

Utility Function Based-Mixed Integer Nonlinear Programming (MINLP) Problem Model of Information Service Pricing Schemes

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Abstract. The development of the internet in this era of globalization has increased fast. The need for internet becomes unlimited. Utility functions as one of measurements in internet usage, were usually associated with a level of satisfaction that user get for the use of information services used specifically relating to maximize profits in achieving specific. There are three internet pricing scheme used, that is flat fee, usage based and two part tariff by applying pricing scheme Internet by using one of the utility function is Cobb-Douglas with monitoring cost and marginal cost. The internet pricing scheme will be solved by LINGO 13.0 in form of non-linear optimization problems to get optimal solution. internet pricing scheme by considering marginal and monitoring cost of Cobb Douglass utility function, the optimal solution is obtained using the either usage-based pricing scheme model or two-part tariff pricing scheme model for each services offered, if we compared with flat-fee pricing scheme. It is the best way for provider to offer network based on two part tariff scheme. The results show that by applying two-part tariff scheme, the providers can maximize its revenue either for homogeneous and heterogeneous consumers.

Key words : Cobb Douglass, flat fee, usage based, two-part tariff, monitoring cost, marginal cost.

I. INTRODUCTION

The development of the internet in this era of globalization has increased rapidly. The need for internet usage becomes unlimited; this becomes a challenge for internet service provider companies or ISP in satisfy their customers. According to [1] in making the model of internet pricing scheme, there should be a match between the price given and the satisfaction obtained by consumer. The level of satisfaction can be related to the utility function. Utility functions were usually associated with a level of satisfaction that user get for the use of information services used specifically relating to maximize profits in achieving specific [2]. So well-known utility function used by researchers are Cobb-Douglas [3] Quasi-Linear, Perfect Substitute utility function [4] and a function of bandwidth [5] [6-8] utility functions. There are three internet pricing scheme used, that is flat fee, usage based and two-part tariff by applying pricing

scheme Internet by using one of the utility function is Cobb-Douglas modified utility function to maximize benefits to the ISP [9]. Wu and Bunker [9] chose three internet pricing scheme that are flat fee, usage based and two-part tariff by applying modified utility function of Cobb Douglass in order to maximize the profits by ISP.

QoS (Quality of Service) is the transmission rates, error rates and other level of measurable characteristics to support the level of progress at a service provider [10]. Basically, QoS allows to provide better services for specific requests. To show the efficiency of ISP in service there must be an interaction between price and QoS [11]. Based on the research [12] the optimal internet pricing scheme for homogenous consumer case and heterogeneous consumer (High end and Low end) is obtained in utility functions of Cobb-Douglas with usage based scheme model by ignoring the monitoring cost and marginal cost. From the analysis result done by [13] it is found that utility function of Cobb-Douglas produce maximum profit for ISP with usage based pricing scheme for homogenous consumer and heterogeneous (High end and Low end) with monitoring cost and marginal cost. Based on the research conducted [14] based on the models application on the each traffic data it's find that the use of Cobb-Douglas utility function optimal pricing scheme more than the Quasi Linear utility function.

In general, the marginal cost are defined as the costs adjusted to the level of production of goods which is resulting differences in fixed costs due to the addition of the number of units produced, while the cost of monitoring is the cost incurred by the company to monitor and control the activities carried out by the agency in managing company. In fact, the marginal cost and the monitoring cost an important in consideration for internet service provider in maximizing profits.

So the contribution of this paper is to formulate the pricing scheme previously discussed differentially into mixed integer nonlinear programming to enable us to seek other option in solving pricing scheme for information services.

II.RESEARCH METHOD

In this paper, the internet pricing scheme will be solved by LINGO 13.0 in form of non-linear optimization problems to get optimal solution. Models are form based on the parameters and variables used to solve the optimization problem. Data required for the model is from the local server data at Palembang. The data used in this research is the data of bandwidth usage on digilib and mail applications with three internet pricing scheme used, those are flat fee, usage based and two-part tariff by considering marginal and monitoring costs based on Cobb Douglass utility function. It takes internet data during peak hours and non-peak hours. The choice of optimal internet pricing scheme with the considering monitoring cost and marginal cost based on the Cobb-Douglass utility function is solved by optimization as a Mixed Integer Nonlinear Programming (MINLP) problem. The solution obtained will help to determine the optimal price on the ISP pricing scheme.

III. RESULT AND DISCUSSION

In this section, the optimization problems are divided into two categories, which are consumer and provider problems.

