indonesian adaptation of the Dutch Realistic Mathematics Education (RME). PMRI was first started in Indonesia as a pilot project of join collaboration between Indonesian and Dutch government in 2001 (Sembiring, Hoogland, & Dolk, 2010). The project was conducted in 4 universities and 12 elementary schools in Java. Since then, more universities and school has participated throughout the whole country.

Treffers (1987) described five tenets of realistic mathematics education which are: (1) the use of contexts, (2) the emergence of models, (3) students' own constructions and productions, (4) interactive instruction, and (5) the intertwining of learning strands. The first tenet, using contextual problems might stimulate students to think of ways to solve the problems. In RME, the point of departure is that context problems can function as anchoring points for the reinvention of mathematics by students themselves (Gravemeijer & Doorman, 1999). A rich and meaningful context is essential to begin the classroom activity.

A good context should allow students to mathematize, for example by using representations and models (English, 2006). Gravemeijer (1994) explained that the contextual situation serves as the starting point of students' to conceptualize a more formal mathematics by modeling the problems. In an educational perspective, modeling requires the students not just to produce or to use models but also to judge the adequacy of those models (Doorman, 2005). In a classroom activity setting, Gravemeijer (1994) explained how a model can serve as *model of* a situation and transforms into *model for* a more formal mathematical reasoning.

The third tenet highlights students' own constructions and productions where Treffers (1987) believed that students' construction stresses on the *action* of the students while students' production stresses on the *reflection* of teacher's didactical activity. The relationship between students' own constructions and productions therefore is not dissociable from the teacher's role. In a realistic mathematics classroom, students construct their own knowledge, guided by the teacher (Treffers, 1987). Freudethal (1991) used the term *guided reinvention* to name such students' construction. In his view, students should reinvent mathematics themselves by repeating the learning process of how the mathematics was invented. Students should experience the learning of mathematics as a process similar to the process by which mathematics was invented. However, with the guidance from the teacher, the process of reinventing mathematics can be made shorter than how it was invented in the history. The guidance can be given in activities that allow and motivate students to construct their own solution procedures.

The fourth tenet emphasizes the interactive classroom environment which promotes classroom discussions as a way of sharing knowledge among the students. An effective classroom discussion should lead students to express their ideas and solutions of the given problems, and at the same time to respond to each other's solutions (Cobb & Yackel, 1996). In such situation, students will be able to negotiate to one another in an attempt to make sense of other's explanation, to justify solutions, and to search for alternatives in a situation in which a conflict in interpretations or solutions have become apparent (Cobb & Yackel, 1996). A discussion should also center on the correctness, adequacy and efficiency of the solution procedures and the interpretation of the problem situation (Gravemeijer, 1994).

The intertwining principle of realistic mathematics education is often called the holistic approach, which incorporates application, and implies that learning strands should not be dealt with as separate and distinct entities (Zulkardi, 2002). With this approach learning mathematics can be more effective, for example learning algebra and geometry can be done simultaneously.

METHOD

This research was conducted in a Junior High School in Karawang, West Java. The experiment took place at a grade 8 classroom. The research used a design research methodology. Gravemeijer and Cobb (2006) explained that design research consists of cycles which have three phases: the preparation phase, the teaching experiment, and the retrospective analysis. The result of the retrospective analysis will add to the local instruction theory of students' development of spatial visualization on volume measurement.

Hypothetical Learning Trajectory

According to Simon (1995) a hypothetical learning trajectory (HLT) is made up of three components, namely the learning goal, the learning activities, and the hypothetical learning process. The goal determines the design of the learning activities. In order to reach the goal, first, researchers need to set up the starting point of the learning, that is the current students' knowledge of the mathematical domain being taught. After that, activities are designed to help students achieve the goals. In designing the activities, the researchers need to make predictions of how students' students understanding will evolve throughout the activities.

In our HLT, we designed sequence of activities in 4 lessons. We used realistic mathematics education (RME) theory in developing the instructional designs in volume measurement of cube. In RME, realistic context is used to stimulate students in constructing their knowledge of the concept. The first lesson's goal is to generate students' awareness of using structures to help them count the number of any given object. Thus, we developed a context of counting the number of white tofu in a plastic bag. This context was chosen because tofu is a very popular Indonesian food that students are already familiar with, and it has a shape of a cube. In the activity the tofu was represented by a paper cube. The use of paper cube served as a *model of* the tofu. We expected students would rearrange the tofu in a structured configuration so that they would be able to count them easier. Once students realized the importance of structuring the objects, we ask students to draw the objects with the help of using isometric grid. By drawing the object we expected students would be able to represent 3D object in perspective drawing when seen from different perspectives.

