Modeling Extreme Rainfall with Gamma-Pareto Distribution

By Herlina Hanum Herlina Hanum

Applied Mathematical Sciences, Vol. 9, 2015, no. 121, 6029 - 6039
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2015.57489

Modeling Extreme Rainfall with Gamma-Pareto Distribution

Herlina Hanum

Department of Mathematics, Sriwijaya University, Indonesia

Aji Hamim Wigena and Anik Djuraidah

Department of Statistics, Bogor Agricultural University, Indonesia

I Wayan Mangku

Department of Mathematics, Bogor Agricultural University, Indonesia

Abstract

Gamma-Pareto is a combination of Gamma and Pareto taking a form of Pareto composed in Gamma. With three parameters, Gamma-Pareto has more flexibility than Gamma or Pareto in accommodating the distribution of real data including rainfall data. Gamma distribution is widely used in various applications, such as modeling rainfall distribution. While Pareto, Generalized Pareto distribution (GPD) or Generalized Extreme Values distribution (GEV) are used to model extreme values, including extreme rainfall. As a development of Pareto distribution, Gamma-Pareto distribution may be considered as an alternative for modeling extreme rainfall. This paper discusses the application of Gamma-Pareto distribution (G-P) in modeling extreme rainfall. The Gamma-Pareto was applied to the monthly rainfall data from Jatiwangi station Jakarta with the observation period from January 1978 to March 2015. The results showed that the Gamma-Pareto was very appropriate for extreme monthly rainfall. For this data set, fitting using Gamma-Pareto was better than using Pareto, Gamma, and GPD distribution. Gamma-Pareto's return level was very good to predict the maximum monthly rainfall.

Keywords: extreme rainfall, Gamma-Pareto distribution, modeling, return level

1. Introduction

Rainfall is an important element of weather and normal rainfall is beneficial for life in the earth. Rainfall which is too much is classified as extreme rainfall. On the other hand, extreme rainfall may cause huge loss in agriculture and fisheries. In order to anticipate the loss we need to forecast the rainfall some periods ahead. The forecasting can be done through rainfall modeling. Modeling rainfall is directed to the suitability of rainfall data with a particular distribution. Corresponding to the expected distribution, the estimation of parameters is done based on rainfall data to be modeled. A good model can be used to forecast rainfall several periods ahead.

Some researches on modeling rainfall are based on Gamma distribution. For example, Husak et al. [6] used the Gamma distribution for monthly rainfall in Africa, while Aksoy [1] used Gamma distribution for daily rainfall. Husak et al. [6] found that Gamma distribution has the capability to represent the variety of distribution shapes. Aksoy [1] noted that Gamma distribution fits very well to daily rainfall data. Furthermore, Sharma & Singh [12] also found that Gamma is the best fit probability distribution for the annual and monsoon season in India. On the other hand Papalexiou et al. [10] noted that Gamma distribution fitting would underestimate the behavior of extreme.

Extreme value distribution is commonly modeled by Pareto, Generalized Extreme Value (GEV), and Generalized Pareto Distribution (GPD). Sharma & Singh [12] also used GEV. They found that GEV has best fit distribution for weekly rainfall data. Modeling of extreme rainfall using GPD, among others, had been conducted by Montfort & Witter [9], and Deidda & Puliga [4]. In Indonesia, Prang [11] used GEV, and Hafid [5] used GPD and Modified Champernowne for modeling daily rainfall from Bogor Dramaga station. The advantage of GPD is that it has 3 parameters, i.e. threshold, shape, and scale. GPD also contains Uniform, Exponential, and Pareto as special cases. However the application of GPD depends on the choice of the threshold parameter. Cabras & Castellanos [3] noted that estimation of GPD parameters is usually not robust with respect to the threshold. Furthermore, Hafid [5] found that GPD estimation tends to be overestimate.