A. Optimization of Consumers' problem

$$\begin{aligned} \text{Max}_{X_i, Y_i, Z_i} U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - PZ_i \\ \text{Subject to:} \end{aligned} \quad (1)$$

$$X_i \leq \bar{X}_i Z_i$$

$$Y_i \leq \bar{Y}_i Z_i$$

$$U_i(X_i, Y_i) - P_X X_i - P_Y Y_i - PZ_i \geq 0$$

$$Z_i = 0 \text{ or } 1$$

B. Optimization of Providers' problem

$$\begin{aligned} \text{Max}_{P_x, P_y} \sum_i (P_x X_i^* + P_y Y_i^* + \\ PZ_i^*) \end{aligned} \quad (2)$$

with $(X_i^*, Y_i^*, Z_i^*) = \text{argmax } U_i(X_i, Y_i) - P_x X_i - P_y Y_i - PZ_i$

Subject to:

$$X_i \leq \bar{X}_i Z_i$$

$$Y_i \leq \bar{Y}_i Z_i$$

$$U_i(X_i, Y_i) - P_x X_i - P_y Y_i - PZ_i \geq 0$$

$$Z_i = 0 \text{ or } 1$$

C. Cobb-Douglass Utility Function

According to [15] the Cobb-Douglass utility function is :

$$U(X, Y) = X^a Y^b ; a, b > 0 \quad (3)$$

with a and b are constants

Table I describes the used traffic data on digilib where usage data is divided based two categories is during peak hours (09.00-16.59 WIB) denoted by X and during non-peak hours (17.00-0859 WIB) denoted by Y .

TABLE I. INTERNET USAGE DATA DURING PEAK AND NON-PEAK HOURS FOR DIGILIB TRAFFIC

	Digilib (byte)	Digilib (kbps)
$\bar{X} = \bar{X}_1$	18,224.03	17.80
\bar{X}_2	13,770.41	13.45
\bar{X}_m	4,123.52	4.03
$\bar{Y} = \bar{Y}_1$	35,720.24	34.88
\bar{Y}_2	28,155.86	27.50
\bar{Y}_m	9,209.93	8.99

Table II describes the used traffic data on mail where again, the usage data is divided based two categories is during peak hours (09.00-16.59 WIB) denoted by X and during non-peak hours (17.00-0859 WIB) denoted by Y .

TABLE II. INTERNET USAGE DATA DURING PEAK AND NON-PEAK HOURS FOR MAIL TRAFFIC

	Mail (byte)	Mail (kbps)
$\bar{X} = \bar{X}_1$	3,687,676.42	3,601.25
\bar{X}_2	1,441,260.93	1,407.48
\bar{X}_m	431,669.78	421.55
$\bar{Y} = \bar{Y}_1$	2,983,577.94	2,913.65
\bar{Y}_2	2,716,046.03	2,652.39
\bar{Y}_m	1,135,254.76	1,108.65

Table III explain the parameters used in the consumer's optimization problem, respectively. Each parameter is used to design the whole improved model.

TABLE III. PARAMETERS FOR CONSUMER'S OPTIMIZATION PROBLEM

Symbol	Meaning
P	The costs incurred when following the services provided
P_x	The price provided by the service provider (ISP) during peak hours (09.00 – 16.59).
P_y	The price provided by the service provider (ISP) during not busy hours. (17.00 – 08.59).
$U_i(X_i, Y_i)$	The utility function from consumer i with X_i is the level of service usage during peak hours and Y_i is the level of service usage during not busy hours.

In Table IV the parameters for the optimization problem of providers are presented. Each parameter is used to set up the model.

TABLE IV. PARAMETERS FOR OPTIMIZATION OF PROVIDER PROBLEM

Symbol	Meaning
$X_i^* = X_{i(P_x, P_y, P)}$: The level of consumer service consumption i during peak hours.
$Y_i^* = Y_{i(P_x, P_y, P)}$	The level of consumer service consumption i during not busy hours.
$Z_i^* = Z_{i(P_x, P_y, P)}$	Consumer variable i to show the participation of the scheme
$U_{i(X_t, Y_t)}$	The utility function from consumer i with X_t is the level of service usage during peak hours and Y_t is the level of service usage during not busy hours.
\bar{X}_t	The highest level consumers i in using the service during peak hours.
\bar{Y}_t	The highest level consumers i in using the service during not busy hours.

Table V describes the decision variables use for consumer's optimization problem. Each variable is designed to fit into the model.