The second lesson aims to develop students skill in building 3D objects correspond to perspective drawing and count the number of cubes in the object. In this lesson, students are given perspective drawing of an object shown its front view, side view, and top view. First, students will be asked to count the number of cubes in the object and then they will be asked to build the object using paper cubes and find out the correct number of cubes used in the object.

The third lesson aims at developing student's ability to estimate the number of cube can be filled in a box as a stepping stone for understanding the concept of volume. Students will be given a box of 5 × 3 × 2 unit cubes and 10 unit cubes. Here, students will estimate the number of cubes needed to fill the box, but since they only have 10 cubes, we wanted them to avoid counting the cubes one by one and expected them to use some strategies such as grouping the cubes by layers or column and move forward by using multiplication. These will lead to the construction of the concept of volume formally, that is length × height × width.

The last lesson's goal is to help students determine the possible length of the sides of a box when the volume is given. In this lesson students will be given 24 unit cubes and will be asked to determine the dimension of a box with a capacity of 24 unit cubes. By using the unit cubes we expected students will be able to rearrange the

 ISBN : 978-602-17465-1-6

structure and find many possibility of the length of each side of the box such as 8 × 3 × 1, 6 × 4 × 1, 6 × 2 × 2, etc.

DISCUSSION

Lesson 1: Drawing the cubes

In the first lesson, we found that the context of counting the number of tofus has stimulated students to use structuring and grouping strategies such as counting by two. This task has provoked students to use of paper cube as the *model of* the tofu. Paper cubes served as a hand-on tool that allowed students to create and recreate different configurations of cube structures. Other flexibility of using paper cubes is that students were able to see the cubes from different perspectives.

The next activity was drawing the configuration of the cubes. Since we found that students had difficulties in perspective drawing, the first drawing activity is to draw one cube in a dot paper (figure 1) and isometric dot paper (figure 2). The drawing in dot paper showed that students did not pay attention on the length of the side of the cube. Students who counted the number of the dot might have known that a cube has congruent sides; however in the perspective drawing in the dot paper, the width of the cube looked longer than the length and the height. On the contrary, using isometric dot helped student in drawing the cube. They count the number of the dots in each side to have the same length. The isometric dot paper helped students to make the perspective drawing.

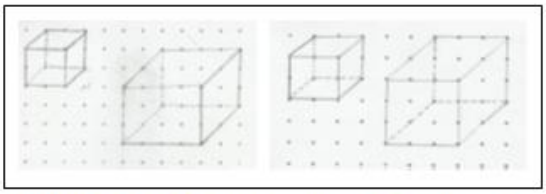


Figure 1: Student's drawing of a cube on dot paper

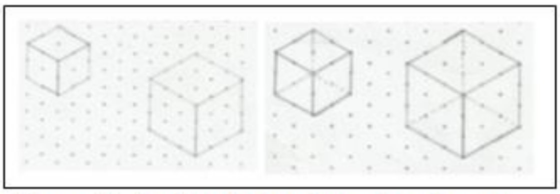


Figure 2: Student's drawing of a cube on isometric dot paper

Next, students are asked to draw the configuration that can be made of 2 and 4 cubes (figure 3). Some students used the paper cube and built the configuration first before drawing it, and other students were able to draw many possible configurations directly. Students who did the later strategy, counted the number of cubes while drawing the configurations. At this activity we found that the drawing with isometric dots helped students to visualize different configurations of cubes. One student was able to visualize a hidden cube in the 4 cubes configuration.

