Design Research on Addition and Subtraction Up to 100 Using Mental Arithmetic Strategies on an Empty Number Line At the 2nd grade of SDN Percontohan Komplek IKIP, Jakarta

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Abstract. *In this design research, a conjectured local instruction theory is developed to help children enhance their mental arithmetic to perform addition and subtraction problems up to 100. Moreover, the development of children's learning process in mental arithmetic both individually and in a social community is analyzed to provide information for the refinement of the theory. This paper highlights mental arithmetic strategies to solve addition and subtraction problems up to 100 on an empty number line. Mental arithmetic can be defined as a way to handle numbers in a handy and flexible way. Contradicts to algorithm, mental arithmetic is characterized with number sense and number relations. Findings from the first phase of this design research show that children at the second grade perform algorithm without sufficient understanding of numbers. Hence, realistic approach that encompasses mental arithmetic strategies on an empty number line is suggested as an alternative to help young children in solving addition and subtraction problems flexibly and meaningfully.*

I. Introduction

> *The Netherlands has about 14 millions inhabitants, against the more than three billions of the US which is 200 times as many. The area of The Netherlands is roughly 40.000 square meters, against the 33.000 square kilometers of the US, which is a thousand times as much. Make comments on this clipping.* (Treffers, 1991)

The above problem was one of test items for students at a teachers' training college conducted by Jacobs (1986) to investigate their arithmetic abilities. In addition, this statement suggests us how numbers play a great deal in everyday life situations. The inability to evaluate such statements involving numbers is classified as one of innumeracy problems. More specifically, Treffers defines innumeracy as inability to handle numbers and numerical data decently and to evaluate statements regarding sums and situations which invite mental processing and estimating (Treffers, 1991). It is also argued by Treffers that one of the causes of innumeracy in primary schools is algorithm when it is taught prematurely and context problems are often neglected. Therefore, the realistic approach in which mental arithmetic strategies and estimation are brought to the front is suggested to be an alternative.

Buys (in van den Heuvel-Panhuizen, 2001) defines mental arithmetic as a way of approaching numbers and numerical information in which numbers are dealt with in a handy and flexible ways and it is characterized by working with number values instead of number digits. Moreover, a framework of number relations and number sense play important parts in doing mental arithmetic. There are considerable researches have been done in the field of flexible mental arithmetic strategies for addition and subtraction up to 100. Beishuizen (1993) stated that both strategies N10 (with an empty number line) and 1010 (splitting tens and ones) enhance the flexibility of students' mental arithmetic. However, the 1010 strategy may cause problems when used with more complex subtraction tasks.

An empty number line is found to be a powerful model to do mental arithmetic strategies flexibly. An empty number line is best to represent children's informal stratgies in counting and it has potential to foster the development of more sophisticated strategies in children (Gravemeijer, 1994). Moreover, Klein (1998) came to a conclusion that providing children with a powerful model like the empty number line, establishing an open classroom culture in which children's solutions are taken seriously, and making teachers aware of both cognitive and motivational aspects of learning, will help every student become a flexible problem solver. It is mentioned that the classroom culture and the proactive role of the teacher play a crucial role in the development of classroom mathematical practice where children's learning processes is viewed from both the individual perspective and the social perspective.

Mental arithmetic is suggested to be learned by children before they learn algorithm (column arithmetic) (Beishuizen, 1993) and it has become more important because it stimulates number sense and the understanding of number relations (McIntosh, Reys, & Reys, 1992, in Klein, 1998). On the other side, a framework of number relations and number sense characterize mental arithmetic strategies. Although considerable research has been devoted to the use of an empty number line as a model for reasoning in doing mental arithmetic strategies, rather less attention has been paid to the development of a framework of number relations to construe flexible mental arithmetic strategies.

Aiming at development of theory and improvement of practice, the research questions below are formulated:

1. How do children develop a framework of number relations to construe flexible mental arithmetic strategies?
2. How do the number relations support the flexibility of doing mental arithmetic strategies in solving addition and subtraction problems up to 100?

