# Realistic Mathematics Education Theory Meets Web Technology

Abstract

***This paper about realistic mathematics education (RME) and Web technology towards mathematics reform in Indonesia. First, reform in mathematics education is briefly described. Second, three main components that influence the reform are discussed: new goals, new content and new theory. Furthermore, the Dutch theory for realistic mathematics education and its characteristics is elaborated. Finally, the ongoing study in developing a Web-based system called CASCADE-IMEI that aims to support student teachers to learn about RME as an innovation in mathematics education in Indonesia is discussed.***

## I. INTRODUCTION

Mathematics education is changing continuously. It can be seen by a movement from traditional curricula to problem oriented curricula or application based curricula (de Lange et al. 1993). Such change or reform in mathematics education can also be seen as a shift away from the transmission of knowledge by teachers towards investigation, construction and discourse (Gravemeijer, 1997). This reform is mostly influenced by changes in ideas on goals, content and the theory for learning and teaching of mathematics.

Concerning the *goals*, there is a growing emphasis on the usefulness of mathematics in daily practice. This trend is fostered by societal changes. The coming information society poses new demands to the citizens as 'mathematical literacy'. These societal changes also have their influence on the *content* of mathematics education in terms of reconsideration of what it means to know and do mathematics. Mathematics is not seen as a ready-made product anymore but it is stressed on the process of doing mathematics. The notion of mastery rules and procedures of mathematics is being exchanged for the idea that students should have a deep understanding of their mathematics and should be able to explain and justify it. Finally after twenty five years of developmental research, a *theory* for learning and teaching of mathematics education called realistic mathematics education (RME) is evolved. RME is strongly related to the constructivist theory (Freudenthal, 1991; Gravemeijer, 1994;de Lange, 1993).

## II. MATHEMATICAL LITERACY: THE NEW GOAL

Mathematics literacy is a new goal that is embedded in the new mathematics curriculum in many countries. The 'traditional goals' often only has two main elements: to prepare for the work-place and for future education; and to understand mathematics as a discipline. A new set of goals were prepared by the Commission of Standards for School Mathematics of the National Council of Teacher Mathematics. It lists ten goals, five the content goals - Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability - explicitly describe the content that students should learn and five the process goals - Problem Solving, Reasoning and Proof, Communication, Connections, and Representation - highlight ways of acquiring and using content knowledge (NCTM, 1989).
The mathematics literacy is further articulated by the NCTM standards by proposing five general goals for students.

- learning to value mathematics. Understanding its evolution and its role in society and the sciences.
- Becoming confident of ones own ability. Coming to trust one's own mathematical thinking and having the ability to make sense of situations and solved problems.
- Becoming a mathematical problem solver. This is essential to becoming a productive citizen and requires experience in solving a variety of extended and non-routine problems.
- Learning to communicate mathematically. Learning the signs, symbols and terms of mathematics.
- Learning to reason mathematically. Making conjectures, gathering evidence and building mathematical arguments.

These goals reflect a shift away from traditional practice; traditional skills are subsumed under more general goals for problem-solving, communication and critical attitude.

## III. REALISTIC MATHEMATICS EDUCATION: THE NEW THEORY

### 3.1 Realistic Mathematics Education

RME is a teaching and learning theory in mathematics education that was first introduced and developed by the Freudenthal Institute in the Netherlands since almost three decades. RME is mostly determined by Freudenthal's view on mathematics (Freudenthal, 1991). Two of his important points of view are: Mathematics must be connected to reality; and mathematics should be seen as human activity. First, mathematics must be close to children and be relevant to every day life situations. However, the word 'realistic', refers not just to the connection with the real-world, but also to problem situations which are real in students' mind. Second, the idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. In this case, the reinvention process uses concepts of mathematization as a guide.
Two types of mathematization, which were formulated explicitly in an educational context by Treffers (1991), are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. Freudenthal (1991) pointed out that horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols. But he adds that the difference between these two types is not always clear cut.
Treffers (1991) classifies mathematics education into four types with regard to horizontal and vertical mathematization:

(1) *Mechanistic*, or 'traditional approach', is based on drill-practice and patterns, which treat the person like a computer or a machine (mechanic). In this approach, both horizontal and vertical mathematization are less or even not used.
(2) *Empiristic approach*, the world is a reality, in which students are provided with materials from their living world. This means that students are faced with situations in which they have to do horizontal mathematization activities. However, they are not prompted to the extended situation in order to come up with a formula or a model.
(3) *Structuralist*, or 'New Math approach' that is based on set theory, flowchart and games that are kinds of horizontal mathematization but they are stated from an 'ad hoc' created world, which had nothing in common with the learner's living world.
(4) *Realistic approach*, a real-world situation or a context problem is taken as the starting point of learning mathematics. And then it is explored by horizontal mathematization activities. This means students organize the problem, try to identify the mathematical aspects of the problem, and discover regularities and relations. Then, by using vertical mathematization students develop mathematical concepts.

### 3.2 The characteristics of RME

Historically, the characteristics of RME are related to the Van Hiele's levels of learning mathematics. According to Van Hiele (cited in de Lange, 1996a) the process of learning proceeds through three levels: (1) real; (2) analysis; and (3) formal.
Traditional instruction is inclined to start at the second or third level, while the realistic approach starts from the first level. Then, in order to start at the first level; that deals with the phenomenon that is familiar to the students, *Freudenthal's didactical phenomenology* that learning should start from a contextual problem, is used. Furthermore, by *guided reinvention and progressive mathematizations* (Treffer, 1991), students are guided didactically to process as efficiently from one level to another level of thinking through mathematization.
Combining the three Van Hiele's levels, Freudenthal's didactical phenomenology and Treffers' progressive mathematization results in the following five tenets or characteristics of RME (Treffers, 1991):

(1) *phenomenological exploration or the use of contexts;*
(2) *the use of models or bridging by vertical instruments;*
(3) *the use of students own productions and constructions or students contribution;*
(4) *the interactive character of the teaching process or interactivity; and*
(5) *the intertwining of various learning strands.*

In the next sections these tenets will be elaborated.

***(1) Phenomenological exploration or the use of contexts***

In RME, the starting point of instructional experiences should be `real' to the students; allowing them to immediately become engaged in the situation. This means that instruction should not start with the formal system. *The phenomena by which the concepts appear in reality* should be the source of concept formation. The process of extracting the appropriate concept from a concrete situation is stated by De Lange (1987) as 'conceptual mathematization'. This process will force the students to explore the situation, find and identify the relevant mathematics, schematize, and visualize to discover regularities, and develop a 'model' resulting in a mathematical concept. By reflecting and generalizing the students will develop a more complete concept. Then, the students can and will apply mathematical concepts to new areas of the real world and by doing so, reinforce and strengthen the concept. This process is called applied mathematization. The two examples of context problems, t-shirt and soda and parent's night, are presented in the next section.

***(2) The use of models or bridging by vertical instruments***

The term model refers to situation models and mathematical models that are developed by the students themselves. This means that the students develop models in solving problems. Four levels of models in designing RME lessons can be distinguished (Gravemeijer, 1994):

*(1)* *the situational level*, where domain-specific, situational knowledge and strategies are used within the context of the situation;
*(2)* *a referential level or the level 'model of'*, where models and strategies refer to the situation described in the problem;
*(3)* *a general level or the level 'model for',* where a mathematical focus on strategies dominates over the reference to the context; and
*(4)* *the level of formal mathematics,* where one works with conventional procedures and notations.