In addition to the modeling of rainfall, Gamma distribution is widely used in various applications. While Pareto distribution is used to model extreme values. Alzaatreh et al. [2] combined Gamma and Pareto taking a form of Pareto composed in Gamma. The combined Gamma and Pareto is introduced under the name of Gamma-Pareto (abbreviated G-P in this paper). With three parameters in G-P, it is expected to expand the scope of data distribution which can be modeled. The scope should be wider than Gamma and Pareto distributions individually. Alzaatreh et al. [2] demonstrated the superiority of G-P compared to other distributions in three different data sets. The data are: the annual floods on the River Floyd (Data1), fatigue life of 6061-T aluminum coupon (Data2), and observed frequency of *Tribolium confusum* strain # 3 (Data3). For Data1 with inverted J-shaped distribution, G-P is better than Pareto distribution, GPD, and Beta-Pareto. Then G-P is better than Beta-Pareto, Weibull, and Pareto distributions in Data2 which spreads almost symmetry. Finally, G-P is better than Gamma, Lagrange-Gamma, and Generalized Normal distributions for Data3 which distribution has a long right tail.

Since the Pareto distribution is commonly used in modeling extreme values, Pareto development in the form of G-P hopefully will increase flexibility in modeling extreme values. With the superiority of the G-P over the Pareto-based distribution, it is expected that G-P can also show goodness in modeling extreme values. In this paper, G-P is applied in modeling extreme rainfall compared to Pareto, Gamma, and GPD.

2. Gamma-Pareto distribution

G-P is one of the distribution in Gamma-Y family developed by Alzaatreh et al [2]. The developing starts with the formation of a cumulative distribution function (cdf) G-P in the form

$$G(y) = \int_0^{-\log(1-F(y))} r(t)dt \qquad (1)$$

with *F(y)* is cdf of Pareto(θ,κ) distribution with pdf $f(y) = \kappa\theta^{\kappa}y^{-(\kappa+1)}, y > \theta$ and cdf $F(y) = 1 - \left(\frac{\theta}{y}\right)^{\kappa}, y > \theta$. Meanwhile *r(t)* is pdf of *Gamma(α,β)* with $r(t) = (\beta^{\alpha}\Gamma(\alpha))^{-1}t^{\alpha-1}e^{-t/\beta}, t \geq 0$. The upper limit of integral *–log(1-F(y))* is one of the upper limits that can be used when Tϵ *(0, ∞)*. From the equation (1) we obtain pdf of G-P is

$$g(y) = \frac{f(y)}{1-F(y)} r(-\log(1-F(y))). \qquad (2)$$

Substituting r *(t)*, *F(y)*, and *f(y)* into (2), then pdf of G-P becomes

$$g(y) = \frac{\theta^{-1}}{\varrho^{\alpha}\Gamma(\alpha)}\left(\log\left(\frac{y}{\theta}\right)\right)^{\alpha-1}\left(\frac{y}{\theta}\right)^{(-1/\varrho-1)} \qquad (3)$$

$\alpha, \varrho, \theta > 0$ And $y > \theta$.

Cumulative distribution function of Gamma-Pareto is $G(y) = \gamma(\alpha, z)/\Gamma(\alpha)$, with $z = \varrho^{-1}\log\left(\frac{y}{\theta}\right)$ and $\gamma(\alpha, z) = \int_0^{v} e^{-z}z^{(\alpha-1)}dz$ is lower incomplete gamma function.

Using equation (3), the likelihood function of G-P is

$$L(\alpha, \varrho, \theta|y) = \prod_{i=1}^{n} \frac{\theta^{-1}}{\varrho^{\alpha}\Gamma(\alpha)}\left(\log\left(\frac{y_i}{\theta}\right)\right)^{\alpha-1}\left(\frac{y_i}{\theta}\right)^{(-1/\varrho-1)}.$$

Then its log likelihood function is

$$logL(\alpha,\varrho,\theta|y) = -n\log\theta - n\alpha\log(\varrho) - n\log\Gamma(\alpha) + (\alpha-1)\sum_{i=1}^{n} log\left(log\left(\frac{y_i}{\theta}\right)\right) - \left(\frac{1}{\varrho}+1\right)\sum_{i=1}^{n} log\left(log\left(\frac{y_i}{\theta}\right)\right).$$

Maximum likelihood estimator of θ is $y_{(1)}$, the minimum value of Y. With fixed value of $\theta = y_{(1)}$, the estimate of ϱ is $\hat{\varrho} = m1/\alpha$ where α satisfies

$$\log(m1) - \log(\alpha) + \psi(\alpha) - m2 = 0,$$

with $m1 = \sum_{1}^{n-n\prime} \frac{log\left(\frac{y_i}{y_{(1)}}\right)}{n-n\prime}$, $m2 = \sum_{1}^{n-n\prime} \frac{log\left(log\left(\frac{y_i}{y_{(1)}}\right)\right)}{n-n\prime}$, n' is the number of data with values of $y_{(1)}$, and $\psi(.)$ is digamma function.