II. Didactical Domain in Addition and Subtraction Up to 100

Children encounter numbers as early as they could recognize more or less in quantity. Treffers explains in Children Learning Mathematics that numbers could have different meanings and functions, such as magnitude (the quantity of five books), order number (the fifth item in a row), measure number (the age of five years), label number (bus number five), calculation number (two plus three is four). At the age of eight (grade two), children should be able to differentiate these meanings of numbers, to compare and order numbers, to recite the number sequence up to twenty, to count up and down from any number up to twenty, to do addition and subtraction for numbers up to twenty.

There are three levels of students thinking in solving addition and subtraction problems up to 100 (see appendix 1). In the low level, children solve addition and subtraction problems by counting one by one. They still don't recognize structure of ten that could help them counting easier. In the next level, children acquire the idea of 'unitizing' which means they understand that a number of objects can build another unit. For example, one pack of T-shirt is twelve T-shirts; one pack of books is ten books. Therefore, if there are 3 packs of books, children understand that 3 packs of books correspond to 30. At the top level, children should already master the previous concepts and strategies before they are taught algorithm.

Most problems that happen in primary school is that teachers skip the 'floating capacity' of the ice berg and they go straight to algorithm. The teaching of algorithm is best after the 'calculation by structuring' is developed in children's thinking.

III. Mental Arithmetic Strategies on an Empty Number Line

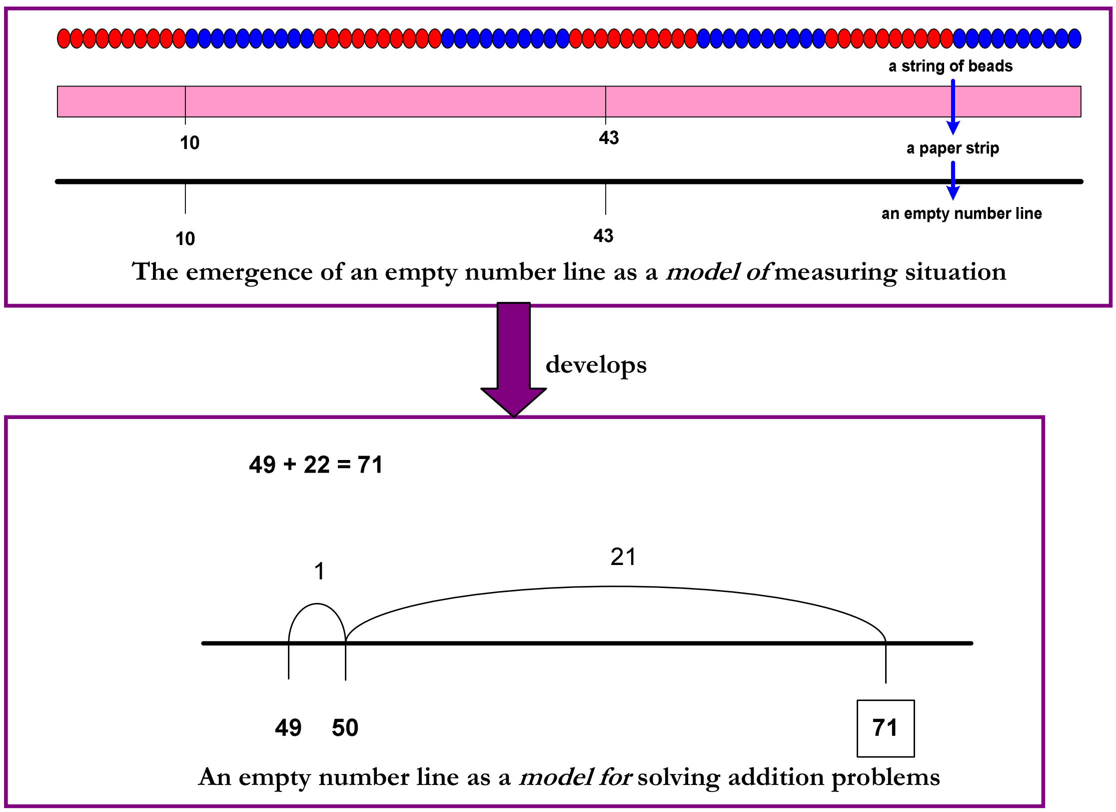
Mental arithmetic is a way of approaching numbers and numerical information in which numbers are dealt with in a handy and flexible way, and characterized by:

1. Working with numbers and not with digits
2. Using elementary calculation properties and number relations
3. Being supported by a well-developed feeling for numbers.
4. Possibly using suitable intermediate notes according to the situation, but mainly by calculating mentally.

There are three basic forms of Mental Arithmetic:
Mental arithmetic by a stringing strategy
Mental arithmetic by a splitting strategy
Mental arithmetic by a varying strategy
(Buys in van den Heuvel—Panhuizen, 2001

A sequence of activities is designed with an aim to develop students' mental arithmetic in solving addition and subtraction problems up to 100. An empty number line-a number line that is presented with no numbers or markers on it- is chosen as a model to represent students' strategies during mental computation. By representing students' strategies on the empty number line, each step in students' thinking can be recorded. Therefore, it allows them to track errors. Moreover, the empty number line provides a visual representation of students' thinking that could engage the classroom community to share and discuss the most sophisticated strategies in solving addition and subtraction problems up to 100.

The empty number line could be introduced through a string of beads that alternate in color every 10 beads in a measuring activity. The empty number line is emerged first as a *model of* a situation (measuring situation) then it develops as a *model for* solving addition and subtraction problems up to 100 using mental arithmetic strategies (see picture 1). Several activities must precede the use of an empty number line as a *model for* solving addition and subtraction problems up to 100, for example: ordering a sequence of numbers up to 100 in a number line (a sequence of ones and a sequence of tens); estimating a location of a number in an empty number line to build number relations; etc.



Picture 1. The emergence of an empty number line

A measuring situation using a string of beads is introduced as the contextual situation. Then, a paper strip will be used to record the result of the measurement. Later, the paper strip is progressively developed into an empty number line that could represent students' strategies in solving addition and subtraction problems involving measuring situation. At the end, students will use an empty number line as a model for solving addition and subtraction problems up to 100 with mental arithmetic strategies in every problem situation.

IV. Design Research Methodology

The purpose of the present research is to contribute to an empirically grounded instruction theory for addition and subtraction up to 100. In the present research we are especially interested in how students can learn to develop a framework of number relations to construe mental arithmetic strategies and how flexible students in solving addition and subtraction problems up to 100 using mental arithmetic strategies in RME oriented education. Design research that aim in developing theories about both the process of learning and the means designed to support that learning (Gravemeijer, 2006, p.18) consists of three main phases, i.e. preparation and design, teaching experiments, and retrospective analysis.

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C.1. Phase 1: Preparation and Design

Gravemeijer explains in *Design Research from a Learning Design Perspective* that the goal of the preliminary phase of a design research experiment from a design perspective is to formulate a conjectured local instruction theory that can be elaborated and refined while conducting the experiment, while from a research perspective is to highlight a crucial issue is that of clarifying the study's theoretical intent.

In the first phase of the design research, mathematical learning goals should be elucidated as well as anticipatory thought experiments in which sequences of learning activities are designed and students' mental activities in engaging the activities are envisioned. Therefore, a conjectured local instruction theory is formulated as a guide in developing thought experiments and implementing instruction experiments. On the other hand, thought and instruction experiments serve the development of local instruction theory.