 ISBN : 978-602-17465-1-6

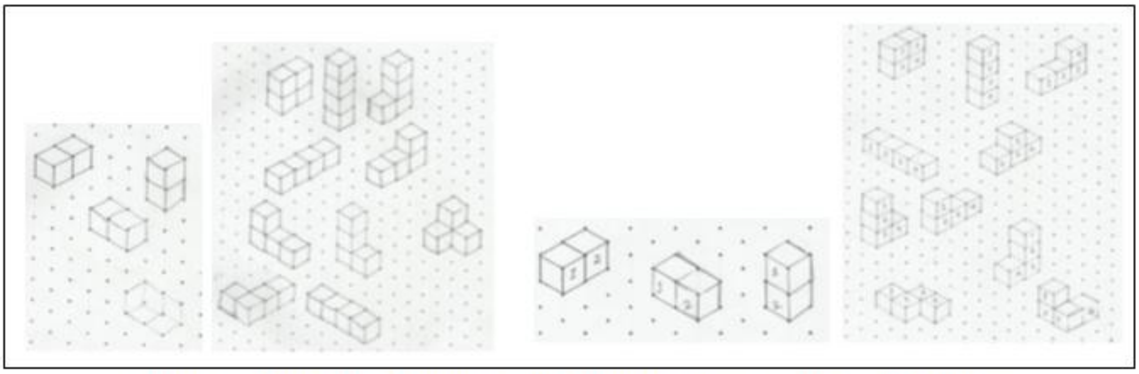
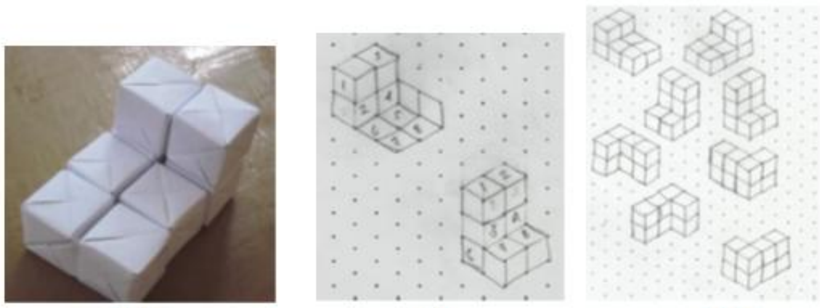


Figure 3: Students' drawing of configurations of 2 cubes and 4 cubes

This activity was continued as follow. The teacher made a building made of 8 paper cubes (Figure 4.a) and asked students to draw it as seen from different perspective but they were not allowed to hold the building. They were forced to use their mental visualization. One student (figure 4.b) had difficulties in drawing the hidden cubes in the building. He did not draw the cubes precisely, but he indicated the cubes by writing the corresponding number of the cubes. Other students were able to draw the building from different perspectives, his drawing shows that he reflected one configuration to get different configuration (figure 4.c).

From students' work we found that they were having difficulties in drawing different perspectives when not having the paper cubes as a manipulative tool. We can say that at this level students were still unable to visualize a mental object; therefore they still need the paper cubes to help them recreate the objects concretely.

The last activity in the lesson is drawing the front, side and top view of an object. In this activity students were only given a perspective drawing and they were asked to draw the front, side, and to view (figure 5). Most students did not understand the task and they needed the paper cubes help them visualize the object. With the help of paper cubes, some students were able to do this task.



(a) (b) (c)