TABLE V. DECISION VARIABLE FOR CONSUMER'S OPTIMIZATION PROBLEM

Symbol	Meaning
X_i	The level of consumption consumer i on the peak hours.
Y_i	The level of consumption consumer i on the non-peak hours.
Z_i	The decision variable which have value 1 if consumers chosen to join the program and have value 0 if consumers didn't.
\bar{X}_t	The maximum level of consumption consumer i on the peak hours.
\bar{Y}_t	The maximum level of consumption consumer i on the non-peak hours.

Table VI describes the decision variables use for provider's optimization problem. Each variable is designed to fit into the model.

TABLE VI. DECISION VARIABLE FOR PROVIDER'S OPTIMIZATION PROBLEM

Symbol	Meaning
P	The cost required to join the service program.
P_x	The price of the service specified by ISP during peak hours.
P_y	The price of the service specified by ISP during non-peak hours.

D. Digilib Traffic Data

In this section we will explain about provider optimization with three pricing scheme are flat fee, usage based and two-part tariff which applied to digilib traffic data for homogeneous consumers and heterogeneous consumers. Digilib traffic data is the traffic consumer bandwidth usage on digilib applications. The parameters value usage for will to show in the table VII-IX.

TABLE VII. PARAMETER VALUES FOR HOMOGENEOUS CONSUMERS FOR DIGILIB TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
A	5	5	5
B	4	4	4
\bar{X}	17.80	17.80	17.80
\bar{Y}	34.88	34.88	34.88

By substituting the parameter values in Table VII, it can be modeled on the homogeneous consumer based on Cobb-Douglas utility function Eq (3) with flat fee pricing scheme equation as shown by the Eq (4):

$$\text{Max } Px X + Py Y + PZ + (X + Y)C - (X^5 Y^4) \quad (4)$$

Subject to:

$$X \leq 17.80 Z \quad (5)$$

$$Y \leq 34.88 Z \quad (6)$$

$$Z = 1 \quad (7)$$

TABLE VIII. PARAMETER VALUES FOR HIGH END AND LOW END HETEROGENEOUS CONSUMERS FOR DIGILIB TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
a_1	5	5	5
a_2	4	4	4
b_1	3	3	3
b_2	2	2	2
\bar{X}_1	17.80	17.80	17.80
\bar{X}_2	13.45	13.45	13.45
\bar{Y}_1	34.88	34.88	34.88
\bar{Y}_2	27.50	27.50	27.50

By substituting the parameters value in Table VIII, flat fee pricing scheme model for High end and Low end Heterogeneous consumers shown at Eq.(7).

$$\begin{aligned} & \text{Max } Px X_1 + Py Y_1 + Px X_2 + Py Y_2 + PZ + (X_1 + Y_1)C \\ & \quad + (X_2 + Y_2)C - (X_1^5 Y_1^3) \\ & \quad + (X_2^4 Y_2^2) \end{aligned} \quad (8)$$

Subject to

$$X_1 \leq 17.80 Z \quad (9)$$

$$Y_1 \leq 34.88 Z \quad (10)$$

$$X_2 \leq 13.45 Z \quad (11)$$

$$Y_2 \leq 27.50 Z \quad (12)$$

$$Z = 1 \quad (13)$$

TABLE IX. PARAMETER VALUES FOR HIGH DEMAND AND LOW DEMAND HETEROGENEOUS CONSUMERS OF DIGILIB TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
a_1	5	5	5
a_2	5	5	5
b_1	4	4	4
b_2	4	4	4
\bar{X}_1	17.80	17.80	17.80
\bar{X}_2	13.45	13.45	13.45
\bar{Y}_1	34.88	34.88	34.88
\bar{Y}_2	27.50	27.50	27.50

Based on the Table IX it can be modelled as flat fee pricing scheme model for high demand and low demand heterogeneous consumers based on Cobb-Douglas utility function shown Eq. (11) as following.

$$\begin{aligned} \text{Max } & PxX_1 + PyY_1 + PxX_2 + PyY_2 + PZ + (X_1 + Y_1)C \\ & + (X_2 + Y_2)C - (X_1^5 Y_1^4) \\ & +(X_2^5 Y_2^4) \end{aligned} \quad (14)$$

Subject to :

$$X \leq 17.80 Z \quad (15)$$

$$Y \leq 34.88 Z \quad (16)$$

$$X_2 \leq 13.45 Z \quad (17)$$

$$Y_2 \leq 27.50 Z \quad (18)$$

$$Z = 1 \quad (19)$$

E. Mail Traffic Data

In this section we will to explain about optimization of provider with three pricing scheme which applied to Mail traffic data for homogeneous consumers and heterogeneous consumers. Mail traffic data is the traffic consumer bandwidth usage on mail service applications. The parameter value usage is shown in Table X-XII.