In order to be able to develop a conjectured local instruction theory, the instructional starting points has to be put in a consideration. Some aspects in the starting points are studying the existing research literatures and use the consequences of earlier instructions to envision instructional activities and conjectured children's learning processes. Carrying out pre-assessment before the teaching experiments such as interviews with the teacher and children, and whole class performance assessments are useful in documenting instructional starting points. Below is the time-schedule in conducting design experiments for addition and subtraction up to 100 using flexible mental arithmetic strategies:

Time	Tasks	Instruments and Tools
1st week of May	❖ Observing the classroom community ❖ Interviewing the teacher about her beliefs ❖ Starting to set up classroom culture for the prospective 2nd graders	video recorder, voice recorder Pictures Questions for interview
2nd week of May	❖ Whole-class performance assessments ❖ Analyzing children's thinking ❖ Interviewing some children ❖ Negotiating with the teacher	Pre-assessment sheets (see appendix E.4) Teacher guide
3rd – 4th week of May	❖ Establishing the classroom culture ❖ Negotiating with the teacher ❖ Adjusting activities based on observation, interview and assessments	The recorded observation of the classroom situation The result of pre-assessment
June-July	❖ Refining Hypothetical Learning Trajectories ❖ Preparing the teaching-learning materials ❖ Preparing children's work sheets ❖ Preparing the teacher's guide	The result of pre-assessment Transcriptions of interviews with the teacher and children The recorded observation of the classroom situation
1st - 3rd week of August	Conducting the teaching experiment and retrospective analysis on a daily basis	Children's work sheets Teacher's guide

C.2. Phase 2: Teaching Experiment

During a teaching experiment researchers and teachers use activities and types of instruction that seem most appropriate at that moment according to the HLT. There will be experienced teachers and assistants that involve in the teaching experiments. The data such as video recording and audio recording will be collected during the teaching experiments. The students' works and field notes will be collected in every lesson, whereas the students' assessments will be held before and at the end of the sequence. Before and after each lesson, some students who will be chosen with intention will be interviewed with the formulated questions

C.3. Phase 3: Retrospective Analysis

After each lesson, we will analyze the data that we get from the class and use the result of the analysis to develop the next design. The results of the retrospective analysis will form the basis for adjusting the HLT and for answering the research questions.

C.4. Reliability and Validity

Reliability is about when other person conducts our research, s/he could trace what steps are in it and what decisions have been made, and it would give almost the same results for him/her. Things to consider in the reliability is data registration (notes, audio or video recording); trackibility (can other person follow the process?); intersubjectivity (there are 2 or 3 people in interpreting data, discussion with others). In Validity, there is internal validity in which triangulation (interview, observation, document analysis) plays important role to test data with HLT. In external validity, the question is can we generalize the results/ theory in other contexts?. While in ecological validity, the experiment is conducted in a real classroom where there are interactions, discussions, real learning processes, etc.

V. A Conjectured Local Instruction Theory

Conjectured local instruction theory is made up of three components: a. mathematical learning goals for students; b. planned instructional activities and the tools that will be used; c. conjectured learning process in which one anticipates how students' thinking and understanding could evolve when the instructional activities are used in the classroom (Gravemeijer, 2004, p.110). This conjectured local instruction theory is revisable and is always developed and generated based on the retrospective analysis of the teaching experiments where the conjectures are put to the test.

Mathematical Learning Goals for Children

Based on an interview with the teacher of the second graders in SD Lab School, children at the end of the first grade at least know the counting sequence up to 100 because they have learnt it in the second semester of the first grade. Moreover, the teacher has already taught algorithm to solve addition and subtraction problem for two digit numbers. Unfortunately, in Netherlands algorithm is suggested for not to be taught to children before the third grade because many researches have found that algorithm may cause innumeracy for children in primary school when it is taught prematurely. Furthermore, the realistic approach that proposes mental arithmetic strategies is proved to be an alternative to prevent innumeracy in primary school children. Therefore, mathematical learning goals for the teaching experiment are formulated as below:

1. Children will develop a framework of number relations to construe flexible mental arithmetic strategies.
2. Children will be flexible in solving addition and subtraction problems up to 100 both in context and in a bare number format using mental arithmetic strategies