3. Data and Method

3.1. Rainfall Data

The rainfall data used represented monthly rainfall data recorded in Jatiwangi station Jakarta. The data were collected from January 1978 to March 2015. There were 1167 observations in the data set. In this research we used Peaks over Threshold (POT) as defined by [8] to determine extreme values. The threshold value used to determine the extreme value is 75^{th} quantile such that the extreme rainfall data exceed 75^{th} quantile.

3.2. Parameter estimation

Extreme rainfall data then fitted with Pareto, Gamma, GPD, and G-P distribution. Parameter estimation is done with the R program. For GPD, estimation used Package *Evir*. While for Pareto, Gamma, and G-P used *mle2* functions in Package *bbmle*.

In this paper parameter estimation of G-P follows the method of Alzaatreh [2]. Firstly, θ is estimated by a minimum value of the data (Y), i.e $y_{(1)}$. So θ is regarded as a fixed value. Then α and ρ are estimated by maximum likelihood estimator without $y_{(1)}$. The initial value for α and ρ are parameter estimator of log $(y/y_{(1)})$ which is assumed to follow Gamma distribution.

3.3. Goodness of fit

Log likelihood, Akaike's Information Criterion (AIC), Mean Absolute Percent Error (MAPE), and Kolmogorov-Smirnov test are used to assess and to compare the goodness of those four distributions. While the quantile plots are used to assess and to compare the goodness graphically. Since the quantile function G-P is currently not available, to create quantile plot of G-P, firstly we need to establish quantile function of G-P distribution.

3.4. Return Level and Forecasting

The next step is to forecast extreme rainfall in the next period with G-P. This step needs return level function of G-P. The return level function of G-P is also to be established first. Then the function is applied to the extreme rainfall to get the maximum rainfall forecast in the specified period.

To get a proper forecast period, eight data were established with a different length of the observation period. These data are the subset of the monthly rainfall data. Each data set is divided into analysis and validation data. For each data we determined monthly extreme rainfall exceed the 75% quantile of analysis data.

For each extreme rainfall data set, we estimated the G-P and GPD parameter. Based on the estimate of these parameter, we determined return levels for the forecast period of 4, 6, 9, 12, 18, 24, and 36 months for each of the data. MAPE value is calculated based on the difference between the return level and the maximum value of data validation in the forecast period specified. Forecast period with little MAPE value is regarded as a good forecast period. Furthermore, the forecast is based on extreme value of the complete data for the forecast period selected.

4. Result and Discussion

4.1. Quantile Function of Gamma-Pareto$(\boldsymbol{\alpha}, \boldsymbol{\varrho}, \boldsymbol{\theta})$

One way to check the conformity of data with a particular distribution is quantile plot which is formed from pairs of actual and predicted values using a particular distribution. The predicted value is obtained from quantile function of used distribution. For Gamma-Pareto distribution, quantile function is obtained from the inverse of cumulative distribution function G(y) as follows.

$$G(y) = \rho = \gamma(\alpha, z)/\Gamma(\alpha)\ ,\ \rho\epsilon[0,1],\ \mathrm{z} = \varrho^{-1}\log\left(\frac{y}{\theta}\right)$$
$$\Rightarrow\ \rho\Gamma(\alpha) = \gamma(\alpha, z)$$
$$\Rightarrow\ \gamma^{-1}\big(\alpha, \rho\Gamma(\alpha)\big) = \mathrm{z} = \varrho^{-1}\log\left(\frac{y}{\theta}\right)$$
$$\Rightarrow\ \varrho\gamma^{-1}\big(\alpha, \rho\Gamma(\alpha)\big) = \log\left(\frac{y}{\theta}\right)$$
$$\Rightarrow\ \hat{y} = \theta\ \exp(\varrho\gamma^{-1}\big(\alpha, \rho\Gamma(\alpha)\big))$$

where $\gamma^{-1}\big(\alpha, \rho\Gamma(\alpha)\big)$ is the inverse of lower incomplete gamma function with parameter α and variable $\rho\Gamma(\alpha)$.