***(3) The use of students own productions and constructions***

Students should be asked to 'produce' more concrete things. De Lange(1987) and Streefland (1991) stress the fact that, by making 'free production', students are forced to reflect on the path they themselves have taken in their learning process and, at the same time, to anticipate its continuation. Free productions can form an essential part of assessment. For example, students may be asked to write an essay, to do an experiment, to collect data and draw conclusions, to design exercises that can be used in a test, or to design a test for other students in the classroom.

***(4) The interactive character of the teaching process or interactivity***

Interaction between students and between students and teachers is an essential part in RME (de Lange, 1996a; Gravenmeijer, 1994). Explicit negotiation, intervention, discussion, cooperation, and evaluation are essential elements in a constructive learning process in which the informal methods of students are used as a lever to attain the formal ones. In this interactive instruction students are engaged in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting.

***(5) The intertwining of various learning strands or units***

In RME (de Lange, 1996a; Gravenmeijer, 1994), the integration of mathematical strands or units is essential. It is often called the holistic approach, which incorporates applications, implies that learning strands can not be dealt with as separate entities; instead, an intertwining of learning strands is exploited in problem solving. One of the reasons is that applying mathematics is very difficult if mathematics is taught 'vertically', that is if various subjects are taught separately, neglecting the cross-connections. In applications one usually needs more than algebra alone or geometry alone.

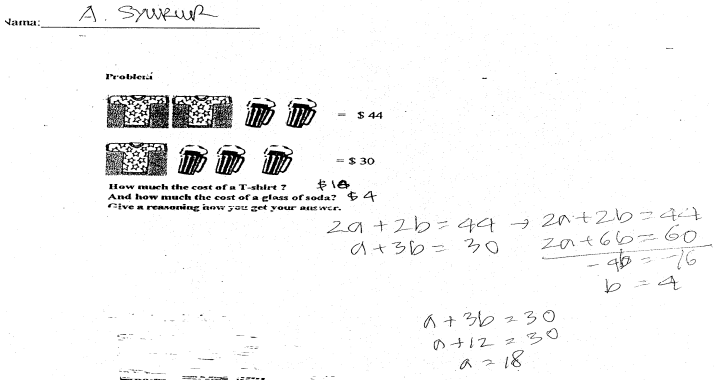
For additional information about these five tenets relating to the designing lessons based on the realistic approach see Zulkardi (1999a).

## IV. MEANINGFUL CONTEXT: THE NEW CONTENT

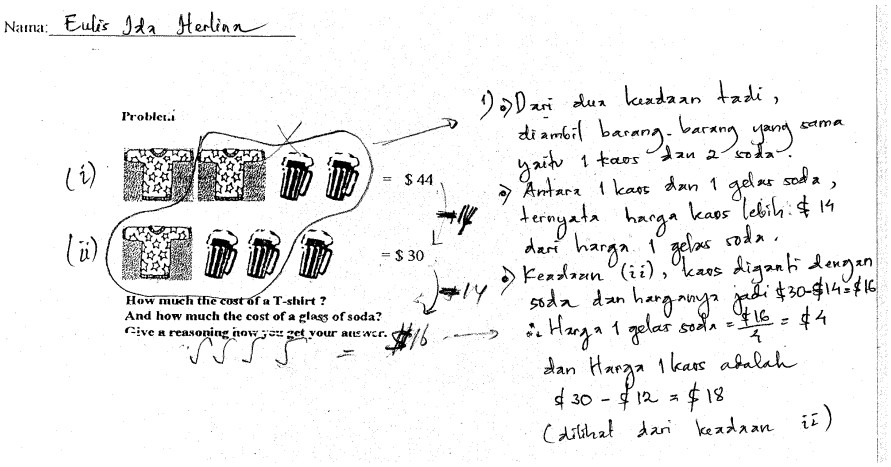
As mention earlier that the first tenet of RME is the use of meaningful context as the starting point of instruction. The following is an example of meaningful context for concept development that was designed for students of ages 12. This problem is very powerful due to the its position in the curriculum and the connections to earlier as well as later activities. It is presented in a visual way (de Lange, 1996a, 1996b):