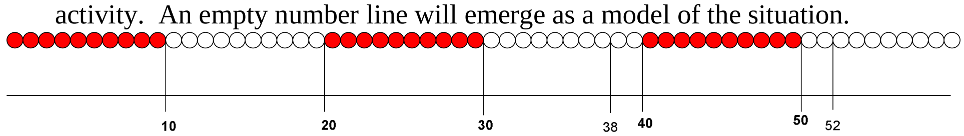
Aimed at the mathematical goals above a sequence of instructional activities and conjectures about children's learning processes are envisioned as a means to support children's development thinking.

Planned Instructional Activities

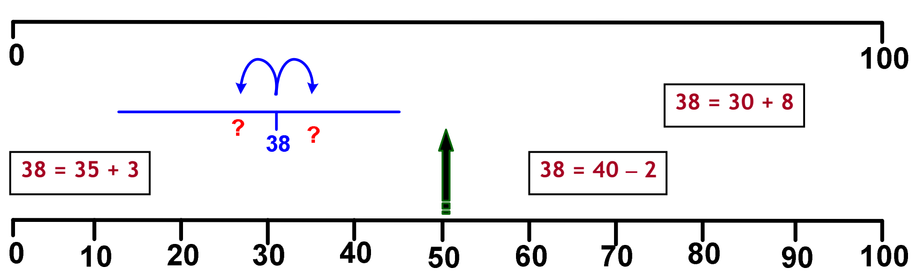
Unit 1 : Celebrating the 63rd Indonesian Independence Day
Developing the context; combinations that make ten; structurizing; place value patterns when adding groups of ten.

Unit 2 : Measuring
Measurement activity within the context 'celebrating the 63[rd] Indonesian Independence Day highlights students' conceptions of measuring and their strategies in counting.

Unit 3 : The Emergence of an Empty Number Line
A math congress allows students to discuss and share their strategies from the previous activity. An empty number line will emerge as a model of the situation.



Unit 4 : Exploring Number Relations
Exploring number relations in an almost empty/empty number line through some activities and games to construe mental arithmetic strategies.



Unit 5 : Exploring Addition
A mini lesson on double-digit additions within the context
Unit 6 : Developing Addition Strategies
The emergence of an empty number line as a model for solving double-digit addition problems focusing on jumps of tens strategy.
Unit 7 : Developing Addition Strategies
The emergence of an empty number line as a model for solving double-digit addition problems focusing on jumps via tens strategy.
Unit 8 : Exploring Subtraction
Subtraction as taking away and subtraction as adding on.
Unit 9 : Developing Subtraction Strategies
Subtraction as taking away on an empty number line
Unit 10: Developing Subtraction Strategies
Subtraction as adding on on an empty number line

This sequence of activities is conjectured to help children enhance their mental arithmetic in solving addition and subtraction problems up to 100. This local instruction theory is provisional, which means it still can be refined depend on the situation on day to day basis. The students' learning process is analyzed to refine the local instruction theory.

VI. Discussions

The empty number line has more flexibility in mental strategies and it fits with the linear, counting-type solution methods (the stringing strategy/ N10) rather than the collection-type methods (the splitting strategy/ 1010). However, the latter method usually becomes a natural way or children and it also gives a base for the column algorithm. On the other hand, the empty number line and associated jump strategy must be introduced thoughtfully and with well chosen examples to avoid the inappropriate use of an empty number line that could cause an empty number line to be a less sophisticated and meaningless strategy,

VII. Conclusion

The design research attempts to develop Local Instruction Theory (LIT) an addition and subtraction up to 100 by designing a sequence of activities that help children enhance their mental arithmetic in solving addition and subtraction problems up to 100. An empty number line is proved to be a powerful model to represent children's thinking in doing mental arithmetic strategies. Developing mental arithmetic using realistic approach is expected to prevent innumeracy.