For $\rho_i = (\frac{i}{N} + 1)$, with positive fixed value of $\Gamma(\alpha)$, $\gamma^{-1}\big(\rho\Gamma(\alpha)\big)$ is an increasing function of i. This function does not accommodate the ties values of real data. This is the reason why the quantile function tends to be over estimate at some values being closed to maximum of Y when Y has so many ties.

4.2. Fitting Results

There were 112 observations which exceed a threshold value of 75th quantile or 366.4 mm/month. The minimum value of the extreme rainfall data was 366.9 mm/month. This value became the estimator of θ for Pareto and Gamma-Pareto, and the value of threshold for GPD. The estimate of parameters, and the goodness of fits are presented in Table 1.

Table 1. Parameters estimation by four distributions for Monthly Extreme Rainfall

Distribution	Parameters' estimate	Log likehood	AIC	MAPE	K-S test (D, P-value)
Pareto	θ=366.9 κ=3.276.0	-674.63	1351.26	0.0728	0.1607 0.1108
Gamma	α = 25.4979 β = 0.0502	-673.86	1351.71	0.0347	0.1161 0.4375
GPD	ξ = -0.3873 β= 228.48	-664.43	1332.86	0.0445	0.1250 0.3457
Gamma-Pareto	θ=366.9 α = 1.1915 ϱ = 0.3568	-657.68	1318.44	0.0205	0.0536 0.9971

Based on the value of MAPE, G-P distribution was very good to fit the data. Conformity is indicated by the value of MAPE that less than 10%. According to Lewis[7] 0-10% MAPE value shows a very good prediction. From the value of log likelihood, AIC, and MAPE it can also be seen that G-P can improve suitability of Gamma and Pareto. Furthermore, G-P had a better fit than GPD. This is indicated by the values of log likelihood, AIC, and MAPE of G-P distribution which are smaller than the values for three other distributions. Smaller log likelihood, AIC, and MAPE implies that the estimation value is closer to the data. Kolmogorov-Smirnov test also indicates that the data are almost perfectly fitted by G-P.

Suitability of data to G-P distribution can be observed from Figure 1. In the pdf plot, the pdf of G-P well follows the ups and downs of the data histogram. While the pdf of Pareto and GPD only follow when the histogram's peak decreases, and even then not exactly at the peak of the histogram. As we can see in the quantile plot, G-P has very good fit until the value of 700. It means that G-P gives very good estimate for 95% of the lower values of the data. While GPD prediction is overestimate for 60% of upper values of the data.

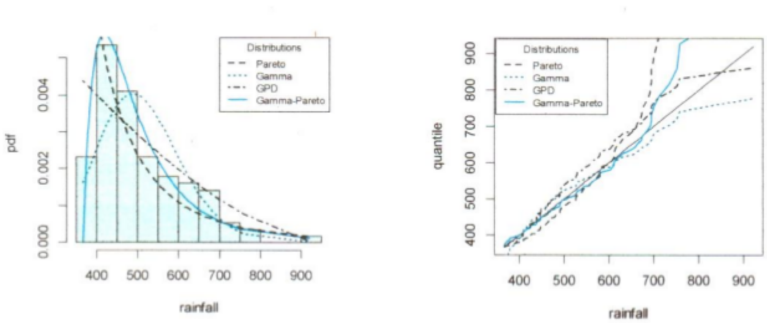

Figure 1. Data fitting with G-P distribution

4.3. Return Level Function of Gamma-Pareto

In order to forecast maximum rainfall in a given period, we require the return level function. Return level, y_m, is a high threshold to be exceeded in m period, with probability $p = 1/m$. In another word, y_m is the level that is expected to be exceeded once in a period of m Mallor et al [8]. Using a threshold u and unconditional probability of Y i.e. $\delta_u = P(Y > u)$, conditional probability of Y i.e, $P(Y > y \mid Y > u) = 1 - G(y)$, becomes $P(Y > y) = \delta_u(1 - G(y))$. For a certain period m,

$$p = \frac{1}{m} = \delta_u(1 - G(y_m)) \text{ or } G(y_m) = 1 - \frac{1}{m\delta_u}.$$