**How much the cost of a T-shirt ?**
**And how much the cost of a glass of soda?**
**Give a reasoning how you get your answer.**

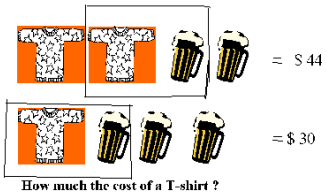
This problem I used as an assessment problem in a workshop that followed by junior high school teachers in Bandung. Before the problem was stated the participants were told to use the informal method. However, it appeared that most of them (see A. Syukur's solution) solve the problem using the formal method of linear algebra.




However, one of the teacher (see Herlina's solution) solved the problem using the informal method.



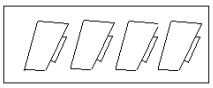

Students of ages 12 at the school come with very ingenious and different solutions. They are not familiar with the algebraic nature of the problem and are not hindered by this algebraic knowledge that makes the problem so difficult to teachers. Instead of using 2a + 2b = 44 and a + 3b = 30, they solve the problem just using common sense reasoning. Students operate differently:




How much the cost of a T-shirt ?
And how much the cost of a glass of soda?
Give a reasoning how you get your answer.

one t-shirt costs $ 18 because 1 t-shirt and 1 soda are $ 22. This leaves 2 sodas in the lower picture and $ 8 so 1 soda is $ 4, and 22-4 = 18 so one t-shirt costs $ 18


How much the cost of a T-shirt ?
And how much the cost of a glass of soda?
Give a reasoning how you get your answer.



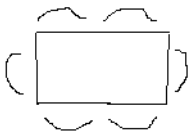
counting on

2 t-shirts 2 cups then
1 t-shirt 3 cups then I do
0 t-shirt 4 cups and the price also counted down $ 44 to $ 30
$ 16. So 1 cup= 16:4 = 4.-

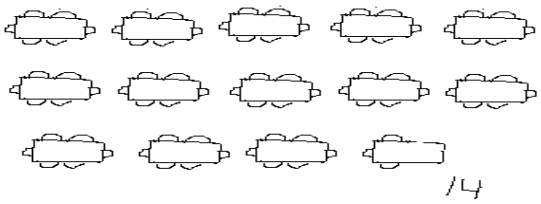
These two solutions are very different in nature: the first one notices that if two t-shirts and two cups are 44, one plus one must equal to 22. They take away one t-shirt and one cup from the second picture which leaves two cups for 8 dollars. Done. The second solution uses regularity as the starting point. The first picture shows 2 + 2, the second 1 + 3, so the 'third' picture must show 0 + 4. The price belonging with the third picture must be 16, fitting in the sequence; 44, 30, 16. The student shows good insight in where the essential part of the reasoning takes place: after noting that 2,2 followed 1,3 leads to 0,4 he writes down: almost ready. Exactly.
Later on students will handle in a formal way systems of equations. At that time they know informally how to reason algebraically, to understand equations, to use the proper mathematical notation and even to understand the concept of variable. But in this case it will take about two years to reach that level of real mathematical understanding (de Lange, 1996b).

The second example, think about a classroom with students of 8 to 9 years of age. The teacher introduces the 'parent night' problem to his students (see also Gravemeijer, 1994; De Lange, 1996b):
*Tonight 81 parents will visit our school. Six parents can be seated at each table*
*How many tables do we need?*

The teacher makes a small sketch on the blackboard:

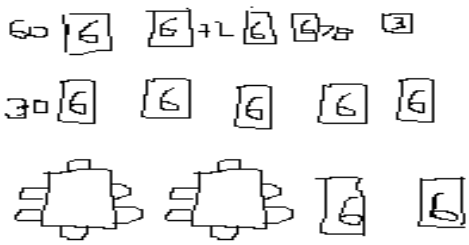


The students start to work on the problem in groups 3 and 4. The teacher walks around in the classroom, asking small questions about the process of solving the problem. The students were eager to engage in this process. After about ten minutes the teacher ends this part of the lesson. Students are asked to show and explain their solution. They vary quite a bit. Austin just copied as many as of the teacher's sketch as he needed to seat the parents:



Another student, Ani, started out the same way, but after drawing two tables he shifted to more schematic representation: a rectangle with number 6 on it. After drawing two of those 'tables' he realized that five tables would add up to 30. So, via 30 to 60 and then on to 72 and 78. And finally he added the last three chairs.

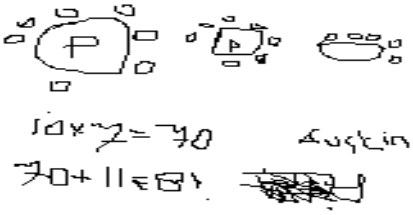
A third student, Alhariz, went a step further in mathematizing the problem. Although he also started to draw the table from the blackboard as a model he immediately schematized the problem and used his very recent knowledge of multiplication, by using multiplies of 6. He wrote down: 6 x 6 = 36, then doubled the 36 to 72, and added another two tables to get a capacity of 84.
If we look from distance at these three different solutions (and of course there were many more) we notice a different level using 'real' mathematics in this 'real-world' problem. Many



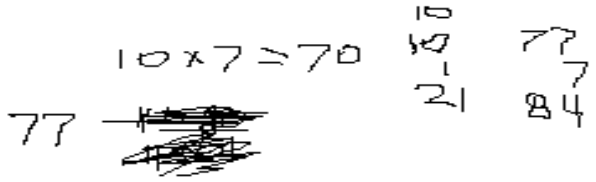
teachers would even argue that the first solution no mathematics has been used at all. But visualizing and schematizing are also important mathematizing tools that can be very powerful. The third solution makes the mathematics more visible and will be considered to have a 'higher' level.
After the whole classroom discussion without making too many explicit recommendations of wich solution is the best, the teacher continued by presenting the next problem:
*The 81 parents will be served coffee, of course. Each pot holds 7 cups of coffee. How many pots wee need?*

From a mathematical point of view exactly the same problem. Instead 81:6 we now have to deal with 81:7. Not so for the students. In the first place the context prohibits an easy pictorial solution: the students have a hard time drawing the coffee pots as Alhariz' s solution shows:

Alhariz, who used the most formal solution in the table problem now goes completely to mental arithmetic with visual support. Austin, who ad the most simple solution with the tables, tries to use the schema again with the pots. He represents them in the same way: a series of cups around the pot.



But after two pots he seems to realize the discussion about multiplication as a means to speed things up and he jumps to: 10 x 7 = 70 and adds: 70 + 11 = 81, which give him 12 pots. The work of yet another student, Yan, shows a typical solution for this second round of the parents problem:




No visualization, but multiplication. Within one lesson one can easily see the progress that the students have made towards solving this class of real contextual problems and mathematizing in order to develop new mathematical concepts, in this case division.

## V. CASCADE-IMEI: A STUDY IN THE FIELD OF REFORM IN MATHEMATICS EDUCATION IN INDONESIA

### 5.1 Mathematics Education in Indonesia need to be reformed

Based on the literature review in the preliminary study of CASCADE-MEI (see also Manan, 1998; Zulkardi, 1999b) mathematics education especially in the secondary level need to be reformed. Nowadays, mathematics education in Indonesia is categorized into the mechanistic-structuralist approach. While many countries already change their approach from mechanistic-structuralistic to the new approach, realistic approach, it is clear that in Indonesia mathematics education need to be changed as well.