VIII. References

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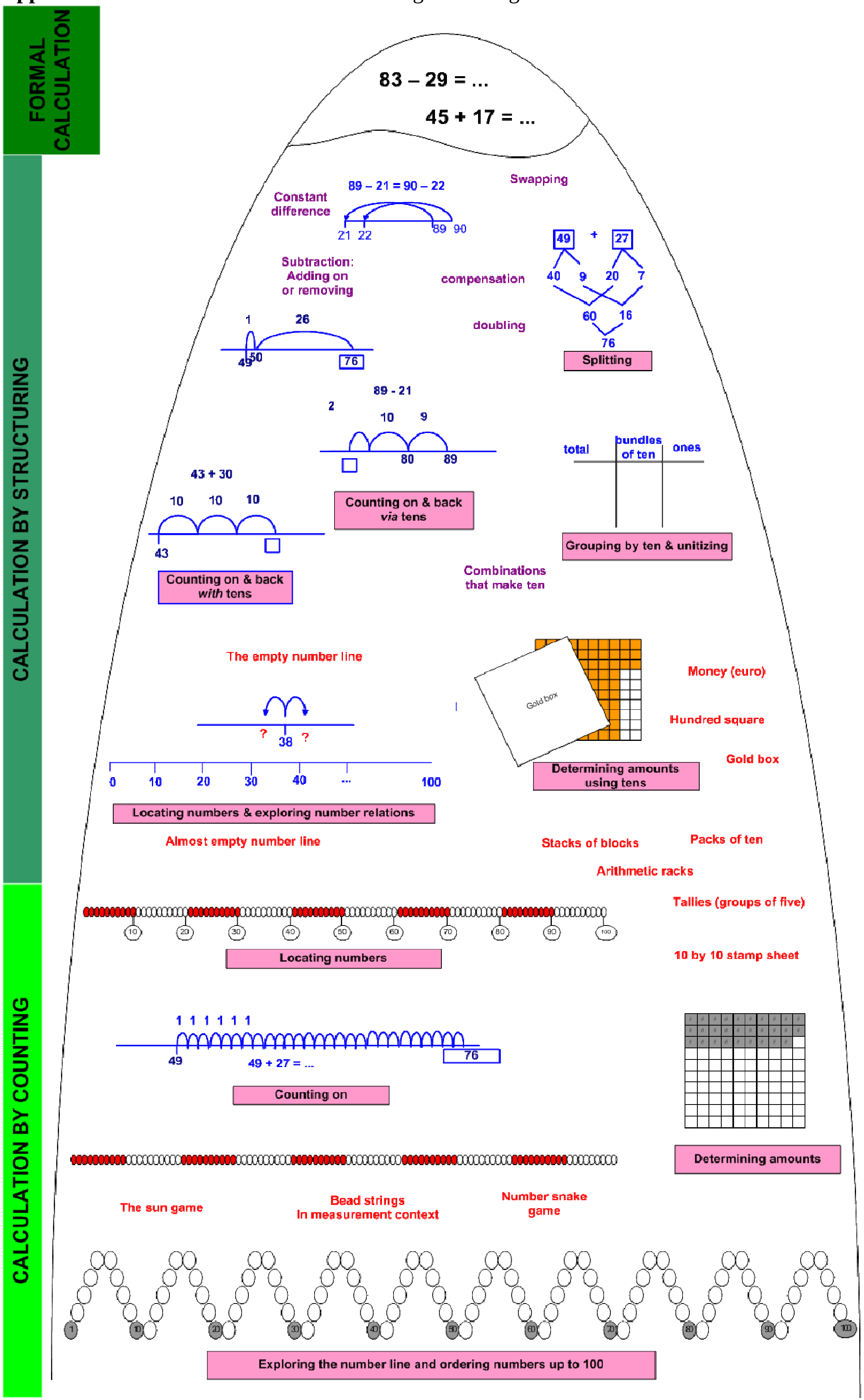
Appendix 1. The Level of Children's Thinking in Solving Addition and Subtraction Problems.



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