If Y follows G-P with $G(y_m) = \frac{\gamma(\alpha, z_m)}{\Gamma(\alpha)}$ where $z = \varrho^{-1}\log\left(\frac{y}{\theta}\right)$, then

$$\frac{\gamma(\alpha, z_m)}{\Gamma(\alpha)} = 1 - \frac{1}{m\delta_u} \text{ or } \gamma(\alpha, z_m) = \Gamma(\alpha)\left(1 - \frac{1}{m\delta_u}\right).$$

With $z_m = \varrho^{-1}\log\left(\frac{y_m}{\theta}\right) = \gamma^{-1}\left(\alpha, \Gamma(\alpha)\left(1 - \frac{1}{m\delta_u}\right)\right)$ then

$$y_m = \theta \exp\left(\varrho\gamma^{-1}\left(\alpha, \Gamma(\alpha)\left(1 - \frac{1}{m\delta_u}\right)\right)\right).$$

The inverse of lower incomplete gamma function ($\gamma^{-1}(.)$) is only defined in positive area. Therefore, for G-P, $\Gamma(\alpha)\left(1 - \frac{1}{m\delta_u}\right)$ must be positive. Since $\Gamma(\alpha)$ is positive then $m\delta_u$ should be at least one. In another word, the period m is at least $1/\delta_u$. Meanwhile, δ_u may be estimated by the proportion of the number of extreme value (n) to the overall amount of data (N). So the period m must be at least N/n. For example, if the 90^{th} quantile is used as a threshold, then the proportion of extreme value or δ_u is 0.1. With $\delta_u = 0.1$, then forecasting period *m* must be at least 10. This period would be appropriate for daily rainfall data as maximum rainfall can be predicted for the next 10 days. If we use monthly rainfall data, the return level of Gamma-Pareto can only forecast the maximum rainfall for 10 months period. It means no forecasting for period less than 10 months. So for shorter-term forecasting it requires greater proportion of extreme values. In another word, if the threshold value is used, we should take a smaller threshold value.

4.4. Application of Gamma-Pareto for Forecasting Extreme Rainfall

Maximum rainfall that may occur in the future may be predicted by the return level. In this paper we use the return level of G-P compared with GPD. The monthly extreme rainfall exceeding 75^{th} quantile threshold is calculated for each group, and then the estimation of G-P and GPD parameter is determined. Grouping data, threshold values, and parameter estimator for GP and GPD are presented in Table 2.

Table 2. Grouping data, threshold values, and parameter estimator for G-P and GPD

Group	Analyze Data	75^{th} Quantile	Validation Data	Gamma Pareto			GPD	
				θ	Α	Ρ	Ξ	β
1	Jan 1978-Mar2005	376.85	Apr2005-Mar2008	378.5	2.0574	.1389	-.2338	165.54
2	Jan 1978-Mar2006	373.80	Apr2006-Mar2009	375.2	2.0629	.1429	-.2483	171.23
3	Jan 1978-Mar2007	373.80	Apr2007-Mar2010	375.2	2.0574	.1406	-.2366	166.39
4	Jan 1978-Mar2008	373.80	Apr2008-Mar2011	375.2	1.9290	.1498	-.2397	166.67
5	Jan 1978-Mar2009	373.80	Apr2009-Mar2012	375.2	1.9301	.1521	-.2534	171.68
6	Jan 1978-Mar2010	371.50	Apr2010-Mar2013	371.7	1.7633	.1695	-.2547	172.48
7	Jan 1978-Mar2011	371.50	Apr2011-Mar2014	371.7	1.7658	.1677	-.25.01	170.13
8	Jan 1978-Mar2012	372.05	Apr2012-Mar2015	372.4	1.9456	.1529	-.2537	170.61

The values of 75^{th} quantile for all groups is almost similar, namely 371-377 mm/month. This means that the analyzed data for each group has a value that exceeds 371 mm/month. The estimator of θ in G-P is a minimum observation value that exceeds the threshold for each group. Since the threshold value is not always the value of the observation, then θ is not necessarily to be equal to the threshold value.