TABLE X. PARAMETER VALUES FOR HOMOGENEOUS CONSUMERS FOR MAIL TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
a	5	5	5
b	4	4	4
\bar{X}	3,601.25	3,601.25	3,601.25
\bar{Y}	2,913.65	2,913.65	2,913.65

By substituting the parameters value in Table X, it can be modelled on the homogeneous consumer based on the (3) equation of Cobb-Douglas utility function as shown by the Eq. (19):

$$\begin{aligned} \text{Max } & Px X + Py Y + PZ + (X + Y)C - \\ & (X^5 Y^4) \end{aligned} \quad (19)$$

Subject to:

$$X \leq 3,601.25 Z \quad (20)$$

$$Y \leq 2,913.65 Z \quad (21)$$

$$Z = 1 \quad (22)$$

TABLE XI PARAMETER VALUES FOR HIGH END AND LOW END HETEROGENEOUS CONSUMERS OF MAIL TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
a_1	5	5	5
a_2	4	4	4
b_1	3	3	3
b_2	2	2	2
\bar{X}_1	3,601.25	3,601.25	3,601.25
\bar{X}_2	1,407.48	1,407.48	1,407.48
\bar{Y}_1	2,913.65	2,913.65	2,913.65
\bar{Y}_2	2,652.39	2,652.39	2,652.39

By substituting the parameters value in Table XI, Model with flat fee pricing scheme for High end and Low Heterogeneous consumers is shown in Eq. (23)

$$\begin{aligned} \text{Max } & PxX_1 + PyY_1 + PxX_2 + PyY_2 + PZ + (X_1 + Y_1)C \\ & + (X_2 + Y_2)C - (X_1^5 Y_1^3) \\ & +(X_2^4 Y_2^2) \end{aligned} \quad (23)$$

Subject to :

$$X_1 \leq 3,601.25 Z \quad (24)$$

$$Y_1 \leq 2,913.65 Z \quad (25)$$

$$X_2 \leq 1,407.48 Z \quad (26)$$

$$Y_2 \leq 2,652.39 Z \quad (27)$$

$$Z = 1 \quad (28)$$

TABLE XII. PARAMETER VALUES FOR HIGH DEMAND AND LOW DEMAND HETEROGENEOUS CONSUMERS FOR MAIL TRAFFIC

Parameters	Pricing Scheme		
	Flat Fee	Usage Based	Two Part Tariff
a_1	5	5	5
a_2	5	5	5
b_1	4	4	4
b_2	4	4	4
\bar{X}_1	3,601.25	3,601.25	3,601.25
\bar{X}_2	1,407.48	1,407.48	1,407.48
\bar{Y}_1	2,913.65	2,913.65	2,913.65
\bar{Y}_2	2,652.39	2,652.39	2,652.39

Based on the Table XII it can be modelled with flat fee pricing scheme for heterogeneous consumers' high demand and low demand like stated in Eq. (29):

$$\begin{aligned} Maks = & PxX_1 + PyY_1 + PxX_2 + PyY_2 + PZ + (X_1 + Y_1)C \\ & + (X_2 + Y_2)C - (X_1^5 Y_1^4) \\ & +(X_2^5 Y_2^4) \end{aligned} \quad (29)$$

Subject to :

$$X_1 \leq 3,601.25 Z \quad (30)$$

$$Y_1 \leq 2,913.65 Z \quad (31)$$

$$X_2 \leq 1,407.48 Z \quad (32)$$

$$Y_2 \leq 2,652.39 Z \quad (33)$$

$$Z = 1 \quad (34)$$

Table XIII-XV show the result by LINGO 13.0 of three optimal pricing schemes for each homogeneous consumer and heterogeneous consumer on the digilib and mail traffic based on Cobb-Douglas utility function with monitoring cost and marginal cost.