Figure 4: Drawing 8-cube building

 ISBN : 978-602-17465-1-6

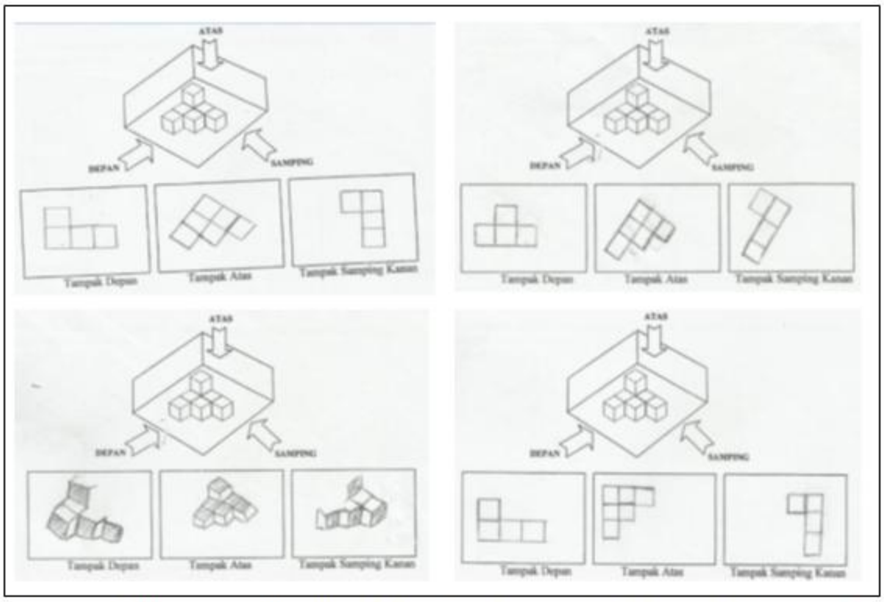


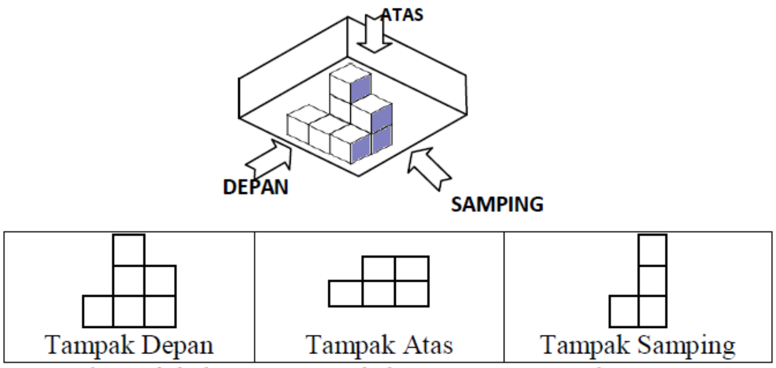
Figure 5: Students' drawing of the front, side and top view of an object

In general, in the first lesson we found that students have not yet developed a spatial visualization of mental objects. They still need the paper cubes as a manipulative tool to help them build the concrete objects. When the concrete objects are available, students are able to project them in the perspective drawing.

Lesson 2: Building and counting

In this activity students were asked to build a building made of unit cubes when a perspective drawing is given (Figure 6) and then count the number of unit cubes needed. Students related the previous activity, i.e. the front, side and top view of a building to do this task.

1. a. Bangunlah susunan kubus berikut ini dengan menggunakan kubus satuan!



b. Berapa banyak kubus satuan pada bangun ruang tersebut?

Figure 6: Building a figure from a perspective drawing

They started building the figure from their chosen perspective; the front view and the top view then arranged the unit cubes in layers or row. The group that started from the front view, they put 3 cubes in the first row, and put another 5 cubes behind it (Figure 7.a). This corresponded to how they count the cubes (Figure 7.b).

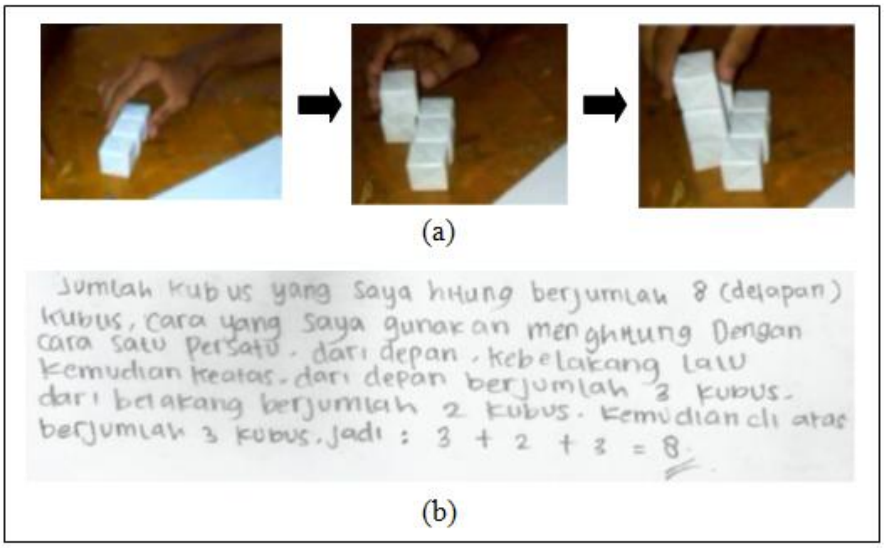


Figure 7: Building an object and counting the cubes

We found that the perspective drawing activity in the previous lesson has built a strong base for students in doing this task. The ability to visualize an object from different perspectives has enabled them to resemble the object from their own perspective. In doing this, students decomposed the objects into smaller groups (layers, columns, or rows). This grouping helped students to count the number of cubes in the figure; that is by adding the number of cubes in each group. This is an important stepping stone for students to construct the concept of volume.

Lesson 3: Predicting the volume of a box

In this lesson, students were given 10 unit cubes and were asked to determine the number of cubes needed to fill out a box of 5×3×2. Most students use grouping strategies, putting the cubes in rows or columns and used the number of cubes in each row or column to visualize the empty space, then finally used addition or multiplication to find out the total number of cubes (Figure 8.a). However, we found one group that came up with the multiplication strategy. They only used 8 cubes and put them in each edge of the box, these cubes representing the length, width and height of the box (Figure 8.b).