As a consequence of reform, alternative curricula, alternative textbooks and teaching materials, and alternative forms of assessment (de Lange, 1993; Van den Heuvell-Panhuizen, 1996; Fullan, 1991) are essential. Selter (1997) pointed out that these are all mediated through the teacher, specifically through teacher's beliefs about how to organize and facilitate children's learning of mathematics. Consequently, teachers are a key players in educational change. In this context, teacher training (pre service as well as in service) plays a vital role. Reform efforts will seriously be hampered, if reform in teacher training is not regarded as being as urgent as reform at schools. This implies that reform can be started from teacher training.

As Gravemeijer (1997) pointed out, it is not easy to support teachers in order to make our common reform efforts successful. Teacher training should not primarily be an apprenticeship training that provides recipes and methods that are directly applicable in the classroom, but it should first and foremost assist prospective teachers in developing their autonomy. This implies to support them in increasing their degree of awareness: about mathematics, about children's mathematical learning, about the quality of teaching mathematics, and so forth (Selter, 1997). This means that a learning environment is needed that contains supports with which student teachers can enrich their knowledge about content as well as didactic mathematics.

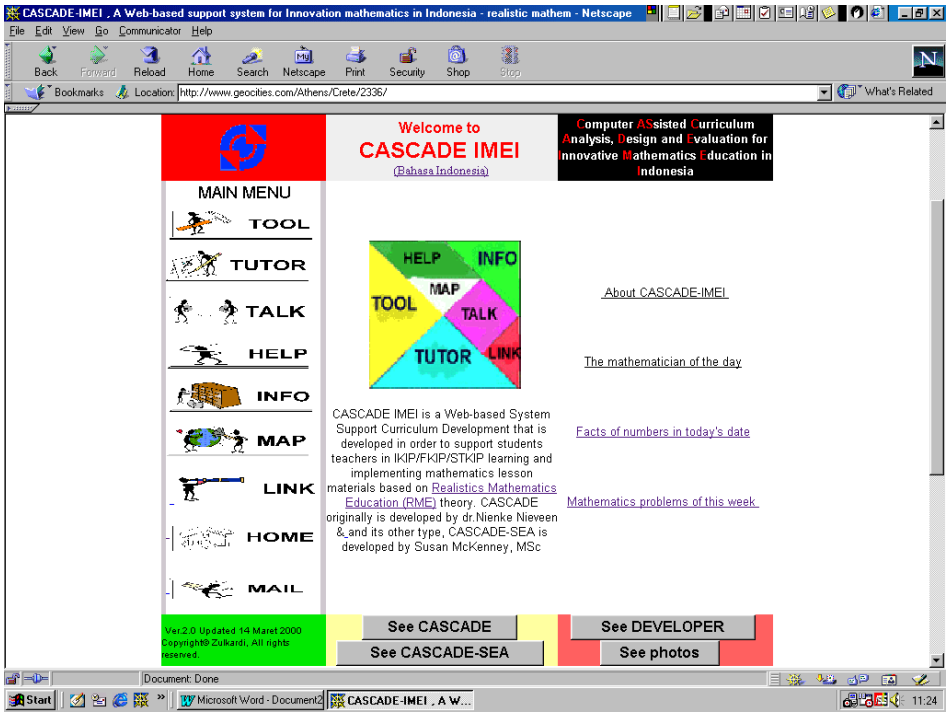
### 5.2 CASCADE-IMEI: Theory meets technology

Information and communication technology (ICT) tools currently permeate almost every professional domain. Web technology is starting to get used in education as well. Teachers can use this technology to store information such as lessons or problems, then can access these materials from many places in the world, and discuss and share problems and experiences after using the materials in the classroom.
CASCADE-IMEI stands for Computer Assisted Curriculum Analysis, Design and Evaluation for Innovation in Mathematics Education in Indonesia. The main purpose of this study is to develop a web-based system which provides resources, training opportunities and communication tools for student teachers in Indonesia, learning RME as an innovation in

mathematics education. This study builds on the previous studies about computer support system (see Nieveen, 1997) and number of studies about realistic mathematics education (de Lange , 1987; Gravemeijer, 1994, Streefland, 1991; Van den Heuvel-Panhuizen, 1996 ). The study is guided by the following main research question:
*"what role can a Web-based system play in supporting student teachers learn RME as an innovation in mathematics education in Indonesia?"*