TABLE XIII. LINGO 13.0 SOLUTION OF DIGILIB TRAFFIC FOR HOMOGENEOUS CONSUMER

Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Model class	NLP	NLP	NLP
State	Local optimal	Local optimal	Local optimal
Objective	359.07	1,047.48	1,057.94
Infeasibility	0	0	0
Iterations	19	24	24
Extended Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Update Interval	2	2	2
GMU(K)	20	21	21

Table XIII shown the solution of LINGO 13.0 of three optimal pricing schemes for homogeneous consumer on digilib traffic data based on Cobb-Douglas utility function. From the Table it can be seen that the two-part tariff pricing scheme it the most optimal with an objective value is 1,057.94.

TABLE XIV. LINGO 13.0 Solution FOR HIGH END AND LOW END HETEROGENEOUS CONSUMERS OF DIGILIB TRAFFIC

Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Model class	NLP	NLP	NLP
State	Local optimal	Local optimal	Local optimal
Objective	635.58	1,327.96	1,337.96
Infeasibility	0	0	0
Iterations	25	36	35
Extended Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Update Interval	2	2	2
GMU(K)	22	22	21

Table XIV shown the solution of LINGO 13.0 of three optimal pricing schemes for heterogeneous consumer on digilib traffic data based on Cobb-Douglas utility function. From the Table it can be seen that the two-part tariff pricing scheme it the most optimal compared to flat fee and usage based with an objective value is 1,337.96. If the classification is based on the heterogeneous consumers on willingness to pay in digilib traffic, then Table XIV explains the higher benefit is obtained again in two part tariff pricing scheme.

TABLE XV. LINGO 13.0 SOLUTION OF MAIL TRAFFIC FOR HOMOGENEOUS CONSUMER

Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Model class	NLP	NLP	NLP
State	Local optimal	Local optimal	Local optimal
Objective	29,146.5	87,409.5	87,419.5
Infeasibility	0	0	0
Iterations	21	31	31
Extended Solver Status	Pricing Scheme		
	Flat Fee	Usage Based	Two part tariff
Update Interval	2	2	2
GMU(K)	20	21	21

Table XV shown the solution by LINGO 13.0 of three optimal pricing schemes for homogeneous consumer on mail traffic data based on Cobb-Douglas utility function. From the Table XV can be concluded that the two-part tariff pricing scheme it the most optimal among the three pricing scheme with an objective value is 87,419.5. By applying two part tariff scheme, again the provider gain more benefit compared to others.

Table XVI explain the recapitulation of result conducting by using LINGO 13.0, as we can see that for highest revenue obtained by the provider if they offer two part tariff pricing scheme, for high end and low end user for each traffic data in network.

TABLE XVI. RECAPITULATION OF DIGILIB TRAFFIC DATA

Data	Type of Pricing	Consumers		Income
		Homogeneous	Heterogeneous High end & Low end	
Digilib	flat-fee			
	Usage based	Homogeneous	1,047.47	
		Heterogeneous High end & Low end	1,327.96	3,699.20
	Two-part Tariff	Heterogeneous High demand & Low demand	1,323.76	
		Homogeneous	1,057.94	
		Heterogeneous High end & Low end	1,337.96	3,729.67
		Heterogeneous High demand & Low demand	1,333.66	

From Table XVI the maximum solution is obtained by applying the two part tariff scheme for heterogeneous high and low consumers due to the variety of willingness to pay of the consumers in applying the network. The two part tariff scheme can be considered a best option for provider to be promoted due to the subscription fee and usage based scheme that allow provider to maintain its network

TABLE XVII. RECAPITULATION OF MAIL TRAFFIC DATA

Data	Type of Pricing	Consumers		Income
Mail	flat-fee	Homogeneous	29,146.5	
		Heterogeneous High end & Low end	55,670.48	140,487.39
		Heterogeneous High demand & Low demand	55,670.41	
	Usage based	Homogeneous	87,409.52	
		Heterogeneous High end & Low end	113,933.7	315,276.62
		Heterogeneous High demand & Low demand	113,933.4	
		Homogeneous	87,419.52	
	Two-part Tariff	Heterogeneous High end & Low end	113,943.7	315,306.62
		Heterogeneous High demand & Low demand	113,943.4	

From Table XVII the maximum solution is obtained by applying the two part tariff scheme for heterogeneous high and low consumers due to the variety of willingness to pay of the consumers in applying the network.

IV. CONCLUSION

Based on the optimization result of the internet pricing scheme by considering marginal and monitoring cost of *Cobb Douglass* utility function, the optimal solution is obtained using the either *usage-based* pricing scheme model or *two-part tariff* pricing scheme model for each services offered, if we compared with *flat-fee* pricing scheme. It is the best way for provider to offer network based on two part tariff scheme.

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