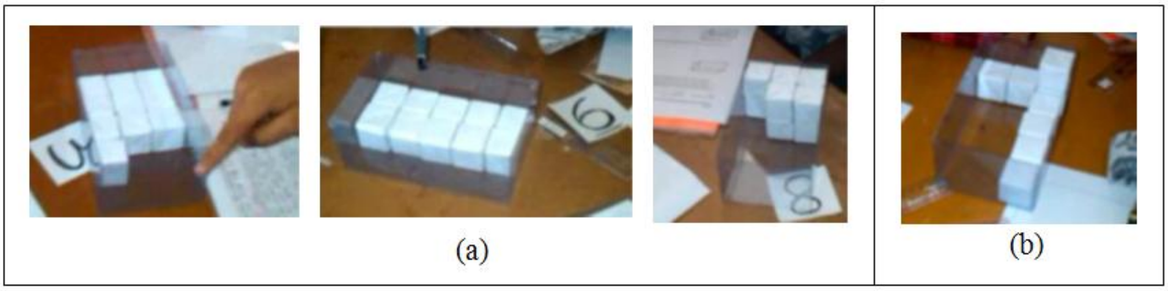


Figure 8: Students' strategies of predicting the volume of a box

We found that small group work and discussion played an important role in this level of the learning. Students tried different configurations of the unit cubes, such as arranging by columns of 3 cubes, or 3 by 2 cubes. These arrangements left much empty space that it was difficult for them to predict the number of cubes needed to fill out the remaining empty space. By discussion within the group, they rearranged the cubes and found that grouping the cubes in rows of 5 was easier. Throughout this process, students has built a social learning culture, that is working in group, discussing and justifying their personal reasoning to the group and obtaining an agreement for the group. We found that this positive learning culture helped students construct their understanding of volume.

In terms of using the paper cubes as a manipulative learning tool, we found that in this level of the learning, the paper cubes has transformed from a *model of* in to a *model for*. The students have built an understanding that the cubes are now used as the unit of the volume of the box. The grouping strategies still played a very important role in students' reasoning of the volume. Students who were using additive strategy found a difficulty in visualizing the remaining empty space. However, they managed to find an easier way by filling out the longest side of the box. The grouping also supported students who were using multiplicative strategy. By grouping the cubes in lesson 3, they have developed a higher level of thinking by seeing the multiplicative structure in the box. Thus, volume of the box is *length* × *width* × *height*.

Lesson 4: Determining the dimension of a box

In this activity students were given 24 cubes and were asked to determine the dimension of a box that can be filled with all 24 cubes. Students used the paper cubes and built various arrangements. They drew those arrangements and could summarize that the dimension of the box might vary but the volume stay the same (Figure 9).

This activity was followed by working on an exercise in a worksheet. Students were given the same problem with bigger number of cubes, but this time they did not have the paper cube to work with. They had to use their spatial visualization for drawing the possible boxes. The students' did not find any difficulties in doing this task. We found that this activity has strengthened students' understanding of the concept of volume. They have constructed the formula of volume in the previous activity with concrete paper cubes, and were able to move to an abstract level by using their spatial visualization to draw the different configurations and use the formula of volume.

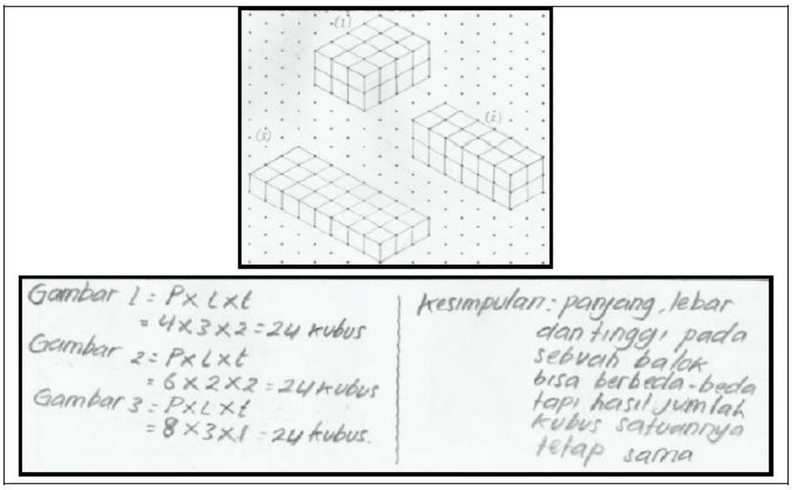


Figure 9: Possible dimensions of a box of 24 cubes

CONCLUSION

The result of this research added to the development of local instructional theory of how students develop their spatial visualization on building understanding of the concept of volume in a RME learning environment. The use of context plays an important role in developing students' awareness of structured objects. This awareness will lead to grouping strategies when counting the number of unit cubes in a given object.