With a threshold value of 75^{th} quantile means that the proportion of data being analyzed as an extreme value is 25%. So the estimator of δ_u is 0.25. Based on the parameters estimator in Table 2, we determined return level for the 4, 6, 9, 12,18, 24, and 36 months using G-P and GPD respectively. The return level of G-P and GPD for all the forecast periods in all groups of data are presented in Table 3.

Table 3. Return Level and Forecast value of G-P and GPD for Some Forecast Periods

Extreme rainfall	Data Analysis	Forecasting for the i-th month ahead						
		4	6	9	12	18	24	36
Actual value	1978-Mar2005	260.2	260.2	290.5	684.3	684.3	684.3	684.3
	1978-Mar2006	318.7	318.7	407.6	441.3	441.3	585.0	694.5
	1978-Mar2007	379.3	379.3	379.3	585.0	585.0	694.5	694.5
	1978-Mar2008	172.0	172.0	694.5	694.5	694.5	694.5	694.5
	1978-Mar2009	217.5	217.5	360.5	610.3	610.3	610.3	612.3
	1978-Mar2010	416.0	416.0	416.0	566.9	612.3	612.3	612.3
	1978-Mar2011	612.3	612.3	612.3	612.3	612.3	612.3	671.7
	1978-Mar2012	152.5	152.5	494.2	539.8	539.8	671.7	671.7
G-P	1978-Mar2005	378.5	449.4	494.4	525.0	568.4	599.7	645.3
	1978-Mar2006	375.2	448.0	494.3	525.8	570.6	603.1	650.3
	1978-Mar2007	375.2	446.4	491.7	522.6	566.3	597.9	643.9
	1978-Mar2008	375.2	444.5	490.5	522.3	567.5	600.5	648.6
	1978-Mar2009	375.2	445.7	492.6	525.0	571.2	604.9	654.2
	1978-Mar2010	371.7	440.0	488.7	522.9	572.2	608.5	662.1
	1978-Mar2011	371.7	439.4	487.5	521.3	569.9	605.8	658.6
	1978-Mar2012	372.4	443. 6	490.8	523.4	569.9	603.8	653.4
	MAPE (%)	56.13	77.4	28.2	14.7	11.7	6.9	5.7
	1978-Mar2015	-	-	**464.9**	**502.2**	**556.8**	**597.2**	**657.2**
GPD	1978-Mar2005	376.9	440.9	499.1	537.2	586.8	619.2	661.3
	1978-Mar2006	373.8	439.8	499.6	538.4	588.7	621.4	663.8
	1978-Mar2007	373.8	438.1	496.6	534.8	584.4	616.8	658.9
	1978-Mar2008	373.8	438.2	496.6	534.8	584.3	616.6	658.5
	1978-Mar2009	373.8	440.0	499.7	538.4	588.5	621.1	663.1
	1978-Mar2010	371.5	437.9	497.9	536.8	587.0	619.6	661.7
	1978-Mar2011	371.5	437.1	496.4	534.9	584.8	617.2	659.1
	1978-Mar2012	372.1	437.8	497.1	535.6	585.4	617.9	659.4
	MAPE (%)	55.85	75.1	28.9	13.2	10.5	6.2	4.8
	1978-Mar2015	-	-	**476.2**	**518.6**	**572.6**	**607.3**	**651.5**

G-P has slightly greater MAPE than GPD in forecasting maximum rainfall. However, both of them have MAPE less than 15% in the forecast period at least 12 months. This means that forecasting results in such periods are considered good to approach the true values. While period of 9 months has sufficient forecast. So 9, 12, 18, 24, and 36 months are considered as good forecasting periods.