CASCADE-MEI, a rich learning environment, will be developed based on the philosophy of RME theory which is represented in a number of support tools such as *infobase* (various topics of RME lessons, teacher guides, assessment problems and student productions), *communication tools* (communication and discussion using e-mail and mailing list) and *training opportunity* (a course about the introduction to the RME).

Now, this is end of the second year of a four year study of CASCADE-IMEI. A number of mathematics lessons have been developed and researched as well as the course. These tentative result can be seen in the Dikti homepage
(http://www.dikti.org) in the 'free space' part (see figure the Website below).




## VI. CONCLUSION

In this paper the new perspective on the goals, the content and the theory for teaching and learning of mathematics education have been discussed and illustrated. These are combined on the ongoing study CASCADE-IMEI that uses Web technology as a tool in order to support mathematics student teachers in teaching mathematics to the students in the school.
If the Netherlands is taken as an example of a successful country in reforming the mathematics education in the level primary and secondary school then the following considerations should be taken in to account:

- mathematical literacy should be added to the curriculum;
- the content of curriculum should be changed into the application based curriculum;
- mechanistic-structuralistic approach should be moved to the realistic approach; and
- teacher training (pre service and in service) should change as well.

CASCADE-IMEI is focusing on these considerations by providing a rich learning environment using Web towards mathematics reform in Indonesia.

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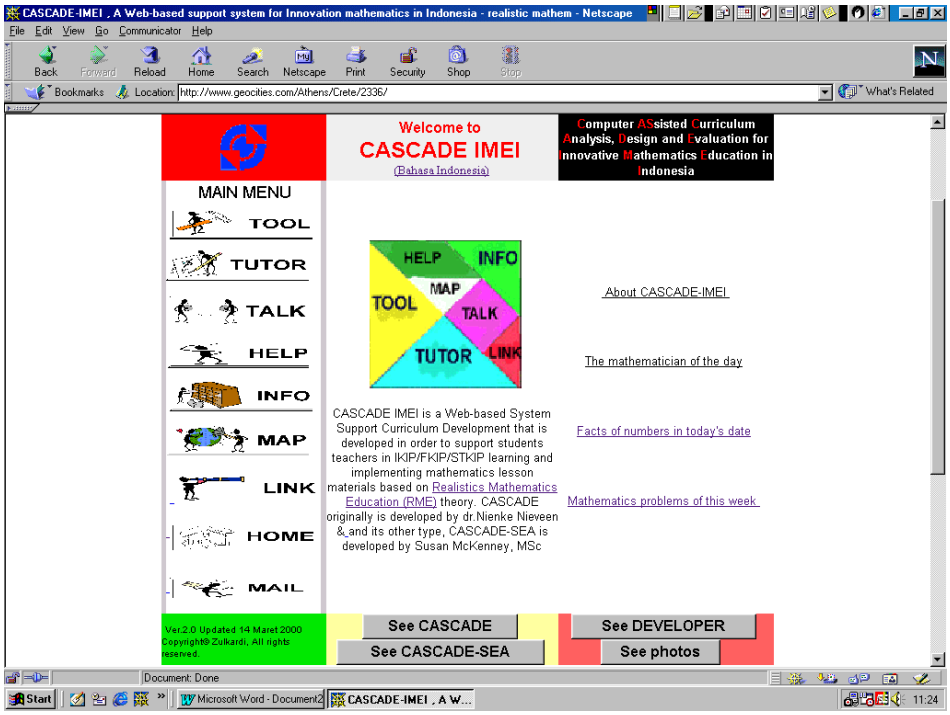



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