The learning tools can support students' development of spatial visualization skill. We found that the use of isometric dot supported students in developing projective drawing skill. Through drawing activities students have developed their spatial visualization skills such as seeing and drawing an object from different perspectives, knowing how the front, side and top view of an object look like.

The building activity has supported students' understanding of the concept of volume. As students worked with paper cubed to build a 3D object, they used grouping strategy such as groups of rows, column, or layers in counting the number of cubes needed. This grouping led to additive strategy.

The use of paper cubes has supported students throughout the whole lessons. At first, the paper cubes served as a *model of* the object that being observed. As the level of students' spatial thinking developed, the paper cubes transformed into a *model for* the volume of a box. Finally, students constructed their understanding of the formula of volume of a cube, and they could work without paper cubes anymore.

An interactive learning atmosphere was also an important aspect in the students' learning. We found that throughout small group work and classroom discussion students have learned both individually and collectively by posing and justifying their arguments, listening to different answer used by others, evaluate and discuss an agreement of which is the best strategy to choose.

REFERENCE

Amin, S. M., Julie, H., Munk, F., & Hoogland, K. (2010). The Development of Learning Materials for PMRI. In R. Sembiring, K. Hoogland, & M. Dolk (Eds), *A Decade of PMRI in Indonesia*, Bandung, Utrecht: APS International.

Ben-Haim, D., Lappan, G., & Houang, R. T. (1985). Visualizing Rectangular Solid Made of Small Cubes: Analizing and Effecting Students' Performance. *Educational Studies in Mathematics*, 16: 389 – 409.

Bennett, A. B. & Nelson, L.T. (2001). *Mathematics for Elementary Teachers a Conceptual Approach*. New York: The McGraw-Hill Companies, Inc. p. 679.

Clements, D. H., & Battista, M. T. (1992). Geometry and Spatial Reasoning. *Handbook of Research on Mathematics Teaching and Learning: A Project of The National Council of Teachers of Mathematics*. New York: Macmillan Library Refence, 444.

Cobb, P., & Yackell, E. (1996). Constructivist, emergent, and sociocultural perspective in the context of developmental research. *Educational Psychologist, 31(3/4)*, 175-190.

Doorman, L. M. (2005). *Modeling motion: from trace graphs to instantaneous change*, Utrecht: CD-β Press.

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide, *Educational Studies in Mathematics*, 63, 303-323.

Freudenthal, H. (1991). *Revisiting mathematics education. China lecture*, Doordrecht: Kluwer Academic Publisher.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*, Utrecht: CD Beta Press.

Gravemeijer, K. P. E., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example, *Educational Studies in Mathematics 39.*

Gravemeijer, K. P. E., & Cobb, P. (2006). Design research from a learning design perspective, In J. Van Den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational Design Research* (pp. 17-51). New York: Routledge.

Revina, S., Zulkardi, Darmawijoyo, & van Galen, F. (2011). Spatial Visualization Tasks To Support Students' Spatial Structuring In Learning Volume Measurement. *IndoMS. J.M.E* Vol. 2 No. 2. Pp. 127 – 146.

Sembiring, R. K., Hoogland, K., & Dolk, M. (2010). Introduction to: A Decade of PMRI in Indonesia. In R. Sembiring, K. Hoogland, & M. Dolk (Eds), *A Decade of PMRI in Indonesia*, Bandung, Utrecht: APS International.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective, *Journal for Research in Mathematics Education, 26*, 114-145.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction – The Wiskobas project.* Dorderecht, the Netherlands: Reidel Publishing Company.

Van de Wall, J. (2007). *Elementary and Middle School Mathematics. Teaching Developmentally.* USA: Pearson Education, Inc. 388.

Zulkardi. (2002). *Developing a learning environment on realistic mathematics education for Indonesian student teachers*. University of Twente, Enschede: Doctoral dissertation.

This paper can be cited as Zulkardi(2013). Proceeding The First South East Asia Design/Development Research (SEA-DR) International Conference, Sriwijaya University, Palembang, April 22nd-23rd.