Based on the good periods we determined the return level of monthly extreme rainfall data of January 1978 to March 2015. The return level of the G-P and GPD for the forecast period 9, 12, 18, 24, and 36 months are presented in the last row of G-P and GPD in Table 3. For a period of 9, 12, 18 and 24 months maximum monthly rainfall forecast from G-P is lower than the GPD. Although, both maximum monthly rainfall predicted until February 2017 are about 600 mm/month. While the forecast until February 2018 are exceeded 650 mm/month.

5. Conclusion

Gamma-Pareto is very appropriate for modeling extreme monthly rainfall. The return level of Gamma-Pareto distribution is also suitable for forecasting the maximum rainfall of several periods ahead. The minimum period of forecasting with Gamma-Pareto is *N/n*, that is the number of observations divided by the number of extreme values in the data. Until February 2017, the maximum rainfall predicted for Jatiwangi station is about 600 mm/month.

References

[1] H. Aksoy, Use of Gamma Distribution in hydrological analysis, *Turkish Journal of Engineering and Environmental Sciences*, **24** (2000), 419-428.

[2] A. Alzaatreh, F. Famoye and C. Lee, Gamma-Pareto Distribution and application, *Journal of Modern Applied Statistical Methods*, **11** (2012), no. 1, 78-94.

[3] S. Cabras, M. E. Castellanos, An Objective Bayesian Approach for Threshold Estimation in the Peaks Over Threshold Model, *Technical Report* 2010.10 RIESGOS-CM.

[4] R. Deidda and M. Puliga, Modeling daily rainfall with Generalized Pareto Distribution, G*eophysical Research Abstracts*, **7** (2005), 09164.

[5] M. Hafid, *Pendugaan Nilai Ekstrim Menggunakan Sebaran Champernowne Termodifikasi, Sebaran Pareto Terampat, dan Nilai Gabungan (studi kasus curah hujan harian Dramaga Bogor),* Institut Pertanian Bogor, 2013.

[6] G. J. Husak, J. Michaelsen, C. Funk, Use of the Gamma Distribution to represent monthly rainfall in Africa for drought monitoring applications, *International Journal* of *Climatology*, **27** (2007), no.7 935-944. http://dx.doi.org/10.1002/joc.1441

[7] C. D. Lewis, *Industrial and Business Forecasting Methods*, Butterworths, London, 1982. http://dx.doi.org/10.1002/for.3980020210

[8] F. Mallor, E. Nualart, and G. Onner, An introduction to statistical modeling of extreme values. Application to calculate extreme wind speeds, *Hogeschool Universteit Brussel Research Paper* 2009/36, (2009), 45.

[9] M. A. J. Van Montfort and J. V. Witter , The Generalized pareto distribution applied to rainfall depths, *Hydrological Sciences Journal*, **31** (1986)**,** no. 2, 151-162. http://dx.doi.org/10.1080/02626668609491037

[10] S. M. Papalexiou, D. Koutsoyiannis, C. Makropoulos, How extreme is extreme? An assessment of daily rainfall distribution tails, *Hydrology and Earth System Sciences*, **17** (2013), no. 2, 851-862. http://dx.doi.org/10.5194/hess-17-851-2013

[11] J. D. Prang, *Sebaran Nilai Ekstrim Terampat Dalam Fenomena Curah Hujan Ekstrim* [tesis], Institut Pertanian Bogor, 2006.

[12] M. A. Sharma & J. B. Singh, Use of probability distribution in rainfall analysis, *New York Science Journal,* **3** (2010), no. 9, 40-49.

Received: August 2, 2015; Published: October 1, 2015

Modeling Extreme Rainfall with Gamma-Pareto Distribution

ORIGINALITY REPORT

95%

SIMILARITY INDEX

PRIMARY SOURCES

1	Herlina Hanum, Aji Hamim Wigena, Anik Djuraidah, I Wayan Mangku. "Modeling extreme rainfall with Gamma-Pareto distribution", Applied Mathematical Sciences, 2015 Crossref	2312 words — 59%
2	www.m-hikari.com Internet	1404 words — 36%

EXCLUDE QUOTES	OFF	EXCLUDE MATCHES	OFF
EXCLUDE BIBLIOGRAPHY	ON		